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LIGHT PSEUDOSCALAR AND SCALAR PARTICLES IN QUARKONIUM RADIATIVE DECAYS: QCD SUM RULES ESTIMATES

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ԹԵԹԵՎ ՓՍԵՎԴՈՍԿԱԼՅԱՐ ԵՎ ՍԿԱԼՅԱՐ ՄԱՍՆԻԿՆԵՐԸ ՔՎԱՐԿՈՆԻՈՒՄԻ ՌԱԴԻԱՑԻՈՆ ՏՐՈՀՈՒՄՆԵՐՈՒՄ. ՔՔԴ ԳՈՒՄԱՐՆԵՐԻ ԿԱՆՈՆՆԵՐԻՑ ԲԽՈՂ ԳՆԱՀԱՏՈՒՄՆԵՐԸ:

ՔՔԴ գումարների կանոններից $O(\alpha_s)$ ճշտությամբ ստացված են $J/\psi\to P_\gamma$, S_γ և $\Upsilon\to P_\gamma, S_\gamma$ ռադիացիոն տրոհումների լայնքերի գնահատումները՝ մոդելից անկախ, որտեղ P-ն S-ը ակսիոնի /Հիգսի բոզոնի/ տիպի թեթև փսեվդոսկալյար /սկալյար/ մասնիկ է: ՔՔԴ կանխագուշակումների տարբերությունը Վիլչեկի ոչ ռելյատիվիստական բանաձևից այդ լայնքերի համար չնչին է:

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Л.С.ДУЛЬЯН,А.Ю.ХОДЖАМИРЯН

ЛЕГКИЕ ПСЕВДОСКАЛЯРНЫЕ И СКАЛЯРНЫЕ ЧАСТИЦЫ В РАДИАЦИОННЫХ РАСПАДАХ КВАРКОНИЯ: ОЦЕНКИ ИЗ ПРАВИЛ СУММ КХД

Из правил сумм КХД с точностью $O(\alpha_s)$ получены модельно-независимые оценки ширин радиационных распадов $J/\psi \to P\gamma, S\gamma$ и $\Upsilon \to P\gamma, S\gamma$, где $P(S)$ - легкая псевдоскалярная (скалярная) частица типа аксиона (хиггсовского бозона). Отклонение предсказаний КХД от нерелятивистской формулы Вильчека для этих ширин оказалось незначительным.

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LIGHT PSEUDOSCALAR AND SCALAR PARTICLES IN QUARKONIUM RADIATIVE DECAYS: QCD SUM RULES ESTIMATES

From QCD sum rules the model-independent estimates of widths of radiative decays $J/\psi \to P\gamma, S\gamma$ and $\Upsilon \to P\gamma, S\gamma$ are obtained with $O(\alpha_s)$ accuracy, where $P(S)$ is a light pseudoscalar (scalar) particle of the axion (Higgs boson) type. Deviation of QCD predictions from the Wilczek nonrelativistic formula for these widths turned out to be inessential.

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1. Introduction

In many elementary particle models with spontaneous symmetry breaking new light pseudoscalar bosons (P) of the axion type or light scalar bosons (S) of Higgs type are predicted weakly interacting with matter. In most of these models the interaction of these particles with the quarks is proportional to the quark mass. Therefore one of the hopeful ways to discover new bosons is [1] the search for radiative decays of quarkonium $J/\psi \to P\gamma, S\gamma$ or $\Upsilon \to P\gamma, S\gamma$.

The simple formula for the widths of these decays suggested by Wilczek [1] has the same form for pseudoscalars and scalars:

$$\Gamma_W(V \to P(S)\gamma) = \frac{m_Q^2}{2\pi\alpha x_Q^2} \Gamma(V \to \mu^+\mu^-) \qquad (1)$$

at $m_{P(S)} \simeq 0 \ (\ll m_Q)$, where $V = J/\psi, \Upsilon, \ldots \ (Q = c, b, \ldots)$, x_Q is the model-dependent constant $\sim G_F^{-1/2}$.

Recall that this formula reflects a similarity of two different processes of nonrelativistic quarkonium annihilation. Indeed, both processes $V \to P(S)\gamma$ and $V \to \mu^+\mu^-$ proceed at small distances $\sim 1/m_Q$. Hence: 1) their diagrams can be calculated with free and nonrelativistic quark lines; 2) their amplitudes are proportional to the same quantity $R_S(0)$, the quarkonium S-wave function at the origin.

In connection with the fact that J/ψ and Υ radiative decays experiments have already achieved the level of the widths predicted by (1), recently the corrections to this formula were discussed: a) radiative $\sim \alpha_s$ corrections [2,3] , b) relativistic ones which are due to the fact that in real quarkonia $\langle v/c \rangle$ is not small [4] . In both cases of P- and S-particles each correction turns out to reduce the ratio $\Gamma(V \to P(S)\gamma)/\Gamma(V \to \mu^+\mu^-)$ by a factor from 0.8 to 0.5.

In this work we would like to call attention to the fact that the calculation of the radiative widths $J/\psi, \Upsilon \to P\gamma(S\gamma)$ in the limit $m_{P(S)} = 0$ (really at $m_{P(S)} \ll m_Q$) can be carried out in a purely relativistic way with an accuracy of $O(\alpha_s)$ and quite independent of the nonrelativistic approach to quarkonium. This is possible due to the QCD sum rules (SR) method [5] which has been proved to be successful in the calculation of masses, leptonic and radiative widths of the quarkonium levels (see, e.g. recent review [6]). Moreover, it turns out that SR for the considered decays need no separate derivation. They are obtained immediately as a byproduct of SR for radiative transitions in quarkonium [7-10] . The starting object for all these SR is a three-current correlator $\langle j\, j_\mu j_\nu \rangle$ of current $j_\nu = \bar{Q}\gamma_\nu Q$ generating the quarkonium vector levels with currents j_μ and $j = i\bar{Q}\gamma_5 Q$ $(\bar{Q}Q)$ corresponding to interaction of Q-quarks respectively with the photon and bosons $P(S)$.

The main observation the present work is based on consists in the fact that there exists a similarity between this correlator determining the $V \to P(S)\gamma$ decay amplitude and the two-current diagonal correlator $\langle j_\nu j_{\nu'} \rangle$ which determines the leptonic width $\Gamma(V \to \mu^+\mu^-)$. On the one hand, due to the QCD asymptotic freedom both correlators can be calculated in the region of current zero masses as a sum of a bare diagram and $O(\alpha_s)$ diagrams. On the other hand, the physical representations of both correlators are saturated by one and the same set of states: the resonances with the quantum numbers of J/ψ or Υ ($Q = c$ or b). Owing to the mentioned similarity, the ratio of the radiative decay widths to the leptonic width can be calculated from the QCD SR with accuracy up to $\sim 10\%$ if all main contributions to the correlators are known. Practically, we could achieve such an accuracy only for the decays to a pseudoscalar and photon. As to the decays to a scalar particle and the photon, the accuracy here is limited so far, since the perturbative α_s-corrections to the corresponding three-current correlator have not been calculated yet. This is a separate problem which represents considerable calculational difficulties and is out of scope of this work.

We present at once the list of calculated widths of the quarkonium radiative decays to the light bosons normalized to the corresponding Γ_W widths determined in (1) (the latters are simply leptonic widths with a coefficient $\sim m_Q^2 / x_Q^2$ depending on the light boson model):

$$\Gamma(J/\psi \to P\gamma) = (1.2 \div 1.35)\,\Gamma_W(J/\psi),$$
$$\Gamma(\Upsilon \to P\gamma) = (1.15 \div 1.2)\,\Gamma_W(\Upsilon), \tag{2a}$$

$$\Gamma(J/\psi \to S\gamma) = (0.8 \div 0.9)\,\Gamma_W(J/\psi),$$
$$\Gamma(\Upsilon \to S\gamma) = (0.85 \div 1.0)\,\Gamma_W(\Upsilon). \tag{2b}$$

We emphasize again that the results given in (2) are not corrections to the Wilczek formula (1) but represent completely independent estimates from the QCD.

The rest of this paper contains derivation of the SR and calculation of the resulting estimates (2). This procedure does not differ in essence for decays to pseudoscalars and scalars. Therefore, both types of J/ψ decays will be considered simultaneously in Section 2. In Section 3 we'll give the corresponding results for Υ decays.

2. The QCD Sum Rules for $J/\psi \to P\gamma, S\gamma$ Decays.

So, let us consider a three-current correlator

$$\Delta^{P(S)}_{\mu\nu}(\kappa,q) = \int dx\,dy\,\exp[-i(\kappa x - qy)]\langle 0|T\{j_{P(S)}(0)j_\mu(x)j_\nu(y)\}|0\rangle, \tag{3}$$

where $j_\mu = \bar{c}\gamma_\mu c$ is the electromagnetic c-quark current corresponding to the emission of the real photon ($\kappa^2 = 0$); $j_P = i\bar{c}\gamma_5 c$ ($j_S = \bar{c}c$) is the current of c-quark interaction with massless pseudoscalar (scalar) boson $P(S)$ $((q-\kappa)^2 = 0)$, $j_\nu = \bar{c}\gamma_\nu c$ is the current corresponding to creation of $J/\psi, \psi', \ldots$ -mesons (at $q^2 = m_\psi, m_{\psi'}, \ldots$).

In the region $q^2 \simeq 0$ (really at $q^2 \ll 4m_c^2$), where m_c is the c-quark current mass, the correlator (3) is a vacuum fluctuation at small ($\sim 1/m_c$) distances and can be calculated in QCD. Fig. 1a,b,c presents various types of QCD diagrams up to $O(\alpha_s)$. For SR we'll need a few first derivatives of (3) at $q^2 = 0$ point, precisely, the derivatives of invariant amplitudes $\Delta^{P,S}(q^2)$, containing the whole dynamical information on the correlator (3):

$$\Delta^P_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} \kappa_\alpha q_\beta \Delta^P(q^2), \qquad (4)$$
$$\Delta^S_{\mu\nu} = [\delta_{\mu\nu}(\kappa q) - \kappa_\nu q_\mu] \Delta^S(q^2).$$

In Refs.[7-10] the same correlator was considered in a more general case, $(q-\kappa)^2 \neq 0$, in connection with the calculation of $J/\psi \to \eta_c \gamma$ (j^P current) and $\chi_0 \to J/\psi \gamma$ (j^S current) radiative decay widths. Using the calculations made there we obtain the expansion of $\Delta^{P,S}(q^2)$ at $q^2 = 0$ up to $O(\alpha_s)$:

$$\frac{1}{n!}\frac{\partial^n}{\partial q^{2n}} \Delta^P(q^2)\Big|_{q^2=0} = \frac{3}{2\pi^2}\frac{(n!)^2}{m_c^{2n+1}(2n+2)!}\left\{1 + c_n^P \phi + \frac{\alpha_s}{\pi} d_n^P\right\}, \qquad (5a)$$

$$\frac{1}{n!}\frac{\partial^n}{\partial q^{2n}} \Delta^S(q^2)\Big|_{q^2=0} = \frac{3}{\pi^2}\frac{(n!)^2}{m_c^{2n+1}(2n+4)!}\left[(2n+3)(n+1)+1\right] \times \qquad (5b)$$
$$\times \left\{1 + c_n^S \phi + \frac{\alpha_s}{\pi} d_n^S\right\},$$

where $\phi = \frac{4\pi}{9} \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} |0\rangle (4m_c^2)^{-2}$ is the well-known gluon condensate density coefficient [5], so that $c_n^{P,S}\phi$ is a relative contribution of the Fig. 1b type diagrams corresponding to the c-quark interaction with the gluon condensate. According to the results of [8-10] :

$$c_n^P = -\frac{n(n+1)}{2n+3}\,[\,n^2+5n+6\,], \qquad (6a)$$

$$c_n^S = -\frac{(n+1)(n+2)}{2n+5}\left[(2n+3)(n+1)+1\right]^{-1}\left[2n^4+17n^3+53n^2+68n+24\right] \qquad (6b)$$

More complicated is the situation with the coefficients $d_n^{P,S}$ determining the contribution of the two-loop $O(\alpha_s)$ diagrams with gluon exchanges (of type shown in Fig. 1c). The only information we have are the first four coefficients: $d_1^P = 3.92$, $d_2^P = 2.76$, $d_3^P = 2.04$ and $d_4^P = 1.33$, which were calculated in [9] along with other coefficients determining the perturbative $O(\alpha_s)$ correction to the SR moments for $J/\psi \to \eta_c \gamma$ decay. In order to estimate d_n^P at $n > 4$, we use the following trick. Let us represent the $O(\alpha_s)$ diagram contribution (Fig.1c plus all similar diagrams), which we denote by $\Delta_1^P(q^2)$, in the form of a dispersion integral:

$$\frac{1}{n!}\frac{\partial^n}{\partial q^{2n}}\Delta_1^P(q^2)\Big|_{q^2=0} = \frac{1}{\pi}\int_{4m_c^2}^{\infty}\frac{Im\,\Delta_1^P(s)}{s^{n+1}}\,ds \qquad (7)$$

Recall, that the bare triangle diagram of Fig.1a has an imaginary part:

$$Im\,\Delta_0^P(s) = \frac{3m_c}{2\pi s}\,\ln\frac{1+v}{1-v}\,\theta(s-4m_c^2), \qquad (8a)$$

$(v=\sqrt{1-4m_c^2/s})$. For completeness, we present the corresponding expression for the scalar current case:

$$\mathrm{Im}\,\Delta_0^S(s) = \frac{3m_c}{2\pi s}\left[(2-v^2)\ln\frac{1+v}{1-v} - 2v\right]\vartheta(s-4m_c^2). \quad (8b)$$

Written in terms of the c-quark velocity v, the integral (7) at large n contains a factor $(1-v^2)^n$, which damps the $\mathrm{Im}\,\Delta_1^P$ asymptotics at $v\to 1\ (s\to\infty)$. On the other hand, the dominant contribution to the imaginary part of the diagram $O(\alpha_s)$ at $v\to 0$ has a Coulomb-like form $\sim 1/v$ (for details see [6]), so in general

$$\mathrm{Im}\,\Delta_1^P(s) = \alpha_s\,\mathrm{Im}\,\Delta_0^P(s)\left[\frac{a}{v} + b + \dots\right]. \quad (9a)$$

Restricting ourselves to the first two terms of this expansion, we determine the coefficients a and b "empirically", substituting (9a) into (7) and equating at $n=3,4$ the integration result with the known values of $\frac{\alpha_s}{\pi}d_3^P$, $\frac{\alpha_s}{\pi}d_4^P$, respectively. Here we should take into account also the α_s-correction to the c-quark mass [5,6] . The latter mass in the dispersion representation (7) is implied to be normalized "on the mass shell", whereas during the explicit calculation of the Fig.1 QCD-diagrams the Euclidean mass $m_c(p^2=-m_c^2)$ is taken. Finally we obtain:

$$a \simeq 2.7 \qquad b \simeq -2.8 \quad (9b)$$

The comparison of (7) (wherein (9) is substituted) at $n=1,2$ with $\frac{\alpha_s}{\pi}d_1^P$, $\frac{\alpha_s}{\pi}d_2^P$ convinces us that the omitted in (9) unknown terms of expansion in positive powers of v are inessential. The difference is 30% and 8%, respectively. At $n>4$

this difference must substantially decrease. Thus, we can rather precisely estimate the coefficients d_n^P at $n > 4$ (up to those n values at which the subsequent term $\sim (\alpha_s / v)^2$ becomes essential). Unfortunately, for the triangle diagram with scalar current $O(\alpha_s)$ correction has not yet been calculated at all. As a rough estimation, below we shall simply assume $d_n^S \simeq d_n^P$ taking into account the fact that for two-current correlators $\langle j^S j^S \rangle$ and $\langle j^P j^P \rangle$ these corrections are close to each other [6] .

Using dispersion relation over q^2 and unitarity, we can easily obtain physical representation of the correlator (3) in the form:

$$\Delta_{\mu\nu}^{P(S)} = \sum_{\psi} \frac{\langle 0 | j^{P(S)} j_\mu | \psi \rangle \langle \psi | j_\nu | 0 \rangle}{q^2 - m_\psi^2} + \dots , \qquad (10)$$

where in a sum over all ψ -resonances, e.g. for J/ψ , the matrix element $\langle J/\psi | j_\nu | 0 \rangle \equiv g_\psi^{-1} m_\psi^2 \psi_\nu$ determines leptonic width:

$$\Gamma(J/\psi \to \mu^+ \mu^-) = \frac{4\pi \alpha^2 Q_c^2}{3} g_\psi^2 m_\psi \qquad (11)$$

(ψ_ν is the J/ψ polarization vector), while the matrix elements

$$\langle 0 | j^P j_\mu | J/\psi \rangle \equiv \varepsilon_{\mu\nu\alpha\beta} k_\alpha q_\beta \psi_\nu m_\psi^{-1} A(J/\psi \to P\gamma) , \qquad (12)$$

$$\langle 0 | j^S j_\mu | J/\psi \rangle \equiv [\delta_{\mu\nu} (kq) - k_\nu q_\mu] \psi_\nu m_\psi^{-1} A(J/\psi \to S\gamma)$$

determine the radiative widths we are interested in

$$\Gamma(J/\psi \to P(S)\gamma) = \left(\frac{m_c}{x_c}\right)^2 \frac{\alpha Q_c^2 m_\psi}{24} \left| A(J/\psi \to P(S)\gamma)\right|^2 \qquad (13)$$

in the limit $m_{P,S} = 0$. The continuum contribution $(D\bar{D},\dots)$ in (10), denoted by dots we replace as usual by dispersion integral taken from the imaginary part (8) with respect to the α_s -correction (9) over the region $s > s_0$, where $s_0 > 4m_D^2$.

Differentiating (10) at $q^2 = 0$ and equating the obtained expression

$$\frac{1}{n!}\frac{\partial^n}{\partial q^{2n}} \Delta^{P(S)}(q^2)\Big|_{q^2=0} = \frac{g_\psi A(J/\psi \to P(S)\gamma)}{m_\psi^{2n+1}} \Big\{ 1 + \sum_{\psi^i} \left(\frac{g_{\psi^i}}{g_\psi}\right) \times \frac{A(\psi^i \to P(S)\gamma)}{A(J/\psi \to P(S)\gamma)} \left(\frac{m_\psi}{m_{\psi^i}}\right)^{2n+1} + \dots \Big\} \qquad (14)$$

with the r.h.s. of (5a) ((5b)), we come to the set of SR moments. In their physical part at large n the J/ψ contribution dominates. Being equipped with experimental data [11], in the sum over highest ψ -resonances we take into account five levels: $\psi^i = \{\psi(3685),\ \psi(3772),\ \psi(4030),\ \psi(4160),\ \psi(4415)\}$ with the corresponding constants g_{ψ^i} derived from leptonic widths according to (11). Further, it is natural to assume

$$\frac{A(\psi^i \to P(S)\gamma)}{A(J/\psi \to P(S)\gamma)} \simeq \frac{g_{\psi^i}}{g_\psi} . \qquad (15)$$

This relation is valid in any nonrelativistic quarkonium model because, as mentioned above, both sides of it are proportional to the same ratio $R_S^i(0)/R_S(0)$. At the same time it is obvious that gluon or relativistic corrections will not change (15) significantly. Finally, since we use sufficiently high moments of SR in order to derive the amplitude $A(J/\psi \to P(S)\gamma)$, the accuracy of (15) is inessential.

Compare now the obtained SR to the well-known SR for the two-current correlator $\Pi_{\mu\nu} = \langle j_\mu j_\nu \rangle$ [5,6] (the diagrams of Fig. 1d,e,f):

$$g_\psi^2 \left\{ 1 + \sum_{\psi^i} \left(\frac{g_{\psi^i}^2}{g_\psi^2}\right)\left(\frac{m_\psi}{m_{\psi^i}}\right)^{2n+2} + \cdots \right\} = \tag{16}$$

$$= \frac{3}{2\pi^2}\left(\frac{m_\psi}{m_c}\right)^{2n+2} \frac{(n+2)!\, n!\,(n+2)}{(2n+5)!} \left[1 + b_n \phi + \frac{\alpha_s}{\pi} a_n \right],$$

which, according to (11), determine the leptonic width $\Gamma(J/\psi \to \mu^+\mu^-)$. Eq.(16) corresponds to the initial choice of the correlator:

$$\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - q^2 \delta_{\mu\nu}) q^2\, \Pi(q^2).$$

The difference from the standard choice [5,6] is by one power q^2 or in terms of moments $n \to n+1$, so that the imaginary part in $O(\alpha_s)$ [5] is

$$\mathrm{Im}\, \Pi(s) = \frac{3(3-v^2)v}{2s} \left\{ 1 + \frac{2\pi\alpha_s}{v} - \frac{3+v}{3}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\alpha_s \right\}. \tag{17}$$

The coefficients b_n calculated in [5] determine the gluon condensate contribution (diagrams of Fig.1e type). Dots in l.h.s. of (16) denote the continuum contribution (the integral from (17) with the weight $1/s^{n+1}$ over the region $s > s_0$).

The advantage of QCD SR method is that from (16) as well as from similar SR for other two-current $c\bar{c}$ -correlators all QCD parameters of interest are unambiguously fixed [5,6] :
$m_c = 1.26 \pm 0.02\,\text{GeV}$; $\phi = (1.7 \pm 0.3) \cdot 10^{-3}$, $\alpha_s(m_c) = 0.2 \pm 0.05$. Moreover, if we account for five higher ψ -resonances in the l.h.s. of (16) and choose $\sqrt{s_0} = 4.4$ GeV, we obtain at $1 \leqslant n \leqslant 4$ rather stable prediction for leptonic width:

$$\Gamma(J/\psi \to \mu^+\mu^-) = 4.5 \div 4.6 \text{ keV}. \qquad (18)$$

This result is to be compared with experimental number 4.7 ± 0.3 keV. At $n > 5$ the ϕb_n power correction becomes $> 30\%$, so that SR naturally do not work. The estimated uncertainty of (18) does not exceed 20% and consists of QCD parameter uncertainty as well as of experimental errors of higher ψ -resonance leptonic widths.

Now when we have fixed all parameters, let us divide SR obtained above (i.e. the equality of l.h.s. of (14) and (5)) by SR (16).

Resulting set of combined SR moments is the following:

$$\frac{A(J/\psi \to P(S)\gamma)}{g_\psi} = \frac{2m_c}{m_\psi} R_n^{0P(S)} R_n^{[\psi^i]} \times$$
$$\times \left\{ 1 - (\delta_n^{P(S)} - \tilde{\delta}_n) + (c_n^{P(S)} - b_n)\phi + (d_n^{P(S)} - a_n)\frac{\alpha_s}{\pi} \right\}, \qquad (19)$$

where

$$R_n^{oP} = (2n+3)(2n+5)/[(n+1)(n+2)],$$

$$R_n^{oS} = (2n+5)[(2n+3)(n+1)+1]/[(n+1)(n+2)^2].$$

Taking into account (15), the factor determining higher resonance contribution is

$$R_n^{[\psi^i]} = \frac{1+\sum_{\psi^i}\left(\frac{g_{\psi^i}}{g_\psi}\right)^2\left(\frac{m_\psi}{m_{\psi^i}}\right)^{2n+2}}{1+\sum_{\psi^i}\left(\frac{g_{\psi^i}}{g_\psi}\right)^2\left(\frac{m_\psi}{m_{\psi^i}}\right)^{2n+1}}.$$

$\delta_n^{P(S)}$, $\tilde{\delta}_n$ are the continuum contributions into n-th moment of three-current and two-current SR, respectively.

The obtained SR (19) contain minimal power of m_c - the parameter which is known with limited accuracy. Besides that, due to (15) and to the fact that both triangle and diangle diagrams (the last in option (17)) have the same behavior at $v \to 1$, $(s \to \infty)$ the contribution of higher states in (19) (ψ'-resonances plus continuum) turned out to be $< 10\%$ already at $n \geqslant 2$. Finally in (19) substantial cancellation of power and perturbative $O(\alpha_s)$ corrections to three- and two-current correlators occurs. This circumstance somewhat enlarges the region of reliable moments over n (at $n = 8$ both corrections are still $< 20\%$) as compared with SR (16).

Predictions for $A(J/\psi \to P(S)\gamma)/g_\psi$ ratio following from SR (19) are rather stable. At whole $1 \leqslant n \leqslant 8$ interval of moments the variation of this ratio is $\leqslant 10\%$. Nevertheless, in order to avoid uncertainties due to higher states contamination (small n) and α_s-corrections (too high n), we limit ourselves by $3 \leqslant n \leqslant 6$ interval. In this interval variation of SR predictions is $\leqslant 5\%$.

According to (11) and (13), the ratio $A(J/\psi \to P(S)\gamma)/g_\psi$ obtained in this way determines the J/ψ radiative decay widths in the units of $\Gamma(J/\psi \to \mu^+\mu^-)$ leptonic width:

$$\Gamma(J/\psi \to P(S)\gamma) = \left(\frac{m_c}{x_c}\right)^2 \frac{1}{32\pi\alpha} \left|\frac{A}{g}\right|^2 \Gamma(J/\psi \to \mu^+\mu^-) = \frac{1}{16}\left|\frac{A}{g}\right|^2 \Gamma_W \quad (20)$$

(the Wilczek formula analog in our approach). Substituting into (20) the values of A/g obtained from optimal SR moments $3 \leqslant n \leqslant 6$ we at last come to predictions for $J/\psi \to P(S)\gamma$ radiative decays presented above in (2).

Let us discuss now how one can obtain the analogous estimates for b-quarkonium decays.

3. Estimates for Υ radiative decay widths.

Formally, the derivation of SR (19) carried out above may be repeated for b-quark correlators without substantial changes. It is necessary only to substitute everywhere $c \to b$; $m_c \to m_b$, $J/\psi \to \Upsilon$, $\psi^i \to \Upsilon^i$. However, as it is well known, in reality SR for b-quarkonium work not so well as for charmonium. The $b\bar{b}$-system levels are placed closer in the mass scale, so that at $n \leqslant 10$ it is difficult to isolate reliably the lowest Υ state contribution. Therefore here more than in $c\bar{c}$-case one must trust in the duality of higher states and bare quark loop. On the other hand, the power corrections for not too high moments are negligible being $\sim (m_c/m_b)^4$ of the corresponding charmonium corrections. The main corrections are perturbative ones, their calculated values are reliable in $O(\alpha_s)$ up to $n \leqslant 10$.

At the same time in the considered problem the situation is substantially improved for low and intermediate moments, since a rich experimental information [11] about b-quarkonium vector levels is available. We can in fact take into account five levels, sited higher than Υ. Besides that, $b\bar{b}$-system is more nonrelativistic than $c\bar{c}$-one. Therefore the status of both approximation (9) and relation (15) here is better.

First, let us consider the b-quark analog of SR (16). As it was shown in [12] (see also [6]), from these SR one can extract b-quark mass $m_b(p^2=-m_b^2) = 4.23 \pm 0.05$ GeV. At the same time $\alpha_s(m_c) = 0.2$ corresponds to $\alpha_s(m_b) = 0.15$.

We have repeated the analysis of these SR with account of current experimental information on Υ-resonances. If we take $m_b = 4.21$ GeV, $\sqrt{s_0} = 11.3$ GeV, $\alpha_s = 0.15$, then a very stable value of $\Gamma(\Upsilon \to \mu^+\mu^-) = 1.15 \div 1.2$ keV is obtained in the whole $1 \leqslant n \leqslant 9$ interval of moments. Substituting m_b, $\alpha_s(m_b)$ into combined SR for $A(\Upsilon \to P(S)\gamma)/g_\Upsilon$ ratio (analogous to (19)), we also obtain a stable set of moments with strong reciprocal cancellation of α_s-corrections and higher state contributions. In $4 \leqslant n \leqslant 8$ interval the $(A/g)_\Upsilon$ values following from SR vary in the limits of 2.5% (in $P\gamma$ case) and 5% (in $S\gamma$ case). Substituting these values into corresponding formulae for widths analogous to (20), we arrive at estimates presented in (2) for $\Upsilon \to P\gamma$, $S\gamma$ decays.

Let us note that in the SR moments we have used for $(A/g)_{\Upsilon}$ ratios the contribution of higher states is $< 8\%$ and α_s-correction is $< 15\%$. Recall, that these small values come from the above mentioned cancellation of corresponding contributions into three- and two-current SR. The separate contributions are of course somewhat higher than in J/ψ case.

Thus we come to the conclusion that for radiative Υ decays into light bosons SR seem to be as powerful as for J/ψ decays.

4. Conclusion.

We emphasize that notwithstanding that QCD parameters are known with limited accuracy, the SR moments used above are insensitive to their values due to minimal power of quark mass and successful cancellation of α_s-corrections. The estimated accuracy of predictions presented in (2) is $\sim 10\%$ for J/ψ, $\Upsilon \to P\gamma$ decays and $\sim 20\%$ for J/ψ, $\Upsilon \to S\gamma$ ones. One can be convinced that even $\sim 50\%$ violation of (15) cannot cause variation of these results more than by 10% for J/ψ and 20% for Υ. At the same time such violation is nonrealistic since it destroys stability of SR at lower moments.

Note, that results obtained above demonstrate that from the QCD point of view deviation from nonrelativistic Wilczek formula (1) cannot be substantial. At the same time the J/ψ, $\Upsilon \to P\gamma$ widths are predicted to be somewhat higher than (1), while J/ψ, $\Upsilon \to S\gamma$ must be lower than (1). It can be easily checked that this difference arises already at the level of bare triangle diagrams with pseudoscalar and scalar vertices. The corrections are close to each other and don't remove this relation.

Further, from predictions listed in (2) it follows that $b\bar{b}$ -system is really more nonrelativistic than charmonium, since Υ widths are closer to Γ_W than the corresponding J/ψ widths.

We shall not discuss in details current experimental light boson limits from monochromatic photon searches in J/ψ , Υ decays. Note only that α_s - and relativistic corrections to (1) calculated in [2-4] still leave some room for axion and/or light Higgs existence. According to our results (2), the experimental limits already forbid this possibility.

The problem of discrepancy between radiative width calculations by means of nonrelativism plus corrections and by means of SR remains open, since we yet don't see direct relation between these two approaches. The SR method advocated here has in fact controllable accuracy. Besides that, in this way we may calculate not only the ratio $\Gamma(V \to P(S)\gamma)/\Gamma(V \to \mu^+\mu^-)$ but also leptonic widths themselves, i.e. the absolute normalization of (2). At the same time the nonrelativistic calculation of leptonic widths along with uncertainty of wave function at the origin contains rather uncontrollable value of α_s - correction.

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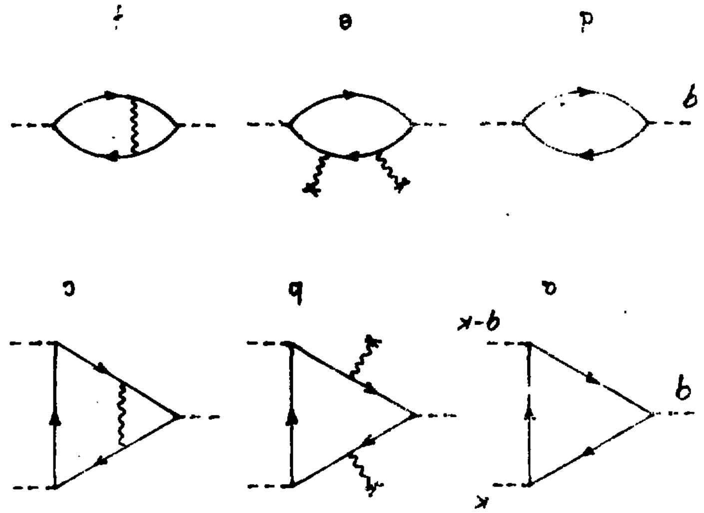

Fig.1. a - diagram of three-current correlator in zeroth order of α_s ; vertices with momenta q , κ , $q-\kappa$ correspond respectively to j_ν , j_μ , j currents; b - one of the gluon condensate diagrams; c - one of the $O(\alpha_s)$ perturbative correction diagrams; d,e,f - corresponding diagrams for two-current correlator.

REFERENCES

1. Wilczek F. Decay of heavy vector mesons into Higgs particles. - Phys.Rev.Lett., 1977, v.39, p.1304-1306.
2. Poljubin I.V., Vysotsky M.I. $V\to a\gamma$ decay rate to order $O(\alpha_s)$. - Preprint ITEP-86-186, Moscow, 1986.
3. Nason P. QCD radiative corrections to upsilon decay into scalar plus gamma and pseudoscalar plus gamma. - Phys. Lett.B, 1986, v.175, p.223-226.
4. Азнаурян И.Г.,Григорян С.Г.,Матинян С.Г. Релятивистские эффекты в распаде $V\to H^0\gamma$. Письма в ЖЭТФ, I986, т.43, с.499-50I.
5. Shifman M.A., Vainshtein A.I., Zakharov V.I. QCD and resonance physics. - Nucl.Phys.B, 1979, v.147, p.385-448.
6. Reinders L.J., Rubinstein H., Yazaki S. Hadron properties from QCD sum rules. - Phys.Reports, 1985, v.127, p.1-81.
7. Khodjamirian A.Yu. Dispersion sum rules for radiative transitions in quarkonium. - Phys.Lett.B, 1980, v.90, p.460-464.
8. Ходжамирян А.Ю. О вычислении ширин распада $J/\psi\to\eta_c\gamma$ в КХД. ЯФ, I984, т.39, с.970-976.
9. Beilin V.A., Radjushkin A.V. QCD sum rules and $J/\psi\to\eta_c\gamma$ decay. - Nucl.Phys.B, 1985, v.260, p.61-78.
10. Дульян Л.С.,Оганесян А.Г.,Ходжамирян А.Ю. Радиационные распады P - уровней чармония в КХД. ЯФ, I986, т.44,с.746 - 755.
11. Particle Data Group. Review of particle properties. - Phys.Lett.B, 1986, v.170, p.1-350.

12. Григорян С.С. Дисперсионные правила сумм и бьютоний. ЯФ, 1979, т.30, с.1407-1411.

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В РАДИАЦИОННЫХ РАСПАДАХ КВАРКОНИЯ : ОЦЕНКИ ИЗ ПРАВИЛ СУММ КХД

(на английском языке, перевод Асланян З.Н.)

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