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MASS CORRECTIONS TO GREEN FUNCTIONS IN INSTANTON VACUUM MODEL

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ԳՐԻՆԻ ՖՈՒՆԿՑԻԱՆԵՐԻՆ ՎԵՐԱԲԵՐՈՂ ԶԱՆԳՎԱԾՆԵՐԻ ՈՒՂՂՈՒՄՆԵՐ ԻՆՍՏԱՆՏՈՆԱՅԻՆ ՎԱԿՈՒՈՒՄԻ ՄՈԴԵԼԻ ՄԵՋ

Գրինի արդյունարար ֆունկցիայում հաշվարկվել են զանգվածների առաջին չանհայտացող ուղղումները ինստանտոնային -անտիինստանտոնային ֆլուկտուացիաների համադրումից բաղկացած ինստանտոնային վակուումի մոդելում: Այդ ուղղումների հաշվառմամբ հաշվվել են մեզոնային հոսանքների հարաբերակցությունները, գտնվել են պսևդոսկալյար օքտետի զանգվածային լուսակը և f_K կաոնային աքսիալ հաստատունի մեծությունը:

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MASS CORRECTIONS TO GREEN FUNCTIONS IN INSTANTON VACUUM MODEL

The *first nonvanishing* mass corrections to the effective Green functions are calculated in the model of instanton-based vacuum consisting of a superposition of instanton-anti-instanton fluctuations. The meson current correlators are calculated with account of these corrections; the mass spectrum of pseudo-scalar octet as well as the value of the kaon axial constant f_K are found.

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МАССОВЫЕ ПОПРАВКИ К ФУНКЦИЯМ ГРИНА В МОДЕЛИ ИНСТАНТОННОГО ВАКУУМА

В модели инстантонного вакуума, состоящего из суперпозиции инстантон-антиинстантонных флюктуаций, вычислены первые неисчезающие массовые поправки к эффективным функциям Грина. Вычислены корреляторы мезонных токов с учетом этих поправок, найдены спектр масс псевдоскалярного октета и величина каонной аксиальной константы f_K .

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1. Introduction

Refs.[1,2] have shown that the instanton fluctuations explicitly violate the QCD γ_5 invariance and may serve as a source of spontaneous breaking of chiral invariance (SVCI) of theory. However, as was shown in a number of works by Shifman et al. (see, e.g. [3]), it is impossible to take into account the large-scale ($>$ 1/GeV) instanton fluctuations by the quasi-classical approximation method. On the other hand, in Ref.[4] it was shown from the phenomenological analysis that the model of vacuum as instanton "fluid" allows to describe successfully many characteristics of the hadron physics. This model has been quantitatively realized by Dyakonov and Petrov [5]. They have shown that the instanton-anti-instanton configurations are dominant nonperturbative fluctuations in the vacuum wave function. The same authors have suggested in Refs.[6,7] a new mechanism of the SVCI based on the symmetry breaking interaction of fermions with external chaotic field as well as worked out an algorithm for calculating the correlation functions in instanton vacuum.

The appearance of massless pion pole in the polarization operator was demonstrated, the mass and axial constant f_π were calculated through the parameters of instanton medium being in perfect agreement with experimental data and results of the current algebra.

In this work, within the approach developed [6,7], we have calculated the first nonvanishing current-mass dependence of three light (u, d, s) quarks in the effective Green functions; the mass spectrum of pseudoscalar octet of mesons as well as the value of the K-meson axial constant f_K are found.

2. Quark Propagator

The general method for calculating the correlation functions in instanton medium proposed by Dyakonov and Petrov consists in the following. The instanton-anti-instanton superpositions must be considered as an external classical field; the correlators in the presence of this field will depend on the characteristics, i.e. dimension ρ, orientation U and centre z of all pseudoparticles. The averaging over statistical ensemble of instantons yields finally exact correlators in instanton vacuum. In this case the averaging is appreciably simplified owing to the following approximations:

1) The packing parameter of the instanton fluid $\bar{\rho}/R$, where R is the mean distance between pseudoparticles is small, $\bar{\rho}/R \approx 1/3$ [5], and therefore the pseudoparticles may be considered uncorrelated.

2) When the number of colors N_c is large, the instanton

distribution as a function of instanton size ρ is very narrow and tends to the δ -shaped function at $N_c \to \infty$, so that in the leading order over $1/N_c$ all sizes of instantons ρ_I may be replaced by their mean values $\bar{\rho}$ [5].

According to above-said, the quark propagator S in instanton-anti-instanton superpositions we'll express through the propagators in the background of individual instantons and expand in Taylor series:

$$S = S_0 + \sum_{n=1}^{\infty} \sum_{I_1 \dots I_n} C_{I_1 \dots I_n} (S_{I_1} - S_0) S_0^{-1} (S_{I_2} - S_0) S_0^{-1} \dots (S_{I_n} - S_0) \quad (1)$$

where S_0 is a propagator of free quark, and coefficients $C_{I_1 I_2 \dots I_n}$ are

$$C_{I_1 I_2 \dots I_n} = (1 - \delta_{I_1 I_2})(1 - \delta_{I_2 I_3}) \dots (1 - \delta_{I_{n-1} I_n}) \quad (2)$$

Isolate in $S_I - S_0$ zero mode contribution which is singular with respect to the quark mass

$$(S_I - S_0)(x,y) = -\frac{\psi_I(x) \ \psi_I^+(y)}{im} + \text{nonzero modes} \quad (3)$$

The contribution of nonzero modes has a finite limit at $m \to 0$ and for light quarks in the first nonvanishing mass approximation it can be neglected compared to the first singular term. Besides, the contribution of nonzero modes must be mostly "absorbed" by renormalization [3].

The substitution of the value $S_I - S_0$ to the propagator exact formula (1) gives

$$S = S_0 - \frac{1}{im} \sum_{n=1}^{\infty} \sum_{I_1 \dots I_n} C_{I_1 I_2 \dots I_n} \psi_{I_1} \frac{V_{I_1 I_2}}{im} \frac{V_{I_2 I_3}}{im} \dots \frac{V_{I_{n-1} I_n}}{im} \psi_{I_n}^+ \quad (4)$$

Here through V_{IJ} we denote the zero mode overlapping integlals:

$$V_{IJ} = \langle \Psi_I | i\partial\!\!\!/ + im | \Psi_J \rangle \qquad (5)$$

Due to helicity properties of zero modes the matrix element

$$T_{IJ} = \langle \Psi_I | i\partial\!\!\!/ | \Psi_J \rangle \qquad (6)$$

differs from zero only for different types of instantons, and the terms proportional to δ_{IJ} vanish at the first term in V_{IJ}, so the coefficients $C_{I_1 I_2 \dots I_n}$ for them must be replaced by unity.

Finally we determine the matrix K:

$$K_{IJ} = im \langle \Psi_I | \Psi_J \rangle (1 - \delta_{IJ}) \qquad (7)$$

Then for the total propagator S we have:

$$S = S_0 + \sum_{I,J} \Psi_I \left(\frac{1}{T + K - im} \right)_{IJ} \Psi_J^{+} \qquad (8)$$

We see that formula (8) differs from the analogous formula in Ref.[6] only by the presence of the term K which is proportional to the quark mass.

The averaging is quite analogous, only here along with instanton-anti-instanton contributions containing T_{IJ} there are present also instanton-instanton contributions among which K_{IJ} should be placed. We shall give here only the result.

Determine F_{IJ} and $\mathcal{D}_{IJ}$ functions.

$$\overline{\left(\frac{1}{T + K - im} \right)}_{IJ} = \begin{cases} -\frac{1}{im}\delta_{IJ} - \mathcal{D}_{IJ} & \text{same charged} \\ -F_{IJ} & \text{opposite charged} \end{cases} \qquad (9)$$

The application of the Dyson method after neglecting the nonplanar contributions which contain additional smallness over $1/N_c$ brings to nonlinear integral equations:

$$F_{IJ}=\frac{T_{IJ}}{(im)^2}+\frac{N\mathcal{E}}{2Vi}\left[\int d^4z_L\, dU_L\, T_{IL}\,\mathcal{D}_{LI}+\int d^4z_M\, dU_M\, K_{IM}F_{MI}\right] \tag{10}$$

$$\mathcal{D}_{IJ}=\frac{K_{IJ}}{(im)^2}+\frac{N\mathcal{E}}{2Vi}\left[\int d^4z_L\, dU_L\, K_{IL}\,\mathcal{D}_{LI}+\int d^4z_M\, dU_M\, T_{IM}\,F_{MI}\right]$$

where we introduce the notations:

$$\mathcal{E}=\frac{1}{m(1-im\gamma)}$$

$$\gamma=\mathcal{D}_{II}$$

V - four-dimensional volume

$N/2$ - the number of instantons (equal to the number of anti-instantons).

By the definition of $\mathcal{D}_{IJ}$ and F_{IJ} functions, the edge pseudoparticles I and J cannot meet inside the sum, i.e. $L\neq I,J;\ M\neq I,J$ so that in K_{IJ} the terms proportional to δ_{IJ} again vanish in integrands. Then in the case of $I\neq J$ they vanish everywhere and equations (10) can be solved explicitly. Look for the Fourier-transformed images in the following form:

$$F_{I\bar{I}}=f(p^2)\,Sp\,U_{\bar{I}}\,p^+U_I^+ \qquad F_{\bar{I}I}=-f(p^2)\,Sp\,U_I\,p^-U_{\bar{I}}^+ \tag{11}$$

$$\mathcal{D}_{IJ}=d(p^2)\,Sp\,U_J\,1_2\,U_I^+ \qquad I\neq J$$

The definition of matrices $p^+, p^-, 1_2$ see in [6].

Solving equations for f and d we obtain

$$f(p^2)=\frac{2VN_c}{N\mathcal{E}}\,\frac{i}{m^2}\,\frac{M(p)}{p^2-2mM(p)+M^2(p)\,\frac{p^2+m^2}{p^2}}$$

$$d(p^2)=\frac{2VN_c}{N\mathcal{E}}\,\frac{i}{m^2}\,M(p)\,\frac{-m+M(p)\,\frac{p^2+m^2}{p^2}}{p^2-2mM(p)+M^2(p)\,\frac{p^2+m^2}{p^2}} \tag{12}$$

Here $M(p)$ is the effective mass of the massless quark in instanton medium and is also determined in [6].

Thus the δ_{IJ} terms change only the self-consistency condition of Eq.(10). Extracting them explicitly in the first term of Eq.(10) for $\mathcal{D}_{IJ}$ in the case of $I=J$, we'll obtain:

$$\gamma=2\int\frac{d^4p}{(2\pi)^4}d(p)-\frac{im}{(im)^2}$$

$$1-2m\mathcal{E}=\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}M(p)\,\frac{-m+M(p)\,\frac{p^2+m^2}{p^2}}{p^2-2mM(p)+M^2(p)\,\frac{p^2+m^2}{p^2}} \tag{13}$$

Eq.(13) defines the function $\mathcal{E}(m,\bar{\rho}/R)$ also in the first approximation over $\bar{\rho}/R$:

$$\mathcal{E}(m)=\frac{\mathcal{E}_0}{\sqrt{1+\left(\frac{m\mathcal{E}_0}{2}\right)^2}+\frac{m\mathcal{E}_0}{2}} \tag{14}$$

$$\mathcal{E}_0=\mathcal{E}(0)=\mathrm{const}\;N_c^{1/2}\;R^2/\bar{\rho}$$

The substitution of the found values of F_{IJ} and $\mathcal{D}_{IJ}$ functions into formula (8) as well as the averaging over the edge pseudoparticles I and J brings for the quark propagator to the expression:

$$S(p)=\frac{\hat{p}+im}{p^2+m^2}-\frac{\hat{p}+i\left(m-\frac{p^2}{M}\right)}{p^2+\left(m-\frac{p^2}{M}\right)^2}, \tag{15}$$

which in the limit $m=0$ reproduces the result of Ref.[6]:

$$S(p)=\frac{\hat{p}+iM}{p^2+M^2}.$$

3. Correlation Functions of Meson Currents

In principle, all correlators must be expressed through $f(p^2)$ and $d(p^2)$ functions, the calculation scheme and hence the functional dependence remaining absolutely unchanged. Therefore, here we only present the calculation results which should be compared to analogous formulae in Ref.[7].

For the connected part of the Π_{con} correlator of pseudoscalar and axial currents we have

$$\Pi_{con}=4N_c^2\frac{V}{N}\;\frac{\Gamma_I(p)\;\;\Gamma_I(-p)}{R_-(p)}\;. \tag{16}$$

The analytical expression $R_-(p)$ as a function of $f(p^2)$ and $d(p^2)$ with account of different masses of quarks has the form:

$$R_-(p)=1+m_1^2\varepsilon_1 m_2^2\varepsilon_2\frac{N}{VN_c}\int(dp_1\,dp_2)\left(d_1 d_2+p_1\cdot p_2 f_1 f_2\right). \tag{17}$$

Here and further, indices 1 and 2 mean the dependence on the corresponding arguments, for example

$$d_1=d(p_1,m_1)\,,\quad d_2=d(p_2,m_2)\,,$$

$$(dp_1,dp_2)\equiv\frac{d^4p_1\,d^4p_2}{(2\pi)^4}\,\delta^4(p_1-p_2-p).$$

Using the self-consistency condition and substituting explicit expressions d and f, present $R_-(p)$ in the form:

$$R_-(P) = m_1\varepsilon_1 + m_2\varepsilon_2 - m_1 \frac{2VN_c}{N}\int\frac{d^4P_1}{(2\pi)^4} M_1\left(P_1^2 - 2m_1M_1 + M_1^2\frac{P_1^2+m_1^2}{P_1^2}\right)^{-1} -$$

$$- m_2\frac{2VN_c}{N}\int\frac{d^4P_2}{(2\pi)^4} M_2\left(P_2^2 - 2m_2M_2 + M_2^2\frac{P_2^2+m_2^2}{P_2^2}\right)^{-1} + \tag{18}$$

$$+ \frac{2VN_c}{N}\int(dP_1dP_2)\frac{(M_1P_{2\mu} - M_2P_{1\mu})^2 + \left(m_1M_1\frac{|P_2|}{|P_1|} - m_2M_2\frac{|P_1|}{|P_2|}\right)^2}{\left[P_1^2 - 2m_1M_1 + M_1^2\frac{P_1^2+m_1^2}{P_1^2}\right]\left[P_2^2 - 2m_2M_2 + M_2^2\frac{P_2^2+m_2^2}{P_2^2}\right]}$$

Calculate $R_-(P)$ at $P^2 \to 0$ in the leading order over small parameters $\bar{\rho}/R$, $m\bar{\rho}$. Remind, that $\bar{\rho}$ = 1/600 MeV and for three light quarks the condition $m\bar{\rho} \ll 1$. is fulfilled. In calculations we should correctly take into account the non-analytical dependence of $\varepsilon(m\bar{\rho}, \bar{\rho}/R)$ on the small parameter $\bar{\rho}/R$ (14). The result has the form:

$$R_-(P) = \beta P^2 + m_1\varepsilon_1 + m_2\varepsilon_2 + \frac{1}{2}(m_1+m_2)\langle\bar{\Psi}\Psi\rangle,$$

$$\beta = \left(\frac{\varepsilon_1+\varepsilon_2}{2\varepsilon_0}\right)^2\left[\frac{2VN_c}{N}\int\frac{d^4K}{(2\pi)^4}\frac{M^2 - \frac{1}{2}MM'|K| + \frac{1}{4}M'^2K^2}{(K^2+M^2)^2} + \right. \tag{19}$$

$$\left. + \frac{2m_1\varepsilon_1 + m_2\varepsilon_2}{\varepsilon_0}\,\frac{VN_c\, M(0)}{N\, 32\pi^2}\right.$$

These formulae together with the found value of $\Gamma_I(0)$

$$\Gamma_I(0) = \frac{\sqrt{\varepsilon_1\varepsilon_2}}{\varepsilon_0}\,\frac{\langle\bar{\Psi}\Psi\rangle}{2N_c} + O(m^2) \tag{20}$$

determine masses and axial coupling constants of $\pi^\pm$ and $K^\pm$ mesons. As to the η_8 particle mass, it can be calculated neglecting mass uncertainties in $|\bar{q}q\rangle$ states of m_q^2 order. Finally for the mass spectrum we have

$$
\begin{aligned}
m_\pi^2 &= -2\,\frac{m_u+m_d}{f_\pi^2}\,\langle\bar\psi\psi\rangle\,, \\
m_K^2 &= -2\,\frac{m_u+m_s}{f_K^2}\,\langle\bar\psi\psi\rangle\,, \\
m_8^2 &= -\frac{2}{3}\,\frac{m_u+m_d+4m_s}{f^2}\,\langle\bar\psi\psi\rangle\,,
\end{aligned}
\tag{21}
$$

which in the SU(2) symmetry approximation restore the Gell--Mann-Okubo identity $(f_K = f_\pi = f)$

$$
3m_8^2 + m_\pi^2 = 4m_K^2
$$

Let us neglect the u and d quark masses, and taking into account the nonanalytic dependence on m_s in the terms (εm_s).

Then for the ratio of axial constants with the use of (14) we have

$$
\frac{f_K^2}{f_\pi^2} = \frac{(\varepsilon+\varepsilon_0)^2}{4\varepsilon\varepsilon_0}\left(1+\frac{8m_s\varepsilon}{\varepsilon_0 f_\pi^2}\,N_c\,\frac{M(0)}{32\pi^2}\right), \tag{22}
$$

where

$$
\varepsilon_0 = \varepsilon(0) = \frac{-\langle\bar\psi\psi\rangle}{\dfrac{\langle F^2\rangle}{32\pi^2}}\,.
$$

Numerically at m_s = 150 MeV

$$
f_K/f_\pi = 1.12
$$

Thus we have shown that the instanton vacuum model describes successfully the physics of pseudoscalar octet of mesons the interpretation of which as pseudogoldstone particles brings to a self-consistent picture. So the meson mass spectrum is described by formulae (21) in agreement with the results of current algebra, and formula (22) explains correctly weak dependence of axial constants on the quark mass.

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МАССОВЫЕ ПОПРАВКИ К ФУНКЦИЯМ ГРИНА В МОДЕЛИ ИНСТАНТОННОГО ВАКУУМА

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