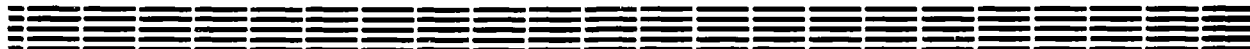


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NON-EXISTENCE OF RENORMALIZABLE N=2
MODELS FOR SCALAR AND SPINOR FIELDS

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ՍԿԱԼՅԱՐ ԵՎ ՍՊԻՆՈՐԱՅԻՆ ԴԱՇՏԵՐԻ $N=2$ ՎԵՐԱՆՈՐՄԱՎՈՐՎՈՂ
ՄՈԴԵԼՆԵՐԻ ԿԱՌՈՒՑՄԱՆ ԱՆՀՆԱՐԻՆՈՒԹՅՈՒՆԸ

Ցույց է տրված, որ անհնար է կառուցել $N=2$ գերհամաչափ վերանորմավորվող, միայն փոխազդող սկալյար և սպինորային դաշտեր պարունակող լազրանոյան:

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NON-EXISTENCE OF RENORMALIZABLE $N=2$ MODELS FOR SCALAR
AND SPINOR FIELDS

Construction of renormalizable $N=2$ supersymmetric Lagrangian containing only interacting scalar and spinor fields is shown to be impossible.

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Р.П.ГРИГОРЯН, И.В.ТЮТИН, Г.Н.ХАЧАТРЯН

ОТСУТСТВИЕ ПЕРЕНОРМИРУЕМЫХ $N = 2$
МОДЕЛЕЙ СКАЛЯРНЫХ И СПИНОРНЫХ ПОЛЕЙ

Показано, что невозможно построить перенормируемый $N = 2$ суперсимметричный лагранжиан, содержащий только взаимодействующие скалярные и спинорные поля.

Ереванский физический институт

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1. Although the supersymmetry [1] during already more than a decade is actively studied by many authors, the interest in it remains unfading. Theories with extended ($N > 1$) supersymmetry are of special interest. While the $N=1$ supersymmetry is well studied and, in particular, in our considered case of interaction of spinor and scalar fields the general renormalizable Lagrangian in terms of superfields is constructed, there are many problems to be solved for $N \geq 2$. In the last years some approaches to construction of such models are worked out. In refs.[2] and [3] by means of introduction of auxiliary "harmonic" variables and using a related approach respectively, there models with $N \geq 2$ supersymmetry off the mass shell are also constructed. However, there arises an infinitely great number of auxiliary fields. Besides, the $N=2$ interaction Lagrangian of the fields of spin 0 and $1/2$ is not renormalizable. Another approach using the language of the nonlinear σ -model [4] allows one to do without introduction of infinite number of auxiliary fields, but in this case too, the $N=2$ Lagrangian of the fields of spin 0 and $1/2$ is not renormalizable.

The attempts to construct a supersymmetric $N \geq 2$ renormalizable model of interaction of spinor and scalar fields had no success, which by itself seriously indicates that

it is impossible to construct such a model. However, it seems to us that there is no explicit proof of such a fact. One familiar to us proof [5] is incorrect, since there, in fact, the existence of a closed algebra of supersymmetry off the mass shell is supposed, which is absent already in the simplest Wess-Zumino model. In the present paper we shall give an explicit proof of the fact, that it is impossible to construct an N=2 supersymmetric renormalizable model which includes only the fields of the spin 0 and 1/2. Here we shall proceed from the fact that the N=1 supersymmetry does exist and we shall search for additional supertransformations for the most general renormalizable N=1 Lagrangian. The final result is, that there are no additional supertransformations.

2. So, consider the most general renormalizable theory of interaction of spinor and scalar fields which has N=1 supersymmetry and is described by the Wess-Zumino Lagrangian [6] :

$$\begin{aligned}
L = & \partial_\mu \phi^{\dagger i} \partial_\mu \phi^i - N^{ij} N^{*jnm} \phi^i \phi^{\dagger l} \phi^m \phi^{\dagger n} + \\
& + \frac{i}{2} \bar{\psi}^i \gamma^\mu \partial_\mu P_+ \psi^i + \frac{i}{2} \bar{\psi}^i \gamma^\mu \partial_\mu P_- \psi^i + \\
& + N^{ijl} (\bar{\psi}^i P_- \psi^j) \phi^{\dagger l} + N^{*ijl} (\bar{\psi}^i P_+ \psi^j) \phi^{\dagger l}
\end{aligned} \tag{1}$$

Here ϕ^i is the set of complex scalar fields, ψ^i is the set of Majorana spinors ($i, j, l, \dots = 1, 2, \dots, n$), the constant N^{ijl} is symmetric over all the indices; $P_\pm = (1/2)(1 \pm \gamma^5)$; there summation is supposed over the repeating indices.

The supersymmetry transformations, leaving the theory action an invariant one, are

$$\begin{aligned}
\delta\phi^i &= \bar{\xi} P_- \psi^i, \\
\delta(P_+ \psi^i) &= -i \partial_\mu \phi^i (\gamma^\mu P_- \xi) + N^{ijk} \phi^j \phi^k (P_+ \xi), \\
\delta(P_- \psi^i) &= -i \partial_\mu \phi^i (\gamma^\mu P_+ \xi) + \bar{N}^{ijk} \phi^j \phi^k (P_- \xi)
\end{aligned} \tag{2}$$

As noted in section 1, our task is to investigate the question whether there exists together with (2) an additional supersymmetry leaving the theory action (1) an invariant one. To solve it, we shall proceed as follows. With account of canonical dimensions of the fields and their Grassmanian parities we write the most general form of the supersymmetry transformation for the fields ϕ^i and ψ^i

$$\begin{aligned}
\delta\phi^i &= C^{ij} (\bar{\xi} P_+ \psi^j) + \mathcal{D}^{ij} (\bar{\xi} P_- \psi^j), \\
\delta\psi^i &= F_1^{ij} \partial_\mu \phi^j (\gamma^\mu P_+ \xi) + F_2^{ij} \partial_\mu \phi^j (\gamma^\mu P_- \xi) - \\
&\quad - \bar{F}_1^{ij} \partial_\mu \phi^j (\gamma^\mu P_- \xi) - \bar{F}_2^{ij} \partial_\mu \phi^j (\gamma^\mu P_+ \xi) + \\
&\quad + S_1^{i\{jk\}} \phi^j \phi^k (P_+ \xi) + S_2^{ijk} \phi^j \phi^k (P_+ \xi) + \\
&\quad + S_3^{i\{jk\}} \phi^j \phi^k (P_+ \xi) + \bar{S}_3^{i\{jk\}} \phi^j \phi^k (P_- \xi) + \\
&\quad + \bar{S}_2^{ijk} \phi^j \phi^k (P_- \xi) + \bar{S}_1^{i\{jk\}} \phi^j \phi^k (P_- \xi),
\end{aligned} \tag{3}$$

where C , \mathcal{D} , F , S are arbitrary matrix coefficients; $S^{i\{jk\}}$ is a matrix coefficient symmetrical over the indices j, k . Also consider that the Lagrangian variation at supersymmetric transformations is reduced to the total derivative of some function of field variables:

$$\delta L = \partial_\mu (\bar{\xi}_\alpha \Delta_\alpha^\mu), \tag{4}$$

where Δ_α^μ is a Grassman-odd value. Write the most general expression for it too, with regard to the field dimensions and the condition of a Hermitian Lagrangian (owing to Majoranity of spinors the structure $\bar{\Delta}_\alpha^\mu \xi_\alpha$ is reduced to $\bar{\xi}_\alpha \Delta_\alpha^\mu$ that is why it is not written in the right hand side of (4)):

$$\begin{aligned}
\bar{\xi}_\alpha \Delta_\alpha^\mu = & \bar{\xi}_\alpha \{ T_1^{ij} (P_+ \psi^i)_\alpha \partial_\mu \phi^j + T_2^{ij} (P_+ \psi^i)_\alpha \partial_\mu \dot{\phi}^j + \\
& + T_2^{*ij} (P_- \psi^i)_\alpha \partial_\mu \dot{\phi}^j + T_1^{*ij} (P_- \psi^i)_\alpha \partial_\mu \phi^j + U_1^{ij} \partial_\mu (P_+ \psi^i)_\alpha \phi^j + \\
& + U_2^{ij} \partial_\mu (P_+ \psi^i)_\alpha \dot{\phi}^j + U_2^{*ij} \partial_\mu (P_- \psi^i)_\alpha \phi^j + U_1^{*ij} \partial_\mu (P_- \psi^i)_\alpha \dot{\phi}^j + \\
& + V_1^{ij} (\sigma^{\mu\nu} P_+ \psi^i)_\alpha \partial_\nu \phi^j + V_2^{ij} (\sigma^{\mu\nu} P_+ \psi^i)_\alpha \partial_\nu \dot{\phi}^j + \\
& + V_2^{*ij} (\sigma^{\mu\nu} P_- \psi^i)_\alpha \partial_\nu \phi^j + V_1^{*ij} (\sigma^{\mu\nu} P_- \psi^i)_\alpha \partial_\nu \dot{\phi}^j + \\
& + X_1^{ij} (\sigma^{\mu\nu} P_+ \partial_\nu \psi^i)_\alpha \phi^j + X_2^{ij} (\sigma^{\mu\nu} P_+ \partial_\nu \psi^i)_\alpha \dot{\phi}^j + \\
& + X_2^{*ij} (\sigma^{\mu\nu} P_- \partial_\nu \psi^i)_\alpha \phi^j + X_1^{*ij} (\sigma^{\mu\nu} P_- \partial_\nu \psi^i)_\alpha \dot{\phi}^j + Y_1^{i\{j\kappa\}} (\gamma^\mu P_+ \psi^i)_\alpha \phi^j \phi^\kappa + \\
& + Y_2^{ij\kappa} (\gamma^\mu P_+ \psi^i)_\alpha \dot{\phi}^j \phi^\kappa + Y_3^{i\{j\kappa\}} (\gamma^\mu P_+ \psi^i)_\alpha \dot{\phi}^j \dot{\phi}^\kappa - \\
& - Y_3^{*i\{j\kappa\}} (\gamma^\mu P_- \psi^i)_\alpha \phi^j \phi^\kappa - Y_2^{*i\kappa j} (\gamma^\mu P_- \psi^i)_\alpha \dot{\phi}^j \phi^\kappa - \\
& - Y_1^{*i\{j\kappa\}} (\gamma^\mu P_- \psi^i)_\alpha \dot{\phi}^j \dot{\phi}^\kappa \} ,
\end{aligned} \tag{5}$$

where T , U , V , X , Y are arbitrary matrix coefficients for the time being.

Now vary the Lagrangian (1) using the expressions (3) for variation of the fields ϕ^i and ψ^i and substitute the

obtained result into the left hand side of (4), and the expression (5) for $\bar{\xi}_\alpha \Delta_\alpha^\mu$ substitute into the right hand side. Equating the expressions at identical field structures in the left and right hand sides, we obtain a set of equations for the matrix coefficients, which after some simplifications are reduced to

1. $N^{jlr} C^{rn} = 0$
2. $N^{jlr} C^{nr} = 0$
3. $N^{irj} D^{lr} - N^{irl} D^{jr} = 0$
4. $-N^{jlr} N^{rnm} D^{mi} + N^{irn} N^{rkj} D^{lk} = 0$
5. $2N^{ijm} D^{mk} - N^{kim} D^{mj} - N^{kjm} D^{mi} = 0$
6. $N^{ijk} D^{lj} - S_1^i\{kl\} = 0$
7. $C^{ij} - iF_2^{ji} = 0$
8. $D^{ij} - iF_1^{ji} = 0$
9. $S_2^{ijk} = S_3^i\{jk\} = 0$

Apparently, this set of equations has at least one solution leading to the supertransformations (2). Existence of N=2 supersymmetry would mean, in fact, that of one more solution of the set (6).

Now let us analyze the set of equations. Consider the eq.1 Multiply both its sides by N^{ijl} and sum up over the indices j and l. We obtain

$$A^{ir} C^{rn} = 0, \quad A^{ir} = N^{ijl} N^{ljr} \quad (7)$$

Treating eq.2 the same way, we obtain

$$A^{ir} C^{nr} = 0 \quad (8)$$

Then, it follows from eqs.(11), which now are written as

$$\lambda'_i C'^{in} = 0, \quad \lambda'_i C'^{ni} = 0 \quad (12)$$

(no summation over i) that at $\lambda'_i = \lambda'_i$

$$C_{\underline{i} \underline{n}} = C_{\underline{i} \underline{n}} = C_{\underline{i} \underline{n}} = 0$$

And when $\lambda'_i = \lambda'_i = 0$, then $C_{\underline{i} \underline{k}}$ is an arbitrary matrix. However, for the values of $\lambda'_i = 0$ we have $N'^{irj} = 0$, which directly follows from the equality

$$\lambda'_{\underline{i}} = N'^{irj} N'^{irj}$$

It also follows from here, that at $\lambda'_i = \lambda'_i$ all the values of N'^{irj} can not simultaneously be equal to zero.

Thus, we obtain that in the section of values of indices where the interaction is absent ($N'^{irj} = 0$), the corresponding matrix block of $C(C_{\underline{i} \underline{k}})$ is an arbitrary one. While in the index section where interaction is present in the theory, the matrix elements of C turn out to be zero.

Now consider the other equations of the set (6). But, first, let us make an important notice. Among the set of fields ϕ^i and ψ^i there may, generally speaking, be found ones which interacting with one part of the fields do not interact with the other. In other words, the whole set of the fields involved in the theory may be subdivided into classes where each field inside the class necessarily interacts with some field (fields), it being only of the same class, and inside of each class there are no subclasses having the same property.

It means that the value N^{ijk} , in the case with the indices numerating fields of different classes, is identically equal to zero. There is, apparently, possible a case when all the fields form one and the only class.

It is easily understood that the set (6) takes place separately for each class of fields. With regard to the notice made, it is clear that it is nontrivial just inside one class, since in case of mixing of fields from different classes the interaction disappears. Moreover, the matrix element of D^{ij} (when the indices i and j numerate fields from different classes) are, as we shall show, equal to zero. In other words, the matrix D is a block diagonal one and its each block corresponds to one class. Consider the equation 3 for this. Expand the set of values the indices r, j, l, \dots may have, into subsets (in accordance with the number of classes equal, e.g., to K) so, that the subset elements numerate fields inside the classes. For instance, by the index $z_\alpha (1 \leq \alpha \leq K)$ will be numerated fields belonging to the class α . Rewrite the equation 3 as:

$$\sum_{\alpha} (N^{iz_\alpha j_\beta} D^{*l_\gamma z_\alpha} - N^{iz_\alpha l_\gamma} D^{*j_\beta z_\alpha}) = 0; \quad \alpha, \beta, \gamma = 1, \dots, K$$

and consider it at $\beta \neq \gamma$. Let $i \in \{i_\beta\}$. With regard to the properties of N^{irj} we have:

$$N^{i_\beta r_\beta j_\beta} D^{*l_\gamma r_\beta} = 0, \quad (13)$$

where summation over r is made only over the class β . For the values of $i \in \{i_\gamma\}$ we obtain

$$N^{i\gamma} r_{\gamma} j_{\gamma} D^{j\beta} r_{\beta} = 0 . \quad (14)$$

The eqs. (13) and (14) may be solved in the same way as eqs. 1 and 2 of the set (6) in presence of interaction. Here we obtain $D^{j\beta} r_{\beta} = D^{j\gamma} r_{\gamma} = 0$ at $\gamma \neq \beta$, i.e. D is a block diagonal matrix.

Our further consideration will be carried out for some one class of interacting fields. Using the eq.3, rewrite the eq. 4. as

$$N^{jlr} (N^{rnm} D^{mi} - N^{inm} D^{mr}) = 0 ,$$

whence

$$N^{rnm} D^{mi} - N^{inm} D^{mr} = 0 \quad (15)$$

This is also easily proved if one follows the way the eqs. 1 and 2 of the set (6) were solved, and takes into account that here - inside one class - all the diagonal elements λ are other than zero.

Introduce the Hermitian matrix

$$Q^{mi} = D^{mi} + D^{mi\dagger}$$

and the anti-Hermitian matrix

$$V^{mi} = D^{mi} - D^{mi\dagger}$$

Combining eq.(15) with the eq.3 of the set (6) for matrices Q and V , we obtain

$$\begin{aligned} N^{rnm} Q^{mi} - N^{inm} Q^{mr} &= 0 \\ N^{rnm} V^{mi} - N^{inm} V^{mr} &= 0 \end{aligned} \quad (16)$$

Consider the first one at first. Using a unitary transformation diagonalizing the Hermitian matrix Q , reduce it to

$$N'^{rni} (\mu_i - \mu_r) = 0,$$

where N'^{rni} is determined by the formula (10), μ_i are the elements of the diagonalized matrix Q . Hence, with regard to the properties of N'^{rni} we find

$$\mu_i = \mu_2 = \mu; \quad Q^{mi} = \mu \delta^{mi} \quad (17)$$

Treating the second equation in (16) the same way, we obtain

$$\nu_i = \nu_2 = \nu; \quad V^{mi} = \nu \delta^{mi}; \quad (18)$$

ν are the elements of the diagonalized matrix V . Note, that the transformation diagonalizing the matrix V does not change the form (17) of the matrix Q . Proceeding from expressions (17) and (18) for the matrix D we obtain

$$D^{ij} = 1/2(\mu + \nu) \delta^{ij}.$$

Introducing the numerical coefficient $(1/2)(\mu + \nu)$ into the definition of the infinitely small parameter ξ (see (3)), the matrix D may be considered to be a unit one: $D^{ij} = \delta^{ij}$. Hence, eq.5 of the set (6) is trivially satisfied which, though, was still apparent from (15).

Thus, we obtain that in the presence of interaction $C^{ij}=0$, $D^{ij} = \delta^{ij}$. Using these equalities, from the set (6) we have also

$$F_2^{ij} = 0, \quad F_1^{ij} = -i\delta^{ij}, \quad S_1^{i\{kl\}} = N^{ijk}.$$

Finally, substituting these expressions into (3) we find that the obtained supersymmetry transformations exactly coincide with the transformations (2).

So, the analysis of the set (6) has shown that the set has the only solution corresponding to the supersymmetry transformations (2). This just means that the $N=2$ supersymmetry is absent in the model considered.

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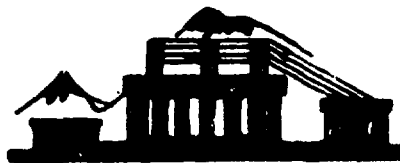
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