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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
YEREVAN PHYSICS INSTITUTE

D.B.SAHAKYAN

NUMERICAL STUDY OF THE CASIMIR EFFECT AT A CRITICAL POINT OF THE d=3 ISING MODEL

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ԵՌԱՉԱՓ ԻԶԻՆԳԻ ՄՈԴԵԼՈՒՄ ԿԱԶԻՄԻՐԻ ԵՐԵՎՈՒՅԹԻ ԹՎԱՅԻՆ ՀԵՏԱԶՈՏՈՒՄԸ ԿՐԻՏԻԿԱԿԱՆ ԿԵՏՈՒՄ

Հետազոտվում է պարբերական եզրային պայմանի դեպքում ազատ էներգիայի կախումը ցանցի չափերից։ Դիտարկվում են դեպքեր, երբ զուգահեռանիստի մի կողմը փոքր է մյուս երկուսից, և երբ երկու հավասար կողմերը փոքր են երրորդից։

Երևանի Ֆիզիկայի Ինստիտուտ
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D.B. SAHAKYAN

NUMERICAL STUDY OF THE CASIMIR EFFECT AT A CRITICAL POINT OF THE d=3 ISING MODEL

The dependence of free energy on the sizes of a lattice with periodic boundary conditions is studied. Cases are considered when one of the parallelepiped's sides is small compared to two others, and then two equal sides are small compared to the third one.

Yerevan Physics Institute
Yerevan 1987

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Д.Б.СААКЯН

ЧИСЛЕННОЕ ИЗУЧЕНИЕ ЭФФЕКТА КАЗИМИРА
В КРИТИЧЕСКОЙ ТОЧКЕ d=3 МОДЕЛИ ИЗИНГА

Изучается зависимость свободной энергии от размеров решетки с периодическими граничными условиями. Рассматриваются случаи, когда одна из сторон параллелипипеда мала по сравнению с двумя другими и тогда две равные стороны малы по сравнению с третьей.

Ереванский физический институт
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In the recent years the strings have attracted great attention of the theorists. There exists a hypothesis [1-3] that the string yields an exact solution of the d=3 Ising model in the critical region. So the study of this model becomes more interesting, since here one can compute the physical quantities with an accuracy hardly achievable at simulation of discrete string models.

The testing of any theory claiming on solution of the given model must be calculation of critical indices known from the numerical experiment. To author's opinion, the calculation of constants in the Casimir effect will be the easiest from the theoretical viewpoint. The method proposed in Ref.[4] allows to calculate a statistical sum with high accuracy. The authors of Ref.[4] offer to measure the free energy change from one temperature to another by comparing the energy distribution functions $P(E,B)$, where $B=1/T$. So, one can easily come to the following formula:

$$\ln \frac{Z(B_1)}{Z(B_2)} = -(B_1 - B_2)E + \ln \frac{P(E,B_2)}{P(E,B_1)} \qquad (1)$$

Here the right-hand side must not depend on the choice of E (if $P(E,B)$ is computed precisely).

For our studied model with a few thousands of spins this method allows to calculate the free energy density with an accuracy $\sim 10^{-4} - 10^{-5}$. Energy distribution after 5000 free iterations was averaged over 50000 subsequent iterations. Here at increasing number of iterations $P(E,B)$ varied by a share of percent (near the maximum point where and only where we used the value of the function $P(E,B)$). At a point $B = 0$ we have $\ln Z = N \ln 2$. Using formula (1) we can compute $\ln Z$ at points with larger values of B, while moving with a step $\Delta B = 0.025$ and finally determining the value of $\ln Z$ at a critical point $B_c \approx 0.22166$.

The table lists the values of $\ln Z - \ln 2$ for various lengths of the parallelepiped. Here formula (1) was used for such values of E at which $P(E,B_1)$ and $P(E,B_2)$ are maximally close. At E varied by a few steps, $\ln Z/N$ varied less than by 10^{-5}. For the case when two sides equal L, and the third one tends to infinity, we obtain:

$$\ln Z/N = \text{const} + C_1/L^d \qquad (2)$$

where $C_1 \approx 0.35 - 0.38$.

The figures at the table show that the length of the third side may be considered infinitely large.

In the case when one side has a length L and two others

are large, we obtain with the accuracy ~ 0.01:

$$C_2 \approx 0.13 \qquad (3)$$

To compute C_2, one needs large lattices and correspondingly larger calculations.

In the case of the d=2 Ising model at a critical point, is obtained as a statistical sum of free massless Majorana fermions, with antiperiodicity condition of wave function at a boundary (in case when the lattice is infinite in opposite direction).

For fermions in d=3 with the same boundary conditions we would obtain:

$$C_2 = 3\zeta(3)/4\pi \approx 0.2869 \qquad (4)$$

this being nearly twice that in formula (3). For the scalar particles we would have:

$$C_2 = \zeta(3)/2\pi \approx 0.1913 \qquad (5)$$

It is likely that the d=3 Ising model is not described by a single free particle.

In conclusion, the author would like to express his sincere gratitude to A.V.Zamolodchikov, A.A.Migdal and S.G.Matinyan for the useful discussions. The author is indebted to V.P.Solakhyan for assistance in the work.

Table of values of $\ln Z(B_c)/N - \ln 2$ for lattices with different lengths of sides

6,6,40	0.08640
6,6,100	0.08652
12,12,10	0.08616
15,15,16	0.08491
4,4,100	0.09064
10,10,16	0.08763
5,20,20	0.08608
5,24,24	0.08592

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