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V.G.GURZADYAN, A.A.KOCHARYAN

ON PURE AND MIXED QUANTUM STATES OF THE UNIVERSE

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ՏԻԵԶԵՐՔԻ ՄԱՔՈՒՐ ԵՎ ԽԱՌՆ ՔՎԱՆՏԱՅԻՆ ՎԻՃԱԿՆԵՐԻ ՄԱՍԻՆ

Հոկինգի քվանտային տիեզերագիտության շրջանակներում քննարկված է Տիեզերքի տոպոլոգիայի փոփոխման խնդիր, որը թույլատրում է տարբերել մաքուր քվանտային վիճակները խառնվածներից: Ցույց է տրված, որ մաքուր վիճակում գտնվող Տիեզերքի դեպքում ոլորտի անցումը Թորի խիստ անհավանական է: Համապատասխան հավանականությունները նույն կարգի են դառնում Տիեզերքի խառն քվանտային վիճակների դեպքում:

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V.G. GURZADYAN, A.A. KOCHARYAN

ON PURE AND MIXED QUANTUM STATES OF THE UNIVERSE

Within the framework of Hawking quantum cosmology a problem of topology change of the Universe allowing to differentiate between pure and quantum states is discussed. It is shown that for the pure state Universe the transition of the sphere to torus is strongly suppressed. The corresponding probabilities become of the same order in case of the mixed states of the Universe.

Yerevan Physics Institute

Yerevan 1987

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В.Г.ГУРЗАДЯН, А.А.КОЧАРЯН

ЧИСТЫЕ И СМЕШАННЫЕ СОСТОЯНИЯ ВСЕЛЕННОЙ

В рамках хоукинговской квантовой космологии обсуждается проблема изменения топологии Вселенной, позволяющая различить чистые квантовые состояния от смешанных. Показано, что при чистых состояниях Вселенной (игрушечная модель) переход сферы в тор сильно подавлен. Соответствующие вероятности становятся одного порядка при смешанных состояниях.

Ереванский физический институт
Ереван 1987

1. Introduction

The wave function of the Universe introduced by Hartle and Hawking [1] enabled them to calculate the probability of quantum state of the Universe with S^3 topology; simultaneously they raised the question of calculation of the corresponding probabilities for different topologies. The latter cases (including torus, hyperbolic space, etc.) were investigated in [2,3]. In these papers only pure quantum states of the Universe were considered.

Hawking [4] and Page [5] independently put forward the hypothesis on mixed states of the Universe, which means transition from the quantum states wave function description to that based on the density matrix. The investigation of minisuperspace models using the density matrix, leads to the conclusion that both descriptions do not give quite different results (probabilities) at least for those models [5]. Therefore, it is of special interest to consider such a problem which, first, allows to estimate the density matrix and second, leads to results sufficiently dependent on whether pure or mixed states are allowed. Before we turn to that problem on the topology change of the Universe, we [illegible] the main difference between Hartle-Hawking wave function and Hawking-Page density matrix.

2. The Wave Function of the Universe

The probability of a quantum state of the Universe with topology Σ , geometry h and matter fields ϕ is defined by the path integral [6]

$$P[\Sigma, h, \phi] = \int_{A[\Sigma, h, \phi]} d[M, g, \varphi] e^{-I[M, g, \varphi]} ,$$

where $A[\Sigma, h, \phi]$ is the set of all oriented compact 4-manifolds with geometry g and matter fields φ , for which there exists an insertion i satisfying the following conditions:

1. $\partial M = \emptyset$,
2. $\tilde{\Sigma} \equiv i\Sigma \subset M$,
3. $\exists M_+, M_- ;\ M = M_+ \# M_- ; \partial M_+ = \tilde{\Sigma} = -\partial M_-$,
4. $i^* g = h$,
5. $\varphi_{\tilde{\Sigma}} = \phi$,

where $\#$ indicates operation of the connected sum of the manifolds, i^* is the differential of mapping i , I is the gravitational action.

The probability $P[\Sigma, h, \phi]$ can be represented in the form [6]

$$P[\Sigma, h, \phi] = \psi_+[\Sigma, h, \phi] \cdot \psi_-[\Sigma, h, \phi] ,$$

where

$$\psi_+[\Sigma, h, \phi] = \psi_-[\Sigma, h, \phi] = \psi[\Sigma, h, \phi] .$$

The function Ψ , called the wave function of the Universe, is equal to*

$$\Psi[\Sigma, h, \phi] = \int\limits_{B[\Sigma, h, \phi]} d[M, g, \varphi]\, e^{-I}, \tag{1}$$

where

$$B[\Sigma, h, \phi] = \{ M, g, \varphi \mid \exists\, i;\ 1)\ \partial M = \tilde{\Sigma},\ 2)\ \tilde{\Sigma} \subset M,\ 3)\ i^* g = h,\ 4)\ \varphi_{\tilde{\Sigma}} = \phi \}.$$

In the case when

$$\Sigma = \sigma + (-\sigma) \equiv \sigma_1 + \sigma_2 \neq \emptyset,$$
$$h = h_{\sigma_1} + h_{\sigma_2},$$
$$\phi = \phi_{\sigma_1} + \phi_{\sigma_2},$$

($h_{\sigma_1}, \phi_{\sigma_1}$ ($h_{\sigma_2}, \phi_{\sigma_2}$) are metric and material fields on $\sigma_1 = \sigma$ ($\sigma_2 = -\sigma$)) gluing the boundaries σ_1, σ_2 one has a compact 4-manifold without boundary:

$$B[\sigma - \sigma,\ h_\sigma + h_\sigma,\ \phi_\sigma + \phi_\sigma] \to$$

$$B_g[\emptyset(\sigma), h, \phi] = \{ M, g, \varphi \mid \exists\, i,\ 1)\ \partial M = \emptyset,\ 2)\ \tilde{\sigma} \subset M,\ 3)\ i^* g = h,\ 4)\ \varphi_{\tilde{\sigma}} = \phi \}.$$

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*Another definition of the wave function see in [7] .

3. The Density Matrix of the Universe

Hawking-Page density matrix is defined as

$$\rho[\Sigma', h', \phi'; \Sigma'', h'', \phi''] = \int_{\mathcal{B}} d[M, g, \varphi]\, e^{-I}, \quad (2)$$

where

$$\mathcal{B} = B[\Sigma' - \Sigma'', h'_{\Sigma'} + h''_{\Sigma''}, \phi'_{\Sigma'} + \phi''_{\Sigma''}].$$

It is interesting that the density matrix ρ can be represented via the wave function ψ. As it follows from (1) and (2), this can be done as follows:

$$\begin{aligned} \rho[\Sigma', h', \phi'; \Sigma'', h'', \phi''] &= \psi[\Sigma', h', \phi'; -\Sigma'', h'', \phi''] \\ &= \psi[\Sigma' - \Sigma'', h' + h'', \phi' + \phi''] . \end{aligned} \quad (3)$$

Then, the probability searched for must be equal to [4,5]

$$P[\Sigma, h, \phi] = \rho[\Sigma, h, \phi; \Sigma, h, \phi]$$

$$= \int_{B_g[\emptyset(\Sigma), h, \phi]} d[M, g, \varphi]\, e^{-I[M, g, \varphi]} .$$

As it is seen, integration is taken not only over manifolds divided into two parts by the 3-boundary, but also by all compact 4-manifolds containing the given 3-manifold: cf. $A[\Sigma, h, \phi]$ and $B_g[\emptyset(\Sigma), h, \phi]$.

This is the main difference between this formulation of probability and the former one. One can come to this conclusion also from other considerations [4]. Assume, that the Universe is created as different non-connected parts (pieces). Then, picking out one of them ("our") one has to integrate over other "unknown" parts.

Thus, assuming that

$$\Sigma = \Sigma_{our} + \Sigma_{unknown},$$

where Σ_{our} is supposed to be connected, one has

$$P(\Sigma_{our}, h_{our}, \phi_{our}) = \int d[\Sigma_{unknown}, h_{unknown}, \phi_{unknown}] \times$$
$$P[\Sigma_{our} + \Sigma_{unknown}, h_{our} + h_{unknown}, \phi_{our} + \phi_{unknown}]. \tag{4}$$

4. Topology Change of the Universe: A Toy Model

Let us turn now to the discussion of the problem which, as distinct of minisuperspace models, leads to results (probabilities) depending on whether pure or mixed states of the Universe are considered, i.e. the wave function or the density matrix is calculated.

Consider the probability of the topology change of the Universe without matter and with cosmological constant. Then 4-torus T^4 of metric

$$ds^2 = 8\pi \ell^2 L^2 \left[d\vartheta_0^2 + d\vartheta_1^2 + d\vartheta_2^2 + d\vartheta_3^2 \right], \; 0 \le \vartheta_i < 1, \quad (5)$$

is a compact gravitational instanton, i.e. satisfies the corresponding Einstein equations ($R_{ik} = 0$).

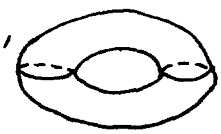
Proceeding from the fact that one can insert both 3-torus T^3 and 3-sphere S^3 into T^4 (of dimension $L \gg 1$), one can calculate the relative probabilities of sphere-torus ($S^3 \to T^3$) and sphere-sphere ($S^3 \to S^3$ with different radii) transitions using the action (for details see [2])

$$I = -\frac{L^2}{8\pi \ell^2} \int_{\partial M} d^3h \; h^{1/2} K ,$$

where K is the second fundamental form of the boundary of 4-manifold (3-sphere and 3-torus).

As it is shown in [2], if the Universe is in a pure state, from the probability of transition from Σ' state to Σ''

$$P_{HH}(\Sigma' \to \Sigma'') = \psi^2(\Sigma' \to \Sigma''); \quad \Sigma' \quad \Sigma'' ; \quad (6)$$



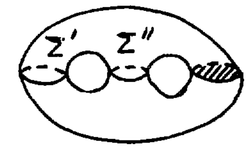
one has the expression

$$\frac{P_{HH}(S^3 \to T^3)}{P_{HH}(S^3 \to S^3)} \simeq e^{-16 L^4} \ll e^{-16} . \quad (7)$$

While deriving this, the volume of 3-torus was taken equal to that of the final 3-sphere. Thus, the transition with topology change is suppressed.

Now let us derive the same transition probabilities in the case when the mixed states are allowed. The transition probability will then be determined by the Hawking-Page density matrix:

$$P_{HP}(\Sigma' \to \Sigma'') = \rho(\Sigma'' - \Sigma'; \Sigma'' - \Sigma');$$



, (8)

or (the shaded region indicates integration over unknown topologies)

$$P_{HP}(\Sigma' \to \Sigma'') = \psi^2(\Sigma'' - \Sigma') + \tilde{\rho}(\Sigma'' - \Sigma')$$

$$= P_{HH}(\Sigma) + \tilde{\rho}(\bar{\Sigma}); \quad \bar{\Sigma} = \Sigma'' - \Sigma',$$

where

$$\tilde{\rho}(\bar{\Sigma}) \simeq \sum_{(M,g) \subset B_g[\phi(\Sigma), h]} e^{-I[M,g]} .$$

In the problem under consideration $M = T^4$ with metric (5) for which

$$I[T^4] = 0 ,$$

so far as $R = \Lambda = 0$ and $\partial M = \phi$.

Hence, the probability ratio searched for turns to

$$\frac{P_{HP}(S^3 \to T^3)}{P_{HP}(S^3 \to S^3)} = \frac{\exp[2(1 - 6\pi^2 R_0^2)L^2] + 1}{\exp[2(R_1^2 - R_0^2)6\pi^2 L^2] + 1}$$

or

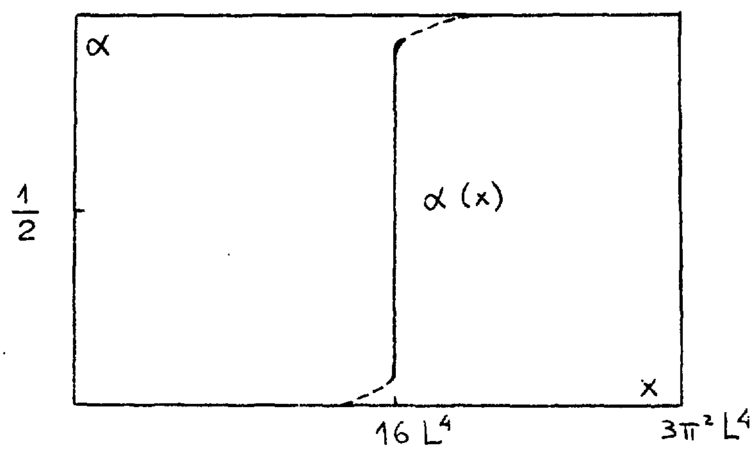
$$\frac{P_{HP}(S^3 \to T^3)}{P_{HP}(S^3 \to S^3)} = \frac{\exp(2L^2 - x) + 1}{\exp(16L^4 - x) + 1} \equiv \alpha(x) , \qquad (9)$$

where

$$12\pi^2 R_o^2 L^2 = x , \qquad 0 < x < 3\pi^2 L^4 ;$$

R_o and R_1 are the radii of the initial and final spheres.

The function $\alpha(x)$ increasing within the whole variation range of x is graphically shown as:



One can see that at $R_1 \leq R_o$, as distinct of the case of pure states, these transitions are equally probable. It is interesting, that at $R_1 = R_o$ $\alpha(x) = 0.5$, i.e. the probability of the sphere to remain unchanged is of the same order as that of turning to a torus (cf. with the Wheeler's ideas on the space-time foam [8]).

Thus, the considered problem on the topology change is an example allowing to differentiate the pure states of the Universe from the mixed ones.

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В.Г.ГУРЗАДЯН, А.А.КОЧАРЯН

ЧИСТЫЕ И СМЕШАННЫЕ СОСТОЯНИЯ ВСЕЛЕННОЙ

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Ереван 36, Маркаряна 2

The address for requests:
Information Department
Yerevan Physics Institute
Markaryan St., 2
Yerevan, 375036
Armenia, USSR

индекс 3624

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ