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ELECTROMAGNETIC WAVE GENERATION WITH HIGH TRANSFORMATION RATIO BY INTENSE CHARGED PARTICLE BUNCHES

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ՓՈԽԱԿԵՐՊՄԱՆ ԲԱՐՁՐ ԳՈՐԾԱԿՑՈՎ ԷԼԵԿՏՐԱՄԱԳՆԻՍԱԿԱՆ ԱԼԻՔՆԵՐԻ ՍԵՐՈՒՄԸ ԼԻՑՔԱՎՈՐՎԱԾ ՄԱՍՆԻԿՆԵՐԻ ԻՆՏԵՆՍԻՎ ԹԱՆՁՐՈՒԿՆԵՐՈՎ

Աշխատանքում ուսումնասիրվում է էներգիայի փոխակերպման բարձր գործակցով էլեկտրամագնիսական դաշտերի սերման հնարավորությունը գլխավոր թանձրուկից՝ արագացվելիքին։ Ստացված են գլխավոր /գեներատորային/ թանձրուկի քաշման փոխակերպման բարձր գործակիցներ տվող նոր տիպեր։ Գլխավոր թանձրուկն իրենից ներկայացնում է թանձրուկների հաջորդականություն, որը բարձրացնում է ինչպես կիլվատերային դաշտով արագացման այդպիսի համակարգի իրականացման հնարավորությունը, այնպես էլ փորձնական լիցքի արագացման տեմպը։

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E.M. LAZIEV, V.M. TSAKANOV, S.S. VAHANYAN

ELECTROMAGNETIC WAVE GENERATION WITH HIGH TRANSFORMATION RATIO BY INTENSE CHARGED PARTICLE BUNCHES

A possibility of electromagnetic wave generation with high ratio of energy transformation from the driving to the accelerated bunch is studied. New types of driving (generator) bunch distributions yielding high transformation ratio are obtained. Here the driving bunch represents a sequence of bunches, which enhances both realizability of such acceleration scheme by a wake field and acceleration rate of the test charge.

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ГЕНЕРАЦИЯ ЭЛЕКТРОМАГНИТНЫХ ВОЛН С ВЫСОКИМ КОЭФФИЦИЕНТОМ ТРАНСФОРМАЦИИ ИНТЕНСИВНЫМИ СГУСТКАМИ ЗАРЯЖЕННЫХ ЧАСТИЦ

В работе исследуется возможность генерации электромагнитных полей с высоким коэффициентом трансформации энергии от ведущего сгустка к ускоряемому. Получены новые типы распределений ведущего (генераторного) сгустка, дающие высокий коэффициент трансформации. При этом ведущий сгусток представляет собой последовательность сгустков, что повышает как реализуемость такой схемы ускорения кильватерным полем, так и темп ускорения пробного заряда.

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Charged particle acceleration in wake fields excited by intense relativistic bunches refers to a class of new methods allowing to have acceleration gradient above 100 MeV/m [1-3]. Clearly, such a scheme may be useful only in case it will allow to accelerate a particle up to energy higher than that of the driving (generator) bunch of particles. For this reason, a necessity arises as to excite electromagnetic waves with high transformation ratio defined as a ratio of maximally possible energy gain of the test charge to initial energy of particles in the driving bunch.

The purpose of this work is to study the structure of excited electromagnetic fields versus the longitudinal shape of the driving bunch in order to obtain high transformation ratio.

Let a point-like relativistic bunch of charged particles with a velocity V_z close to the velocity of light $c\,(V_z \approx c)$ at a time moment $t = 0$ enter some structure (a cavity, diaphragmatic waveguide, plasma) where it induces electromagnetic waves with electric field longitudinal component E_z. If we put $Z=0$ at the arrival point, then the energy acquired (or lost) by the test charge q moving along the same trajectory with a time delay τ will be

$$V_o(\tau) = q\int_0^{\infty} E_z\left(z, t = \tau + \frac{z}{c}\right) dz. \tag{1}$$

The Green function formalism allows to express the loss function $V(\tau)$ for arbitrary longitudinal distribution of driving bunch $P(\tau)$ through the loss function of point-like bunch

$$V(\tau) = \int_0^{\tau} P(\tau') V_o(\tau - \tau') d\tau'. \tag{2}$$

If we assume that the energy exchange process is lasting up to resting (going out of relativism) of the driving bunch particle undergoing maximal deceleration V^-_{min} , then the transformation ratio will be $K = -V^+_{max}/V^-_{min}$, where V^+_{max} is a maximally possible energy gain of the test charge.

An expression for the loss function $V(\tau)$ can be found using for searching for the excited electromagnetic fields the eigenmode expansion method for the case of usual accelerating structures or by solving the kinetic equation in linear approximation for the case of cold isotropic plasma [4] . In both cases a general expression $V_o(\tau)$ for the ultrarelativistic point-like driving bunch takes a relatively simple form:

$$V_o(\tau) = -\eta(\tau) \sum_n U_n \cos \omega_n \tau, \tag{3}$$

where $\eta(\tau) = 0,\ 1/2,\ 1$ at $\tau < 0$ (in front of the bunch), $\tau = 0$, $\tau > 0$ (behind the bunch), respectively; ω_n is frequency of the n-th excited mode; U_n are expansion coefficients determining a contribution of the n-th mode to the wake-wave field. In case of plasma, a single mode on plasma frequency ω_p is excited, and

$$E_z(\tau) = -2\eta(\tau)\frac{Q\omega_p^2}{c^2}K_0\left(\frac{\omega_p}{c}\tau\right)\cos\omega_p\tau, \tag{4}$$

where Q is total charge of the driving bunch, τ is the deviation from the trajectory, $K_0(x)$ is the Bessel function of the II kind [5] . Note, that in front of the bunch electromagnetic fields are absent due to the causality principle. In our further consideration of single-mode approximation we shall not specially mention the plasma case.

From (3), (4) one can readily see that the basic excited mode yields transformation equal to 2. This fact is known as the basic property of excited wake fields [6] . Let us show that with account of all excited modes the transformation ratio in the case of point-like driving bunch does not exceed 2. Indeed, if we introduce a parameter $\alpha = |Q/q|$, then the energy lost by the driving bunch will be $V_1 = \alpha\,|V_0(0)|$, while the energy acquired by the test charge will be $V_2 = V^+_{max} - \frac{1}{\alpha}|V_0(0)|$. Then according to the energy conservation law we have

$$V^+_{max} - \frac{1}{\alpha}\,|V_0(0)| \leqslant \alpha\,|V_0(0)| \tag{5}$$

whence $K \leqslant \frac{1}{\alpha}(1+\alpha^2)$. This relation should hold for all values of α , since the transformation ratio does not depend on charges of the driving and accelerated bunches. The right-hand side is minimal at $\alpha = 1$, so we obtain $K \leqslant 2$.

It can be shown [7,8] that for the symmetric driving bunch the single-mode approximation yields the transformation ratio also equal to 2. And this is achieved in case when maximal deceleration is experienced by average particle of the bunch. In particular, for a uniform bunch of duration T with account of the single excited mode, according to (2) we obtain:

$$V^-(\tau) = -\frac{U_0}{T\omega_0}\sin\omega_0\tau \qquad 0<\tau<T,$$
$$V^+(\tau) = -\frac{2U_0}{T\omega_0}\sin\frac{\omega_0 T}{2}\cos\omega_0\left(\tau-\frac{T}{2}\right) \qquad \tau>T. \tag{6}$$

Here $V^-(\tau)$ corresponds to the loss function inside the bunch, $V^+(\tau)$ - behind the bunch. Fig. 1 shows dependences of acceleration rate and transformation ratios on the bunch duration at the given number of particles in it. At $T=\frac{2\pi}{\omega_0}$ the bunch does not excite the considered mode, since all energy emitted on this mode of the forepart of the bunch is spent for acceleration of its back part. Maximal transformation ratio is 2 being achieved at $T\omega_0=\frac{\pi}{2}(1+4n)$. However, because of the bunch self-acceleration, of practical significance is the first peak (n=0), when the bunch duration is half as large as the period of the excited mode. In this case the acceleration rate falls off nearly twice. Although for real structures the account of all modes leads to a decrease in K (~ 1.5), conditions can be found at which it will be higher than 2. If frequencies of excited modes relate as $\omega_n=(2n+1)\omega_0$ and coefficients U_n coincide, then such structure will yield a transformation $K=\frac{8}{\pi}\left(1+\frac{1}{3}+\frac{1}{5}+\cdots\right)$ [9].

As an asymmetric bunch we'll consider a driving bunch with linear growth of current density $P(\tau)=\frac{2\tau}{T^2}$. For the loss functions we obtain:

$$V^-(\tau) = -\frac{2U_0}{T^2\omega_0^2}(1-\cos\omega_0\tau)$$
$$V^+(\tau) = \frac{2U_0}{T^2\omega_0^2}\left[T\omega_0\sin\omega_0(\tau-T)+\cos\omega_0\tau-\cos\omega_0(\tau-T)\right] \tag{7}$$

Fig. 2 presents dependences of acceleration rate and transformation ratio on bunch duration at a given number of particles in it. At $T=\frac{2\pi N}{\omega_0}$ (N is integer) a transformation ratio is $K=\pi N$. Schematically, the loss functions $V(\tau)$ at $N = 3$ are shown in Fig. 3 (solid line). One can see that the transformation ratio is proportional to the number of identical minima inside the bunch (in the limit $V^-(\tau)$ = const). Note, that for structures where contribution of higher modes is of the order of the principal mode, their account will lead to an increase in the transformation ratio owing to outcoming of loss functions inside the bunch on plateau (Fig. 3a, dotted line). However, for real structures where high modes are quenched it is not valid.

Thus, for single-mode structures we can formulate the following statement: electromagnetic waves with a maximal transformation ratio are generated by a driving bunch all particles of which lose the same energy.

Indeed, let two arbitrary point-like bunches Q_1 and Q_2 move along the same trajectory in the decelerating structure. Then the energy lost by particles of each bunch will be

$$V_1^- = U_0 ,$$
$$V_2^- = \frac{Q_2}{Q_1} U_0 + 2U_0 \cos \omega_0 \tau_0 = \alpha V_1^- \qquad (8)$$

While maximal energy acquired by the test charge will be

$$V^+_{max} = 2U_0 \sqrt{1 + \alpha^2 - 2\alpha \cos \omega_0 \tau_0} . \qquad (9)$$

For the transformation ratio we then have

$$K = \begin{cases} 2\sqrt{1 + \alpha^2 - 2\alpha \cos \omega_0 \tau_0} & \alpha < 1 \\ \frac{2}{\alpha} \sqrt{1 + \alpha^2 - 2\alpha \cos \omega_0 \tau_0} & \alpha > 1 \end{cases} \qquad (10)$$

Maximum in the right-hand side is achieved at $\alpha = 1$, here $K = 2\sqrt{1 + Q_2/Q_1}$. Note, that the ratio Q_2/Q_1 cannot exceed 3. We can show that this is valid for N+1 bunches if for N bunches this condition holds (N is any integer). In this case a transformation ratio is

$$K = 2\sqrt{1 + \sum_{n=2}^{N} \frac{Q_n}{Q_1}} , \qquad (11)$$

where Q_n is total charge of the n-th bunch.

Consider driving bunches whose particles would lose the same energy while flying through the decelerating structure.

From (2) it follows that there is no existent continuous driving bunch whose all particles would have lost one and the same energy. However the loss function inside such bunch can be parametrized as follows:

$$V^-(\tau) = V_0 (1 - e^{-\alpha\tau}) \qquad 0 \leq \tau < T \qquad (12)$$

At $\alpha \to \infty$, $V^{-}(\tau)$ is constant. Driving bunch distribution should then satisfy the integral equation:

$$V_0(1 - e^{-\alpha\tau}) = \int_0^{\tau} P(\tau') V_0(\tau - \tau') d\tau'. \qquad (13)$$

This equation can be solved by means of the Laplace transformation, so for the bunch distribution $P(\tau)$ we find [9] :

$$P(\tau) = P_0 [(\alpha^2 + \omega_0^2) e^{-\alpha\tau} + \omega_0^2(\alpha\tau - 1)] \quad 0 \leq \tau \leq T; \qquad (14)$$

In the limit $\alpha \to \infty$ such bunch yields a transformation $K \approx \sqrt{1 + (\omega_0 T)^2}$. The distribution (14) can be approximated by piece-linear distribution (Fig. 3b). The principal excited mode here yields a transformation $K = \sqrt{1 + (1 - \frac{\pi}{2} + \omega_0 T)^2}$.

For a sequence of N identical point-like driving bunches the condition of equality of particle energy losses holds if their sequence intervals τ_n satisfy the relation $\sin \omega_0 \tau_n = 1/\sqrt{n}$ (n = 1,2,...) (Fig. 4a). In this case a maximal transformation ratio $K = 2\sqrt{N}$ is achieved.

The driving bunch nonstandard distributions considered above offer certain difficulties in their production and acceleration. It is natural therefore to consider a sequence of N point-like bunches with increasing number of particles in them. Here particles of bunches lose the same energy when their recurrence period is $\tau_0 = \frac{\pi}{\omega_0}(2m + 1)$, and the number of particles in the n-th bunch is $N_n = N_1(2n - 1)$. Schematically, the loss functions are shown in Fig. 4b. The transformation ratio according to (11) is $K = 2N$.

The above-obtained formulae for transformation ratios contain various parameters of bunches. Fig. 5 presents dependences of the number of particles participating in wake-field generation on the transformation ratio at the given acceleration gradient. Worsening in the parameters of piece-linear bunch at small K is caused by the influence of the bunch forepart where particles lose different energies (Fig. 4). One can see that most optimal is the sequence of point-like bunches. The advantage of the bunch sequence is also that the total number of particles participating in energy exchange may be considerably increased with a reasonable number of particles left in each bunch.

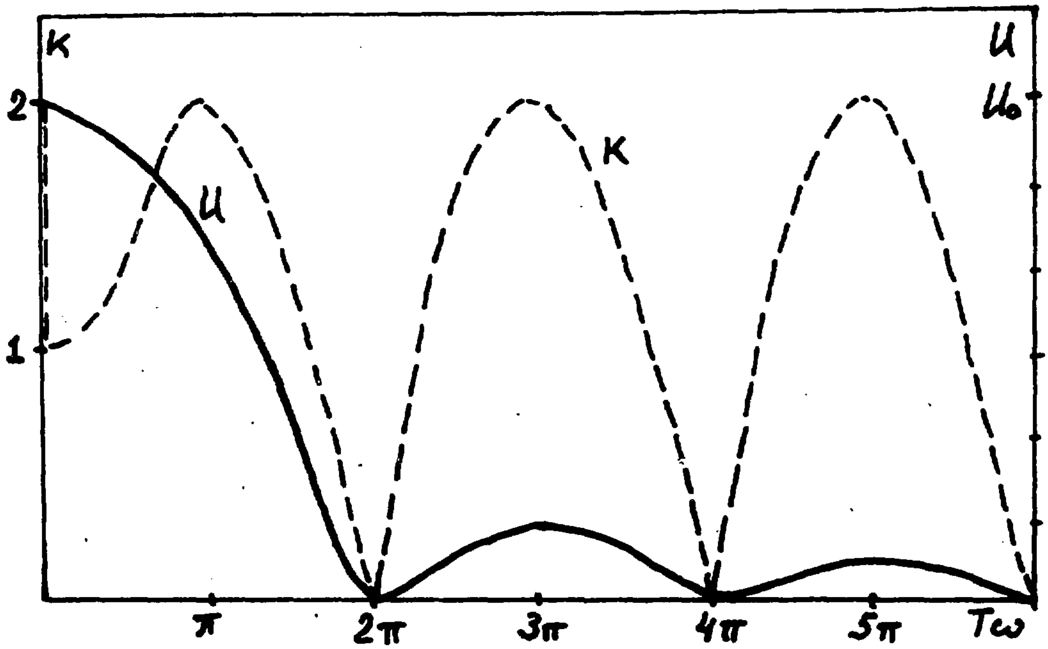

Fig.1

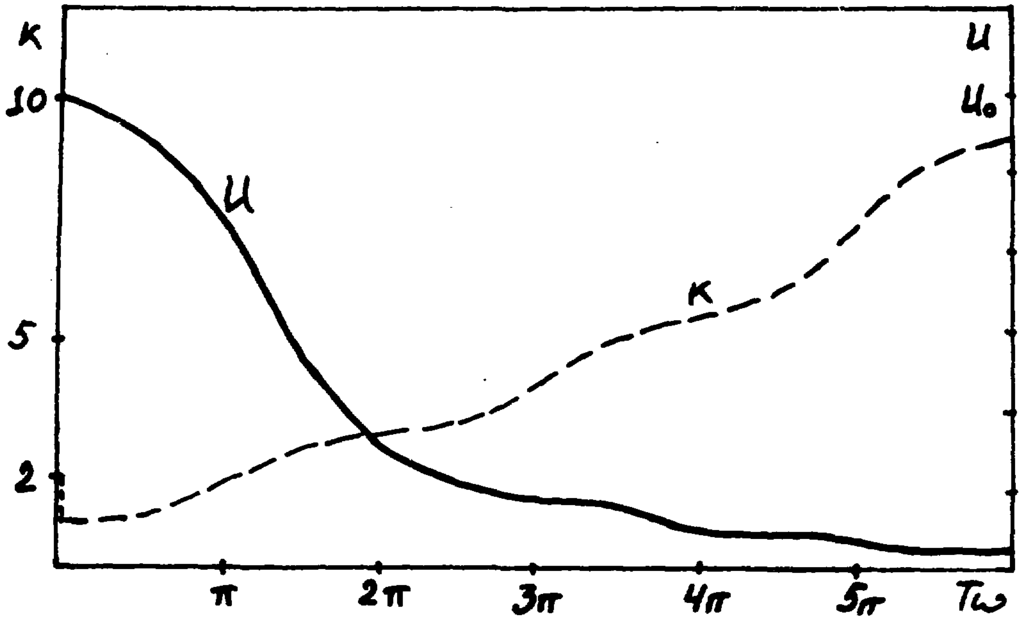

Fig.2

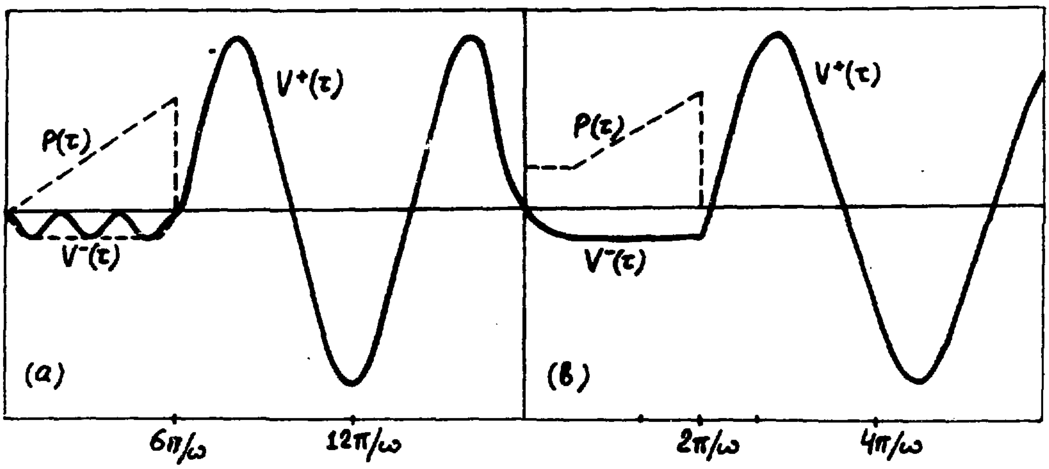

Fig.3

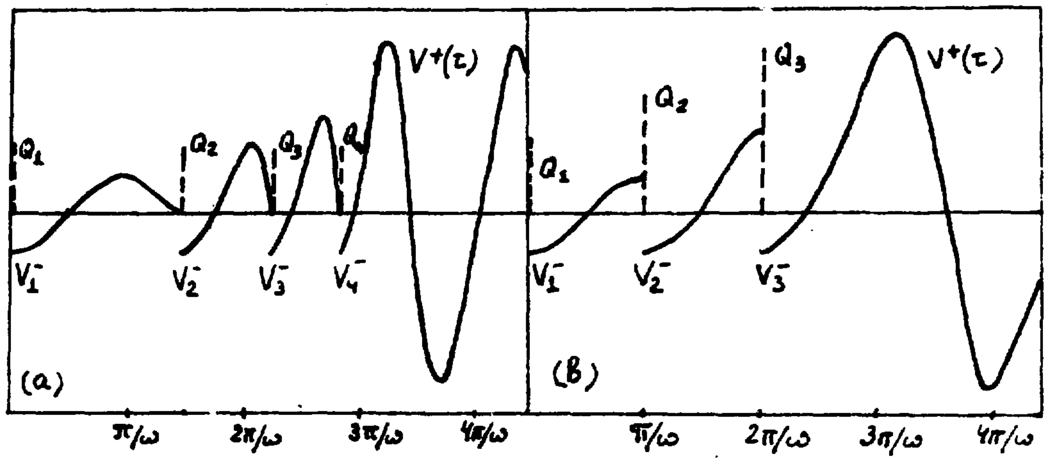
V+(τ)
Q1
Q2
Q3
Q4
V1-
V2-
V3-
V4-
(a)
π/ω
2π/ω
3π/ω
4π/ω
(b)

Fig.4

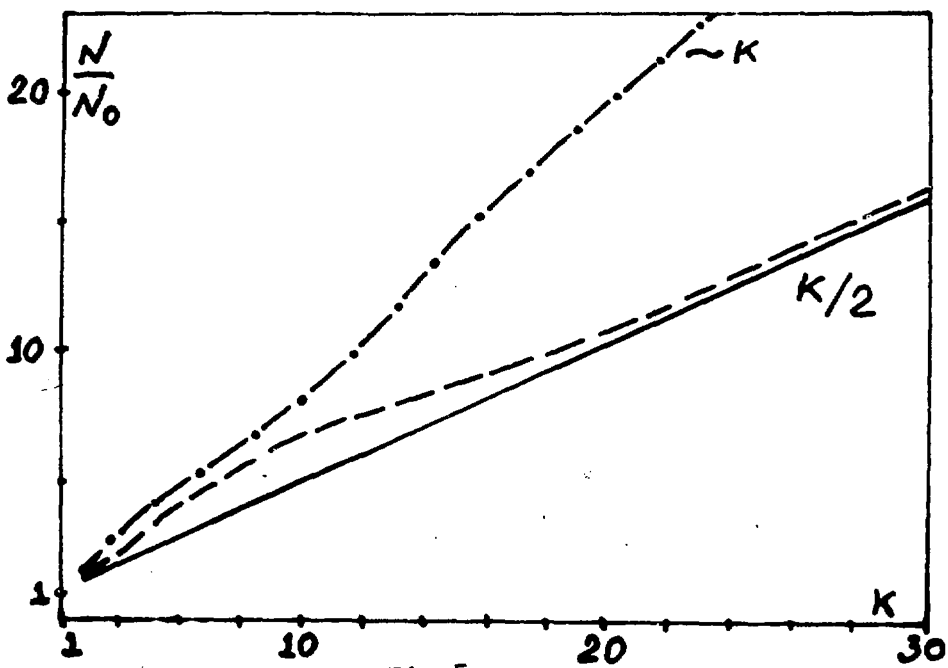
N/N0
20
10
1
~K
K/2
K
1
10
20
30

Fig.5

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ГЕНЕРАЦИЯ ЭЛЕКТРОМАГНИТНЫХ ВОЛН С ВЫСОКИМ КОЭФФИЦИЕНТОМ ТРАНСФОРМАЦИИ ИНТЕНСИВНЫМИ СГУСТКАМИ ЗАРЯЖЕННЫХ ЧАСТИЦ

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