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YEREVAN PHYSICS INSTITUTE

H.M.ASATRYAN, Z.G.BEREZHIANI, A.N.IOANNISSYAN

NEUTRINO MASSES AND MIXING IN SO(10) MODEL

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Г.М.АСАТРЯН, З.Г.БЕРЕЖИАНИ*,
А.Н.ИОАННИСЯН

МАССЫ И СМЕШИВАНИЯ НЕЙТРИНО В
SO(10) - МОДЕЛИ

Изучаются массы и смешивания нейтрино в SO(10) - модели с дополнительной дискретной симметрией в предположении, что фермионы получают массы за счет взаимодействия с 10 - плетом и 126 - плетом полей Хиггса. После учета радиационных поправок к массам правых нейтрино мы получаем три легких майорановских нейтрино с массами в районе $m_1 \approx m_2 \approx (10^{-3}-10^{1})$ эВ и $m_3 = (10^{3}-10^{6})$ эВ в зависимости от величины масштаба M_R нарушения $SU(2)_R$ - симметрии. Вычислены также углы смешивания в заряженном слабом лептонном токе.

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* Институт физики АН Груз.ССР

The proton decay experiments have shown a failure of a simplest grand unified model based on SU(5) group. Among other models the simplest is one based on SO(10) group. As distinct from other grand unified models, the fermion representation of SO(10) just like SU(5) includes only ordinary quarks and leptons as well as right neutrino. Because of the right component of neutrino in SO(10), a nonzero mass of neutrino occurs. The present paper is devoted to a study of neutrino mass and mixing in grand unified SO(10) model.

As was mentioned in Refs.[1-4] , a larger, compared to SU(5), lifetime of proton can be obtained only in those SO(10) breaking schemes which contain left-right intermediate symmetry $SU(2)_L \times SU(2)_R$. However, as shown in Ref.[5] , in this case a problem with the degeneration of heavy quark masses arises if we assume the existence of only one light (with mass $\sim M_W$) Higgs boson connected with breaking of electroweak group. In Ref.[6] an SO(10) model is suggested, where this problem was solved (see also [7]) via the introduction of additional discrete symmetry.

In this work we, on the basis of a model suggested in Ref.[6] and also in [8], shall study questions related to neutrino masses and mixing.

Just like in [6] , discrete symmetry in Higgs sector of theory has a form:

$$54 \longrightarrow 54 \qquad 126 \longrightarrow -1\ 126$$
$$210 \longrightarrow 210 \qquad 10 \longrightarrow 1\ 10 \qquad (1)$$
$$45 \longrightarrow -45$$

Vacuum expectation values (v.e.v.) of these fields violate SO(10) at several steps:

$$SO(10) \xrightarrow{<(1.1.1)_{54}> = M_X} SU(4) \times SU(2)_L \times SU(2)_R \times \hat{P}$$
$$\xrightarrow{<(1.1.1)_{210}> = M_P} SU(4) \times SU(2)_L \times SU(2)_R \equiv G \qquad (2)$$
$$\xrightarrow{<(15.1.1)_{45}> = M_C} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \equiv G_1$$
$$\xrightarrow{<(\overline{10}.1.3)_{126}> = M_R} SU(3)_c \times SU(2)_L \times U(1)_y$$
$$\xrightarrow{<(1.2.2)_{10}> \sim <(15.2.2)_{126}> \sim M_W} SU(3)_c \times U(1)em ,$$

where representation components are indicated according to decomposition over group G.

Further on, we shall assume that electroweak subgroup breaking goes owing to v.e.v. of SU(2) doublets contained in the representations 10 and 126: the use of only 10-plet would lead to equality of masses of charged leptons and down quarks.

In view of this, consider a mass matrix of $\Phi_{10} = (1,2,2)$ and $\Phi_{126} = (1,2,2)$ fields connected with breaking of $SU(2)_L \times U(1)_y$, where we took components of group G decomposition. Mass matrix invariant under G group has a form:

$$\mathcal{L}_M = M_1^2(\Phi_{10}^+ \Phi_{10} + \tilde{\Phi}_{10}^+ \tilde{\Phi}_{10}) + M_2^2 (\Phi_{126}^+ \Phi_{126} + \tilde{\Phi}_{126}^+ \tilde{\Phi}_{126}) +$$
$$M_3^2(\Phi_{10}^+ \tilde{\Phi}_{126} + \tilde{\Phi}_{10}^+ \Phi_{126}) + \text{э.c}$$

$$\Phi_{10} = \begin{pmatrix} \xi_{10}^0 & \eta_{10}^+ \\ \xi_{10}^- & -\eta_{10}^{0*} \end{pmatrix}, \qquad \tilde{\Phi}_{10} = \begin{pmatrix} \eta_{10}^0 & \xi_{10}^+ \\ \eta_{10}^- & -\xi_{10}^{0*} \end{pmatrix}, \qquad (3)$$

$$\Phi_{126} = \begin{pmatrix} \xi_{126}^0 & \eta_{126}^+ \\ \xi_{126}^- & -\eta_{126}^{0*} \end{pmatrix}, \qquad \tilde{\Phi}_{126} = \begin{pmatrix} \eta_{126}^0 & \xi_{126}^+ \\ \eta_{126}^- & -\xi_{126}^{0*} \end{pmatrix},$$

where $\xi_{10} \equiv \begin{pmatrix} \xi_{10}^0 \\ \xi_{10}^- \end{pmatrix}$, $\eta_{10} \equiv \begin{pmatrix} \eta_{10}^0 \\ \eta_{10}^- \end{pmatrix}$, $\xi_{126} \equiv \begin{pmatrix} \xi_{126}^0 \\ \xi_{126}^- \end{pmatrix}$, $\eta_{126} \equiv \begin{pmatrix} \eta_{126}^0 \\ \eta_{126}^- \end{pmatrix}$ are usual $SU(2)_L$ doublets.

Diagonalizing the mass matrix (3) we'll obtain two orthogonal states:

$$\Phi_1 = \cos\varphi\, \Phi_{10} + \sin\varphi\, \tilde{\Phi}_{126}$$
$$\Phi_2 = -\sin\varphi\, \Phi_{10} + \cos\varphi\, \tilde{\Phi}_{126} \qquad (4)$$

these both, generally speaking, must have mass of an order of magnitude M_X (with account of the fact that M_C is close to M_X [6]). Actually, owing to the fact that one of these fields is connected with $SU(2)_L \times U(1)_y$ breaking, then, according to the fine tuning condition, this field (say, Φ_1) must have a mass $\ll M_X$, i.e. M_R . Then obviously, the only doublet of SU(2) whose v.e.v. is connected with electroweak group breaking is contained namely in field Φ_1 (with an accuracy up to M_R^2/M_X^2). Thus, one may consider a mass matrix of doublets which enter Φ_1 separately. With account of terms breaking $SU(2)_R$ -symmetry the mass term for fields ξ, η takes the form:

$$(\alpha M_R^2 + \gamma M_R^2)\xi_1^* \xi_1 + (\alpha M_R^2 - \gamma M_R^2)\eta_1^* \eta_1 \tag{5}$$

Just like in Ref.[6] , when we take account of radiative corrections, there arises a nondiagonal over ξ_1, η_1 term which is connected with discrete symmetry breaking. As a result, we'll come to the following contribution:

$$\frac{\lambda \gamma M_C}{M_X} M_R^2 (\xi_1^* \eta_1 + \eta_1^* \xi_1) \tag{6}$$

Diagonalizing (5), (6), we'll obtain two new orthogonal states ξ, η . One of these states must have a mass $\sim M_W$, namely its v.e.v. is connected with $SU(2)_L \times U(1)_y$ breaking; the other field in accordance with the survival hypothesis for Higgs fields must have mass $\sim M_R$ and zero v.e.v. In what follows we'll believe that Higgs field is ξ - this may always be attained by the choice of sign of γ . Then

$$\begin{aligned} \xi_1 &= \xi \cos\theta - \eta \sin\theta \\ \eta_1 &= \xi \sin\theta + \eta \cos\theta , \qquad \mathrm{tg}\, 2\theta = \lambda \frac{M_C}{M_X} \end{aligned} \tag{7}$$

Turn now to fermion masses. Just like in [8] , we'll determine the action of discrete symmetry for three generations of

fermions as follows:

$$16_{1,3} \to e^{-i\frac{\pi}{4}}\, 16_{1,3} \qquad\qquad 16_2 \to -e^{-i\frac{\pi}{4}}\, 16_2 . \tag{8}$$

Then the Yukawa couplings of fermions with Higgs fields take the form:

$$\sum_{i,j}^{3} 16_i\, 16_j\, (10\,\lambda_{ij} + 126\mu_{ij} + \widetilde{10}\mu'_{ij}) \tag{9}$$

where

$$\lambda_{ij} = \begin{pmatrix} \lambda_1 & 0 & \lambda_4 \\ 0 & \lambda_2 & 0 \\ \lambda_4 & 0 & \lambda_3 \end{pmatrix}, \quad \mu_{ij} = \begin{pmatrix} 0 & \mu_1 & 0 \\ \mu_1 & 0 & \mu_2 \\ 0 & \mu_2 & 0 \end{pmatrix}, \quad \mu'_{ij} = \begin{pmatrix} 0 & \mu'_1 & 0 \\ \mu'_1 & 0 & \mu'_2 \\ 0 & \mu'_2 & 0 \end{pmatrix}$$

With account of (4), (7) we'll obtain mass matrices of fermions in the following form:

$$\begin{aligned} \mathcal{M}_u &= [\lambda_{ij} + \mathrm{tg}\,\theta(\mu_{ij}\,\mathrm{tg}\,\varphi + \mu'_{ij})]\cos\varphi\cos\theta < \xi > \\ \mathcal{M}_d &= -[\lambda_{ij}\,\mathrm{tg}\,\theta + (\mu_{ij}\,\mathrm{tg}\,\varphi + \mu'_{ij})]\cos\varphi\cos\theta < \xi^* > \\ \mathcal{M}_e &= -[\lambda_{ij}\,\mathrm{tg}\,\theta + (-3\mu_{ij}\,\mathrm{tg}\,\varphi + \mu'_{ij})]\cos\varphi\cos\theta < \xi^* > \\ \mathcal{M}_\nu^D &= [\lambda_{ij} + \mathrm{tg}\,\theta(-3\mu_{ij}\,\mathrm{tg}\,\varphi + \mu'_{ij})\}\cos\varphi\cos\theta < \xi > \end{aligned} \tag{10}$$

With respect to the smallness of parameter $\mathrm{tg}\,\theta$ these matrices can be reduced to the form:

$$\mathcal{M}_u = \begin{pmatrix} u & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & t \end{pmatrix} + tg\theta \begin{pmatrix} c & a_1 & 0 \\ a_1 & c & a_2 \\ 0 & a_2 & 0 \end{pmatrix},$$

$$\mathcal{M}_d = \begin{pmatrix} u & 0 & c \\ 0 & c & 0 \\ 0 & 0 & t \end{pmatrix} tg\theta + \begin{pmatrix} 0 & a_1 & 0 \\ a_1 & 0 & a_2 \\ 0 & a_2 & 0 \end{pmatrix},$$

$$\mathcal{M}_e = \begin{pmatrix} u & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & t \end{pmatrix} tg\theta + \begin{pmatrix} 0 & b_1 & 0 \\ b_1 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}, \qquad (11)$$

$$\mathcal{M}_\nu^D = \begin{pmatrix} u & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & t \end{pmatrix} + tg\theta \begin{pmatrix} 0 & b_1 & 0 \\ b_1 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}.$$

As a result, we'll come to the same findings for quark masses and mixings as in Ref. [8] , in particular, to relation $m_b/m_t = tg\vartheta$, this leading to the fact that M_e must be close to M_X [8] . Besides, we'll obtain new results connected with lepton masses and mixings.

Parameters a_1, a_2, b_1, b_2 which enter (11) can be expressed through masses of quarks and leptons and unknown phases [8] . For simplicity we'll consider mass matrices to be real. Then we'll obtain:

$$a_1^2 = ds\left(1 \mp \frac{ub}{dt} \pm \frac{s}{b} + \frac{c}{t}\right), \quad b_1^2 = e\mu\left(1 - \frac{u\tau}{et} + \frac{\mu}{\tau} + \frac{c}{t}\right) \qquad (12)$$

$$a_2^2 = sb\left(\pm 1 + \frac{bc}{st} + 2\frac{c}{t} + 2\frac{b}{s}\frac{c^2}{t^2}\right), \quad b_2^2 = \mu\tau\left(1 + \frac{\tau c}{\mu t} + 2\frac{c}{t} + 2\frac{\tau}{\mu}\frac{c^2}{t^2}\right)$$

$$tg\,\theta = \frac{\pm d \mp s + b}{u + c + t}$$

Turn now to a more detailed study of neutrino mass and mixing. In addition to Dirac masses (11), neutrino may obtain Majorana mass. In our case right neutrino receives Majorana mass owing to Yukawa coupling $\mu_{ij}\, 126 \cdot 16_i \cdot 16_j$ from (9), when v.e.v. ($\sim M_R$) of 126-plet breaks (B-L)-symmetry:

$$\mathcal{M}_\nu^R = \mu_{ij} M_R \qquad (13)$$

Then the mass matrix of neutrino takes the form:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & t \\ u & 0 & 0 & 0 & \mu_1 M_R & 0 \\ 0 & c & 0 & \mu_1 M_R & 0 & \mu_2 M_R \\ 0 & 0 & t & 0 & \mu_2 M_R & 0 \end{pmatrix} \qquad (14)$$

Diagonalizing this matrix we obtain three Dirac neutrinos with masses $uc/\mu_1 M_R$, $t\mu_1/\mu_2$ and $\mu_2 M_R$. Mass of τ-neutrino comes out inapplicably large: $\sim m_t$ ($m_{\nu_\tau exp} < 50$ MeV). This circumstance as well as the fact of appearance of Dirac and not Majorana neutrino in SO(10) scheme are due to the fact that the mass matrix of right neutrino (13) is degenerated. The situation becomes quite different after the account of radiative corrections to the mass matrix of right neutrino. A simplest diagram that makes contribution to masses of right neutrino is given in Fig.1. With respect to this diagram contribution for the mass matrix of right neutrino we obtain:

$$\mathcal{M}'_{\nu_R} = \mu_{ij} M_R + \lambda_{ij} \alpha \frac{\langle \xi_{10} \rangle}{\langle \eta_{126} \rangle} M_R \,, \tag{15}$$

where

$$\alpha \simeq \alpha^2(M_X) \frac{\langle \eta_{126} \rangle}{\langle \xi_{10} \rangle} \varepsilon \approx 10^{-4}$$

$\varepsilon \approx 10^{-1}$ is mixing factor of colored triplets from <u>10</u> and <u>210</u> [9] .

After taking account of radiative corrections we'll have 6 Majorana states, 3 of which are light and 3 are heavy.

The mass matrix of light Majorana neutrino will take the form:

$$\mathcal{M}_\nu = \mathcal{M}^{D}_\nu (\mathcal{M}_{\nu_R})^{-1} \mathcal{M}^{D}_\nu = \frac{\langle \eta_{126} \rangle}{M_R} \left[\alpha^2 uct - \frac{1}{16}\alpha\left[u(a_2-b_2)^2 + t(a_1-b_1)^2\right]\right] \times$$

$$\times \begin{pmatrix} \alpha^2 u^2 ct - \frac{u^2}{16}(a_2-b_2)^2 & -\alpha uct \frac{1}{4}(a_1-b_1) & ut\frac{1}{16}(a_1-b_1)(a_2-b_2) \\ -\alpha uct \frac{1}{4}(a_1-b_1) & \alpha^2 uc^2 t & -\alpha uct \frac{1}{4}(a_2-b_2) \\ ut\frac{1}{16}(a_1-b_1)(a_2-b_2) & -\alpha uct \frac{1}{4}(a_2-b_2) & \alpha^2 uct^2 - t^2 \frac{1}{16}(a_1-b_1)^2 \end{pmatrix} \tag{16}$$

After diagonalization of this mass matrix we obtain three Majorana states with masses:

$$m_1 \simeq m_2 = \frac{\langle \eta_{126} \rangle}{M_R} \frac{4uc}{|a_1-b_1|} = \left(\frac{\langle \eta_{126} \rangle}{10^2\,\text{ГэВ}}\right)\left(\frac{10^{10}\,\text{ГэВ}}{M_R}\right)(1-20)\,\text{эВ} \simeq 20 - 10^{-3}\,\text{эВ}$$

$$|m_1 - m_2| = m_1 \frac{4\alpha ct\,(a_1-b_1)}{t(a_1-b_1)^2 + u(a_2-b_2)^2} = \left(\frac{\langle \eta_{126} \rangle}{100\,\text{ГэВ}}\right)\left(\frac{10^{10}\,\text{ГэВ}}{M_R}\right)(10^{-2} - 1.5)\,\text{эВ} \tag{17}$$

$$m_3 = \frac{\langle \eta_{126} \rangle}{\alpha M_R} t = \left(\frac{\langle \eta_{126} \rangle}{100\,\text{ГэВ}}\right)\left(\frac{10^{10}\,\text{ГэВ}}{M_R}\right)\left(\frac{t}{50\,\text{ГэВ}}\right) 2\,\text{МэВ}$$

if M_R varies within the limits 10^{10}-10^{13} GeV.

Mixing angles in weak charged lepton current are

$$\begin{aligned} \sin\theta_1 &\simeq \frac{1}{\sqrt{2}} \\ \sin\theta_2 &= \frac{1}{\sqrt{2}}\left[\frac{u}{t}\,\frac{(a_2-b_2)}{(a_1-b_1)} + \frac{b_1 b_2}{\tau^2 \frac{c}{t} - b_2^2}\right] \\ \sin\theta_3 &= \frac{u}{t}\,\frac{(a_2-b_2)}{(a_1-b_1)} - \frac{b_2}{\tau} \end{aligned} \tag{18}$$

Whence, with respect to (12) we obtain

$$\begin{aligned} \sin\theta_1 &\simeq \frac{1}{\sqrt{2}} \\ |\sin\theta_2| &\leq 0.015 \\ |\sin\theta_3| &\leq 0.3 \end{aligned} \tag{19}$$

Thus, mixing between two first lepton generations in weak charged current is maximal (45°).

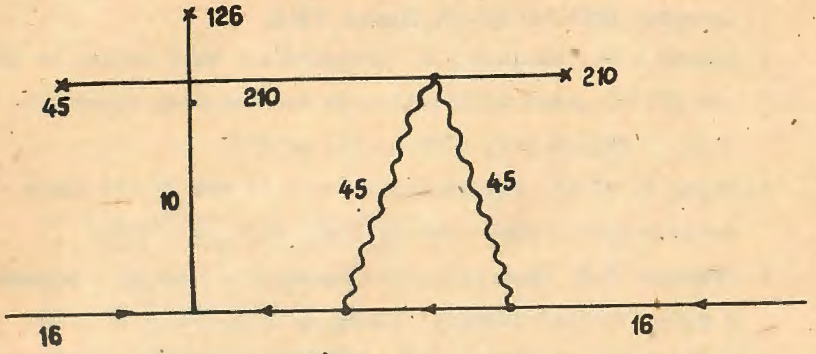
The obtained results do not contradict the experiments on neutrino oscillation detection - the difference in masses among the first two generations of neutrino is very small ($\Delta m_{12} \ll$ 1 eV), and the third generation is mixed weakly with the first two ones ($\sin\theta_2$, $\sin\theta_3 \ll 1$) [10].

Thus, the obtained results agree well with those on measurement of neutrino masses and mixing.

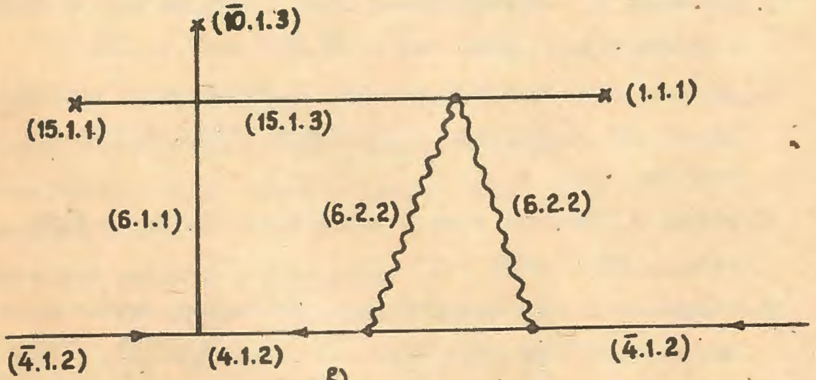
In conclusion the authors (H.M.A. and A.N.I.) would like to express their sincere gratitude to S.G. Matinyan for the useful discussion.

Figure Caption

Fig.1. Diagram that contributes into the mass matrix of right neutrino.
a) in terms of field-representations SO(10)
b) in terms of field-representations $SU(4) \times SU(2)_L \times SU(2)_R$.



a)



б)

REFERENCES

1. Toza J., Branko G.C., Marshak R.E. Constraints on proton lifetime and $\sin^2\theta_W$ in SO(10) grand unified theory. - Phys.Rev.D, 1983, v.28, p.1731.
2. Асатрян Г.М.,Иоаннисян А.Н. Время жизни протона в модели великого объединения SO(10) .ЯФ,1985,т.41,с.1097. Препринт ЕФИ-726(46)-84,Ереван,1984.
3. Gipson J.M., Marshak R.E. Intermediate mass scales in the new SO(10) grand unification in the one-loop approximation. - Phys.Rev.D, 1985, v.31, p.1705.
4. Chang D. et al. Experimental tests of new SO(10) grand unification. - Phys. Rev.D, 1985, v.31, p. 1718.
5. Световой В.Б. Промежуточная симметрия в SO(10) модели и массы тяжёлых кварков. Письма в ЖЭТФ,1983,т.37,с.49.
6. Асатрян Г.М.,Иоаннисян А.Н. Иерархия масштабов нарушения в теории великого объединения SO(10) и массы частиц. ЯФ,1987,т.45. Препринт ЕФИ -883(34)-86, Ереван, 1986.
7. Бережиани З.Г. Нестандартная схема генерации масс в SO(10) и промежуточная правая шкала. ЯФ,1985,т.42,с.1308.
8. Asatryan H.M. Symmetry breaking in SO(10) grand unified theory and quark masses. - Preprint YERPHI-961(11)-87, Yerevan, 1987.
9. Witten E. Neutrino mass in the minimal SO(10). - Phys. Lett., 1980, v.91B, p.81.
10. Vergados J.D. The neutrino mass and family, lepton and baryon number nonconservation in gauge theories. - Phys. Rep., 1986, v.133, N.1-2.

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The address for requests:
Information Department
Yerevan Physics Institute
Markaryan St., 2
Yerevan, 375036
Armenia, USSR

.М.АСАТРЯН, З.Г.БЕРЕЖИАНИ, А.Н.ИОАННИСЯН

МАССЫ И СМЕШИВАНИЯ НЕЙТРИНО В SO(10) - МОДЕЛИ

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