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UNITARITY PROBLEM FOR BACKGROUND SOLUTIONS OF PREGEOMETRICAL STRING FIELD THEORY

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ՈՒՆԻՏԱՐՈՒԹՅԱՆ ԽՆԴԻՐԸ ՆԱԽԱԵՐԿՐԱՉԱՓԱԿԱՆ ԼԱՐԵՐԻ ԴԱՇՏԻ ՏԵՍՈՒԹՅԱՆ ՓՈՆԱՅԻՆ ԼՈՒԾՈՒՄՆԵՐԻ ՀԱՄԱՐ

Ցույց է տրված, որ ունիտարության պայմանը BRST օպերատորի համար չի հետևում ընդհանուր սկզբունքներից, և առաջարկված է մի վարկած, ըստ որի այդ պայմանի բավարարումը յուրահատուկ է նախաերկրաչափական լարերի դաշտի տեսության համար: Վերջավոր չափողականություն ունեցող մոդել է առաջարկված քննարկվող տեսության համար, գտնված են նրա դասական լուծումները: Ունիտարության պայմանը բավարարվում է ոչ բոլոր լուծումների համար: Ֆիզիկական վեկտորները նորովի մեկնաբանված են որպես տեղափոխության գեներատորներ՝ դասական նախաերկրաչափական հավասարումների լուծումների տարածությունում:

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ПРОБЛЕМА УНИТАРНОСТИ ДЛЯ ФОНОВЫХ РЕШЕНИЙ ПРЕГЕОМЕТРИЧЕСКОЙ ТЕОРИИ СТРУН

Присоединенное действие решений классических уравнений прегеометрической теории струн должно задавать операторы BRST [1,2]. В работе отмечено, что условие унитарности теории-неотрицательной определенности скалярного произведения в ядре оператора BRST - не следует из общих соображений, и высказана гипотеза, что выполнение этого условия для произвольных решений прегеометрической теории является характеристическим свойством супералгебры Ли теории поля струн. Предложена конечномерная модель теории поля замкнутых струн [1], в ней явно найдены решения классических уравнений, и показано, что условие унитарности в этой модели выполняется не для всех решений. Дана также новая интерпретация физических векторов теории поля струн как генераторов сдвигов в пространстве решений уравнений прегеометрической теории.

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UNITARITY PROBLEM FOR BACKGROUND SOLUTIONS OF PREGEOMETRICAL STRING FIELD THEORY

The adjoined action of classical equations solutions in pregeometrical string theory gives the BRST operators [1,2] . I is shown that the condition of unitarity - non-negativity of scalar product in kernel of BRST operator - does not follow from general considerations, and a hypothesis is suggested that the fulfilment of this condition for arbitrary solutions of pregeometrical theory is a characteristic property of Lie superalgebra for string field theory. A finite-dimensional model of closed string field theory is proposed; the classical equations of motion are solved and it is shown that the unitarity property is satisfied not for all the solutions. The new interpretation of physical vectors as generators of displacements in the space of the solutions of classical pregeometrical equation is given.

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Introduction

One of the main problems in the theory of strings is the problem of construction of strings field theory. The simplest and adequate formulation is achieved in the so-called pregeometrical string field theory in which one doesn't assume a priori the existence of definite space-time. The pregeometrical theory, based on the field theory of closed strings, was constructed in [1] , and for open string - in [2] .

The pregeometrical theory of [1] is based on the closed field theory of [3] (correspondingly, the work [2] uses [4]). The main object of [3] is Φ - the vector of the Hilbert space H , arising in the BRST - quantization of closed string. H is graded according to the ghost number, and Φ must have the ghost number (-1). The main algebraic properties of theory [3] are the following. In the space H there exist (odd) BRST - charge operator Q , odd bilinear form (scalar product) (,) and operation $*$ which maps $H \times H \to H$.

Action of closed strings field theory constructed in the flat space-time is [3] :

$$S = (\Phi, Q\Phi) + \frac{2}{3} g (\Phi, \Phi * \Phi) \tag{1}$$

The action is invariant with respect to the gauge transformations:

$$\delta\Phi = Q\Lambda + 2g\Phi * \Lambda \tag{2}$$

due to the properties [1,3] :

$$\Phi * \Psi = (-1)^{|\Phi||\Psi|+1} \Psi * \Phi \tag{3}$$

$$\begin{aligned} &\Phi_1 * (\Phi_2 * \Phi_3) + (-1)^{|1|(|2|+|3|)} \Phi_2 * (\Phi_3 * \Phi_1) + \\ &+ (-1)^{|3|(|1|+|2|)} \Phi_3 * (\Phi_1 * \Phi_2) = 0 \end{aligned} \tag{4}$$

$$Q(\Phi * \Psi) = Q\Phi * \Psi + \Phi * Q\Psi (-1)^{|\Phi|} \tag{5}$$

$$(Q\Phi, \Psi) = (-1)^{1+|\Phi|} (\Phi, Q\Psi) \tag{6}$$

$|\Phi|$ in the $(-1)^{|\Phi|}$ means the parity of Φ .

Properties (3), (4) imply that H is the (infinite-dimensional) Lie superalgebra with respect to the operation $*$; in particular, (4) is the Jacobi identity, (5) means that Q is derivation in this superalgebra, (6) means that odd scalar product (,) is invariant.

The property (5) of Q means that the latter is derivation. Then a natural question arises: whether Q is internal derivation? If yes, then the action of Q may be represented as the adjoined action of some element Ψ_0 of considered Lie superalgebra. Ψ_0 must be odd vector due to the odd parity of Q. Nilpotency of Q gives the equation $\Psi_0 * \Psi_0 = 0$ on Ψ_0. These conditions naturally arise in equations of motion of pregeometrical theory based on the action [3] :

$$S = \frac{2}{3g} (\Phi, \Phi * \Phi) \tag{7}$$

The action (1) may be interpreted now as (7) with substitution

$$\Phi = \Psi_0 + g\,\Phi_{qu} \tag{8}$$

where Ψ_0 is the solution of the equations of motion of (7) which are

$$\Psi_0 * \Psi_0 = 0 \tag{9}$$

The (7) with substitution (8) becomes

$$S = 2(\Phi_{qu}, \Psi_0 * \Phi_{qu}) + \frac{2}{3} g (\Phi_{qu}, \Phi_{qu} * \Phi_{qu})$$

It remains to check that the Ψ_0 satisfying (9) and such that $Q = 2\Psi_0 *$, exists. This is proved, up to some subtleties, in [1] . The action (7) differs from (1) essentially in the same way as the action of general relativity $\sqrt{g}\,R$ from the same action, but written with metrics $g_{\mu\nu}$ in the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is flat Minkowski metric. Choosing other solutions of (9) one gets the other operators $Q = 2\Psi_0 *$, among which, probably, there are all the BRST operators arising in the quantization of string in the external gravitational and other fields, satisfying the equations of motion of string. It may happen that (9) has other, exotic solutions having more subtle physical interpretation. In all cases, it is worth to study the properties of the space of solutions of (9). In Sect. 1 we discuss the properties of the solutions of (9) near the fixed solution Ψ_0 . We prove that if we have one-parameter family of (not gauge-equivalent) solutions near Ψ_0 , then the small deviations of this family from Ψ_0 are given by the physical states of corresponding BRST operator $Q = 2\Psi_0 *$, i.e. by the elements of ker Q/ImQ .

The other problem related to (9) is the unitarity problem. It is well-known that the most important property of any BRST operator Q is the fact that the physical subspace ker Q/ImG has positive norm, which means that the S-matrix is unitary on this subspace. For any BRST operator Q arising in the process of quantization of physical system this property is satisfied due to the Fradkin-Vilcovisky theorem [5] . But general operator $Q = 2\Psi_0 *$, which pretends to be BRST operator, arises in a completely different way, and the unitarity property for it a priori may not be satisfied. One may suggest that all the solutions of (9) possess it, and that this fact is a characteristic property of string field theory Lie superalgebra.

In Sect. 3 we have constructed finite-dimensional model of string field theory, possessing many of its algebraic properties.

1. Let Ψ_0 be the solution of (9), i.e.

$$\Psi_0 * \Psi_0 = 0$$

Suppose also that there exists one-parameter family of not gauge-equivalent solutions of (9) with properties

$$\Psi(t) * \Psi(t) = 0, \quad \Psi(0) = \Psi_0 , \quad t \in R$$

Then, expanding $\Psi(t)$ near $t = 0$, we have

$$(\Psi_0 + t\Delta\Psi_1 + \frac{t^2}{2}\Delta\Psi_2 + \dots) * (\Psi_0 + t\Delta\Psi_1 + \frac{t^2}{2}\Delta\Psi_2 + \dots) = 0$$

which leads to equations

$$\Psi_0 * \Delta\Psi_1 = 0$$

$$\Delta\Psi_1 * \Delta\Psi_1 + \Psi_0 * \Delta\Psi_2 = 0$$

. . . .

Defining $Q = 2\Psi_0 *$, we rewrite these equations as

$$Q\Delta\Psi_1 = 0 \tag{10}$$

$$Q\Delta\Psi_2 = -\Delta\Psi_1 * \Delta\Psi_1 \tag{11}$$

The trivial solutions of (10) - $\Delta\Psi_1 = Q\Lambda$ - lead to gauge-transformed Ψ_0 solution. So, nontrivial solutions are in one-to-one correspondence to vectors from physical space ker $Q/\mathrm{Im}\,Q$. This is the main result of this analysis.

The second equation - (11) - means that the square of physical vector must be a null-vector, gauge-equivalent to zero. The problem of finding solutions of (11) is not solved as yet.

2. Let's consider now the unitarity problem for the background solutions of pregeometrical string field theory. Hermiticity and nilpotency of Q don't provide, in general, the unitarity of theory. The last property means that physical space ker $Q/\mathrm{Im}\,Q$ has positive metric. The space ker Q has only non-negative metric.

Since in the pregeometrical approach we get operators Q, which a priori have only the properties of hermiticity and nilpotency, the positivity of the metric in the space ker $Q/\mathrm{Im}Q$ is not guaranteed. This is the unitarity problem for pregeometrical string field theory. Evidently, most satisfactory would be a situation when all the solutions of (9) give the operators $Q = 2\Psi_0 *$ which satisfy the above-mentioned unitarity property.

Presumably, it is the characteristic property of string field theory superalgebra. However, it may also happen that one has to impose some additional conditions on the solutions of (9) to obtain the unitarity solutions only.

The necessary and sufficient condition for non-negative definiteness of metric of ker Q is given in [8]. It is the following: ker Q has non-negative norm iff $\ker_0 Q$ - the set of null-vectors from ker Q is linear space. The simple proof is given in [8]. We have failed to apply this criterion to pregeometrical string field theory [1], and we shall consider the problem in Sect. 3 on a finite-dimensional model.

3. The finite-dimensional model of closed string field theory discussed in this section possesses the main features of original theory. The main idea is to replace the infinite-dimensional Lie superalgebra by the finite-dimensional one (denote it A) with simultaneous change of Z -grading by Z_2 -grading.

According to the properties of original superalgebra, A must have invariant odd scalar product (,). Also, A is real superalgebra, the functionals Φ now will be odd elements of A. All these properties exist in one of the real forms of simple complex Lie superalgebra $Q(n)$ [6] ; that real form has the following realization. The original functionals Φ can be expanded in ghosts zero mode c_0 : $\Phi = (A_0 + A_1) + c_0(B_0 + B_1)$ (indices 0 and 1 mean even and odd parity, respectively). We initiate Φ and this expansion by the matrices:

$$\Phi = \begin{pmatrix} A_0 + A_1 , & i(B_0 + B_1) \\ B_0 - B_1 , & A_0 - A_1 \end{pmatrix} \equiv (A, B) \qquad (12)$$

The Z_2-grading is defined as

$$\deg \begin{pmatrix} A_0 , & iB_1 \\ -B_1 , & A_0 \end{pmatrix} = 0 , \quad \deg \begin{pmatrix} A_1 , & iB_0 \\ B_0 , & -A_1 \end{pmatrix} = 1$$

Matrices A_0 , B_0 have the form

$$\begin{pmatrix} \alpha , & 0 \\ 0 , & \gamma \end{pmatrix} \qquad \begin{matrix} \mathrm{tr}\,\alpha = \mathrm{tr}\,\gamma \\ \alpha^+ = -\alpha , \ \gamma^+ = -\gamma \end{matrix} \qquad (13)$$

A_1 , B_1 are:

$$\begin{pmatrix} 0, & \beta \\ i\beta^+, & 0 \end{pmatrix} \qquad (14)$$

The matrices of the form (13), (14) give the superalgebra $\hat{u}(p,q)$ of traceless superanti-Hermitian matrices.

The desired superalgebra, which we consider as finite-dimensional model of string field theory superalgebra, is defined as

$$\Phi * \Psi = [\Phi, \Psi]_{\pm} \equiv \Phi\Psi + (-1)^{|\Phi||\Psi|+1} \Psi\Phi$$

It is easy to check that such graded commutator also has the form (12). The scalar product is defined as

$$(\Phi,\Psi) \equiv \mathrm{otr}\,\Phi\Psi \equiv \mathrm{str}\left(iA^{\Phi}B^{\Psi} + iB^{\Phi}(A_0^{\Psi} - A_1^{\Psi})\right) =$$

$$= i\,\mathrm{str}\,(A_0^{\Phi}B_0^{\Psi} + A_1^{\Phi}B_1^{\Psi} + B_0^{\Phi}A_0^{\Psi} - B_1^{\Phi}A_1^{\Psi})$$

It is invariant, i.e.:

$$(\Phi, [\Psi, \Lambda]_{\pm}) = (-1)^{|\Phi|(|\Psi|+|\Lambda|)}(\Psi, [\Lambda, \Phi]_{\pm})$$

Now, to obtain Q- operator, we have to solve the equation:

$$2\Psi_0\Psi_0 = [\Psi_0\Psi_0]_+ = 0 \qquad (15)$$

with odd Ψ_0 which has the form:

$$\Psi_0 = \begin{pmatrix} \tilde{Q}, & iK \\ K, & -\tilde{Q} \end{pmatrix}$$

where $\tilde{Q}$ and K are odd and even matrices from $\hat{U}(p,q)$, respectively. Then (15) leads to equations:

$$\tilde{Q}^2 = iK^2 , \quad [K, \tilde{Q}]_- = 0$$

or, parametrizing $\tilde{Q}$ and K as

$$\tilde{Q} = \begin{pmatrix} 0, & q \\ iq^+, & 0 \end{pmatrix} , \quad K = \begin{pmatrix} \ell, & 0 \\ 0, & m \end{pmatrix}$$

we get the equations:

$$qq^+ = -\ell^2 , \quad q^+q = -m^2 , \quad qm - \ell g = 0$$

In analogy with [7] we now assume that the physical space is the space of cohomologies of Q on subspace of vectors, satisfying

$$\frac{\partial}{\partial c_0} \Phi = 0 \qquad (16)$$

which in our model is given by the A_1-type matrices.

The action of $Q = [\Psi_0 , \ ]_\pm$ on Φ is given by

$$Q\Phi = Q(A,B) = (\tilde{Q}A + MB , \ \tilde{Q}B + KA) \qquad (17)$$

where

$$\tilde{Q}A = [Q, A_0]_- + [Q, A_1]_+ , \quad MA = [K, (A_0 - A_1)]_-$$

$$KB = i[K, B_0 - B_1]_+$$

The (17) is analogous to the formula ([7]) for the action of Q-operator $Q = \tilde{Q} + c_0 K + \frac{\partial}{\partial c_0} M$ on the string functional $\Phi = A + C_0 B$:

$$Q\Phi = \tilde{Q}A + MB + C_0(\tilde{Q}B + KA)$$

In the space of A-matrix the even scalar product is defined:

$$(A,B)' = ((A,0),(0,B))$$

Now, from the invariance of this scalar product ((,)) there follows the hermiticity of Q :

$$([\Psi_0,\Phi]_\pm,\Psi) = (-1)^{1+|\Phi|}(\Phi,[\Psi_0,\Psi]_\pm)$$

Then we have

$$(\tilde{Q}A,B)' = ((\tilde{Q}A,0),(0,B)) = (-1)^{1+|\Phi|}((A,0),(0,\tilde{Q}B)) =$$

$$= (-1)^{1+|\Phi|}(A,\tilde{Q}B)'$$

i.e. $\tilde{Q}$ is hermitian in the space of little matrices.

Due to the relation

$$\tilde{Q}\tilde{Q}A = MKA$$

the $\tilde{Q}$ also is nilpotent on the space ker K .

On this space one has to check the criterion of unitarity [8] .

The first nontrivial solution of (15) arises when dimensionality of q, ℓ, m is 2x2.

These solutions are:

1) $\ell = \begin{pmatrix} i\ell_1, & \ell_2 \\ \ell_2^*, & -i\ell_1 \end{pmatrix}$, $m = \begin{pmatrix} -i\ell_1, & m_2 \\ -m_2^*, & i\ell_1 \end{pmatrix}$, $q = \begin{pmatrix} 0, & q_2 \\ q_3, & 0 \end{pmatrix}$

$$|m_2| = |\ell_2|,\ \ell_2^* q_2 + m_2 q_3 = 0,\ |q_2|^2 = |q_3|^2 = \ell_1^2 + |\ell_2|^2,$$

2) $$\ell = \begin{pmatrix} i\ell_1 , \ell_2 \\ -\ell_2^* , -i\ell_1 \end{pmatrix}, \quad m = \begin{pmatrix} i\ell_1 , m_2 \\ -m_2^* , -i\ell_1 \end{pmatrix}, \quad q = \begin{pmatrix} q_1 , 0 \\ 0 , q_1 \end{pmatrix} .$$

$$|m_2| = |\ell_2| , \quad q_1 m_2 = \ell_2 q_4 , \quad |q_1|^2 = |q_4|^2 = \ell_1^2 + |\ell_2|^2 .$$

$$\ell_1 \in R , \; \ell_2 , m_2 , \; q_1 , \; q_2 , q_3 , q_4 \in C$$

Both solutions don't satisfy the criterion of [8] , and hence don't possess the unitarity property.

The other finite-dimensional model of string field theory is given in Ref. [9] .

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