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ՍՊԻՆԱՎՈՐ ԳԵՐՄԱՍՆԻԿ

Կառուցված է տեղային միաչափ և գլոբալ բազմաչափ գերհամաչափությամբ օժտված ռելյատիվիստական մասնիկի տեսություն: Նշված հատկություններն ունեցող առավել ընդհանուր լագրանժյանը կախված է մեկ իրական պարամետրից: Այդ պարամետրի որոշակի արժեքի դեպքում տեսության մեջ ի հայտ է գալիս դերմասնիկի զիգելյան համաչափությանը նման մի լրացուցիչ տեղային համաչափություն: Պարամետրի այսպիսի արժեքի դեպքում երանգանին, ֆիքսված իմպուլսի դեպքում, իրենից ներկայացնում է սպինավոր մասնիկի և գերմասնիկի երանգանների տենզորական արտադրյալը:

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Introduction.

The theories of relativistic particle may be considered as a convenient laboratory for studying different ideas, concerning such important theories as theories of relativistic string.

It is well-known that different theories of relativistic particle differ in a way by which spinning degrees of freedom are introduced. In the present paper we propose a new theory of relativistic particle which has properties of spinning particle and superparticle simultaneously.

The action for a relativistic spinless and massless (we don't consider massive case in this paper) particle is

$$S = \int d\tau \frac{\dot{x}^2}{2e}, \tag{1}$$

where $x^\mu(\tau)$ is a parametrized world-line of particle in the d-dimensional space-time, e is einbein. One may say that (1) is the action for d scalar field "interacting" with one-dimensional "gravity". Correspondingly, (1) is invariant with respect to reparametrization of parameter τ . It is well-known that this gauge invariance permits one to remove the negative norm states, which arise in quantization of time component of x^μ , and eventually (1) describes a **massless spinless relativistic particle.**

One may extend (1) by introduction of d Grassmann variables ψ^μ [1,2,3] which are one-dimensional spinors and d-dimensional vector. In this case the action must possess local supersymmetry which removes the negative norm state arising from time component of ψ^μ . Corresponding action is that of one-dimensional supergravity interacting with d matter supermultiplets [3] . In terms of superfield, depending on τ and one real grassmannian variable $\varkappa$, it reads:

$$S = \int d^2 z \, E \, D_\alpha X^\mu D_\alpha X^\mu ,$$
$$z^M = (z^\mu, z^m) = (\tau, \varkappa) . \qquad (2)$$

$X^\mu(z) = x^\mu(\tau) + \varkappa \psi^\mu$ are scalar superfields. E is superdeterminant of zveibein E_M^A (M, N are world indices and A, B - flat ones). Derivatives D_A are defined as

$$D_A = E_A^M \partial_M \quad , \qquad E_A^M E_M^B = \delta_A^B$$

Action (2) is, evidently, invariant under general coordinate transformation in the superspace z^M , and also under transformations [3] :

$$\delta E_\alpha^M = \varphi E_a^M \ , \qquad \delta E_a^M = 0 \rightarrow$$

$$\delta E_M^\alpha = 0 \ , \qquad \delta E_M^a = - E_M^\alpha \varphi$$

φ is odd.

It is convenient [3] to fix the gauge partially by the requirement of conformally-flatness of E_M^A :

$$E_M^\alpha = \Lambda \bar{E}_M^\alpha \ , \qquad E_M^a = \Lambda^{\frac{1}{2}} \bar{E}_M^a \ ,$$

$$\bar{E}_\mu^\alpha = \bar{E}_m^a = 1 \ , \qquad \bar{E}_\mu^a = 0, \ \bar{E}_m^\alpha = -\varkappa \ .$$

This gauge is stable with respect to combined general coordinate transformation and φ -transformation with parameters:

$$\xi^\mu = a(\tau) + \varkappa \beta(\tau) \ , \qquad \xi^m = \beta(\tau) + \frac{1}{2} \varkappa \dot{a}(\tau)$$

$$\varphi = \dot{\beta} + \frac{1}{2} \varkappa \ddot{a} \tag{3}$$

$$\delta \Lambda = \xi^M \partial_M \Lambda + 2(-1)^M (\partial_M \xi^M) \Lambda \ .$$

In this gauge action (3) becomes

$$S = -\frac{1}{2} \int d^2 z \ \bar{D}_a X \bar{D}_\alpha X \Lambda^{-1} \ ,$$

$$\bar{D}_A = \bar{E}_A^M \partial_M = (\bar{D}_\alpha, \bar{D}_a) = (\partial \ , \ \partial_\varkappa + \varkappa \partial \ ), \tag{4}$$

Transformation (3) leads to the following transformation of derivatives $D_A = E_A^M \partial_M$

$$D_a \rightarrow (1 + \delta(\Lambda^{-\frac{1}{2}})) D_a \ ,$$

$$D_\alpha \rightarrow (1 + \delta(\Lambda^{-1})) D_\alpha \ . \tag{5}$$

The spectrum of (4) is considered (for d=4) in Ref. [4] . The spectrum depends on the choice of the representation of anti-commutation relation on quantum operators ψ^μ : $\{\psi_\mu, \psi_\nu\} = g_{\mu\nu}$. When we represent ψ^μ by γ^μ-matrices, we obtain the theory of massless Dirac (or Majorana, when the condition of reality of wave function is put on) particle.

One may consider also the similar to (4) theory with extended supersymmetry [4], i.e. $\varkappa$ becomes now complex. The spectrum of this theory is studied in [4] . Corresponding Lagrangian is:

$$S = \frac{1}{2} \int d\tau \, d\varkappa \, d\bar{\varkappa} \, E \, D_{\varkappa} X^\mu \bar{D}_{\bar{\varkappa}} X_\mu \, .$$

Another way for introduction of spin degrees of freedom is possible when using d-dimensional spinors θ (odd grassmannian quantities) from the beginning [6] . The action is (all subsequent formulae we shall write down for d=4, although they may be generalized for some other dimensions):

$$S = \frac{1}{2} \int d\tau \frac{1}{e} \left(\dot{x}^\mu - \bar{\theta} \gamma^\mu \dot{\theta} \right)^2 \qquad (6)$$

Except the evident reparametrization invariance, (6) also possesses [7] some fermionic local invariance with parameter which is d-dimensional spinor $\varepsilon(\tau)$:

$$\delta x^\mu = \bar{\theta}\gamma^\mu\theta ,$$

$$\delta\theta = \gamma^\mu \varepsilon \pi_\mu , \qquad \pi^\mu = \dot{x}^\mu - \bar{\theta}\gamma^\mu\dot{\theta} , \tag{7}$$

$$\delta e = 4e\dot{\bar{\theta}}\varepsilon .$$

The possibility which is not studied as yet is to consider the theory which simultaneously has all the symmetries of the Lagrangians (1), (4) and (6). This theory is constructed by us in Sect. 1. We show that there exists a whole one-parameter family of Lagrangians which are locally (in one-dimension) and globally (in d-dimension) supersymmetric, but only for one value of this parameter, $\alpha = -1$, we are able to find the analog of Siegel symmetry (7).

We construct Hamiltonian formalism of the theories of Sect. 1 in Sect. 2 and quantize the theory for $\alpha = -1$. In this case the spectrum is a tensor product, at fixed momentum of spectra of spinning particle and superparticle.

We hope that the result of this paper may be generalized for the theory of fermionic string. Perhaps, if for some such theory the spectrum is the analogous tensor product of spectra of spinning string and superstring (as for the particle at $\alpha = -1$), then the corresponding critical dimension is ten.

It is worthnoting some analogies of our theory with that of Ref.[9] : in our theory naturally arises auxiliary field φ (superpartner of θ) which is even (commuting) spinor quantity. Such a field was introduced in [9] with a quite different purpose of covariant quantization of superparticle.

1. Spinning Superparticle - Lagrangian Formalism.

It is well-known that the form $dx^{\mu}-\bar{\theta}\gamma^{\mu}d\theta$ is invariant under global supersymmetry transformations

$$\delta x^{\mu} = \bar{\varepsilon}\gamma^{\mu}\theta, \qquad \delta\theta = \varepsilon . \tag{8}$$

The other superinvariant form is $d\theta$. Assuming X^{μ} and θ to be the scalar superfields with respect to local one-dimensional susy transformations (3), we can write two actions invariant under (3) and (8):

$$S_1 = \int d^2z\,\Lambda(\dot{X}^{\mu}-\bar{\theta}\gamma^{\mu}\dot{\theta})(DX^{\mu}-\bar{\theta}\gamma^{\mu}D\theta), \tag{9}$$

$$S_2 = \int d^2z\,\Lambda(DX^{\mu}-\bar{\theta}\gamma^{\mu}D\theta)\,D\bar{\theta}\gamma^{\mu}D\theta , \tag{10}$$

$$(\Lambda^{-1}\rightarrow\Lambda, \;\; D = \bar{D}_a) .$$

We note here that all the quantities in our theory are $Z_2 \times Z_2$ graded, contrary to usually used Z_2 grading. To be concrete, our commutation rules are the following: the spinor indices of the same kind ("kind" means one-dimensional or d-dimensional) anticommute, and all other possible pairs of indices commute. For example, the one-dimensional spinor commutes with d-dimensional one, but anticommutes with the quantity which has both kinds of spinor indices.

Local superinvariance under (3) of actions (9) and (10) is evident from the transformation laws (5) of derivatives D_A .

Eventually the most general invariant action has the form $\beta S_1 + \alpha S_2$, but since the overall coefficient is inessential, i.e. only the ratio of α and β is important, we have one-parameter family of invariant actions, with parameter, taking values on $RP^1 \sim S^1$.

Introducing the parametrization for components of superfields in (9), (10):

$$\begin{aligned} X^\mu &= x^\mu + \varkappa e^{-\frac{1}{2}}(\psi^\mu + \bar\theta \gamma^\mu \varphi), \\ \Theta &= \theta + \varkappa e^{-\frac{1}{2}} \varphi, \\ \Lambda &= e + \varkappa e^{3/2} \chi \end{aligned} \tag{11}$$

we can obtain after integration over $\varkappa$ the component action:

$$\begin{aligned} L &= L_1 + \alpha L_2 = \\ &= e(\dot x^\mu - \bar\theta\gamma^\mu\theta)^2 + (\alpha-1)\bar\varphi\gamma^\mu\varphi(\dot x^\mu - \bar\theta\gamma^\mu\dot\theta) + \dot\psi^\mu\psi^\mu + \\ &+ 2(\alpha+1)\dot{\bar\theta}\gamma^\mu\varphi\psi^\mu + e\chi\psi^\mu(\dot x^\mu - \bar\theta\gamma^\mu\dot\theta) + \alpha\chi\psi^\mu\bar\varphi\gamma^\mu\varphi . \end{aligned} \tag{12}$$

One can see that the field φ is non-dynamical, since it enters (12) without derivatives.

It remains to resolve the question of the existence of Siegel-type symmetry like (7). We find the close analog of this symmetry only in the case $\alpha=-1$. In that case (12) reads

$$L = L_1 - L_2 = e\pi^\mu \pi_\mu + \dot{\psi}^\mu \psi_\mu ,$$
$$\pi^\mu = \dot{x}^\mu - \bar{\theta}\gamma^\mu\dot{\theta} - \tfrac{1}{2}\bar{\varphi}\gamma^\mu\varphi + \tfrac{1}{2}\chi\psi^\mu \tag{13}$$

and corresponding Siegel-type transformations are

$$\delta x^\mu = \bar{\theta}\gamma^\mu\delta\theta , \qquad \delta\chi = \delta\psi = 0 ,$$
$$\delta\theta = \gamma^\mu \varepsilon \pi^\mu ,$$
$$\delta e = -4e\dot{\bar{\theta}}\varepsilon , \tag{14}$$
$$\delta\left(\frac{\varphi}{\sqrt{e}}\right) = 0 .$$

Action (13) is also invariant with respect to the following *transformation:*

$$\delta\left(\frac{\varphi}{\sqrt{e}}\right) = \gamma^\mu \tilde{\varepsilon}\pi_\mu , \qquad \delta x^\mu = \delta\theta = \delta\chi = \delta\psi^\mu = 0 ,$$
$$\delta e = 4e\bar{\varphi}\tilde{\varepsilon} . \tag{15}$$

2. Hamiltonian Formalism and Quantization.

Let's construct the Hamiltonian formalism for the Lagrangian in (12).

The momenta are

$$P_e = P_\chi = \bar{P}_\varphi = 0 ,$$
$$p^\mu = \frac{\partial L}{\partial \dot{x}_\mu} = 2e(\dot{x}^\mu - \bar{\theta}\gamma^\mu\dot{\theta}) + (\alpha-1)\bar{\varphi}\gamma^\mu\varphi + e\chi\psi^\mu , \tag{16}$$

$$\bar{P}_\theta = \frac{\partial L}{\partial \dot{\bar\theta}} = -2e(\dot{x}^\mu - \bar\theta\gamma^\mu\theta)\bar\theta\gamma^\mu - (\alpha-1)\bar\varphi\gamma^\mu\varphi\bar\theta\gamma_\mu -$$

$$-2(\alpha+1)\bar\varphi\gamma^\mu\psi^\mu - e\chi\psi^\mu\bar\theta\gamma^\mu = -\bar\theta\gamma^\mu P_\mu - 2(\alpha+1)\bar\varphi\gamma^\mu\psi_\mu .$$

We use the identity

$$\gamma^\mu\varphi\bar\varphi\gamma_\mu\varphi = 0$$

(remember that φ commute with themselves) which is valid in considered dimensions d=4 (and also in d=3,6,10).

For the field ψ^μ Lagrangian (12) is, in fact, already written in Hamiltonian form, and it is well-known that corresponding Poisson bracket has the form:

$$\{\psi^\mu\psi^\nu\} = g^{\mu\nu}. \tag{17}$$

Constraints following directly from definitions (16) are

$$P_e = P_\chi = P_{\dot\varphi} = 0 \;,$$

$$\bar{P}_\theta + \bar\theta\hat{p} + 2(\alpha+1)\bar\varphi\gamma^\mu\psi_\mu = 0 . \tag{18}$$

Hamiltonian is

$$H=\frac{1}{4e}\left(P^2-2(\alpha-1)P^\mu\bar{\varphi}\gamma_\mu\varphi\right)-\frac{\chi\Psi^\mu}{2}\left(P_\mu+(\alpha+1)\bar{\varphi}\gamma_\mu\varphi\right)+$$

$$+\left(\bar{P}_\theta+\bar{\theta}\hat{p}+2(\alpha+1)\bar{\varphi}\hat{\Psi}\right)\lambda_\theta+$$

$$+P_e\lambda_e+P_\chi\lambda_\chi+\bar{P}_\varphi\lambda_\varphi\,,$$

where coefficients in front of constraints are arbitrary functions.

Taking the Poisson bracket of constraints (18) with H, we get:

$$\begin{aligned}
&p^2-2(\alpha-1)p^\mu\bar{\varphi}\gamma_\mu\varphi=0\,,\\
&\Psi^\mu\left(P_\mu+(\alpha+1)\bar{\varphi}\gamma_\mu\varphi\right)=0\,,\\
&\frac{1}{e}(\alpha-1)\bar{\varphi}\hat{p}+\chi\Psi^\mu(\alpha+1)\bar{\varphi}\gamma_\mu-2(\alpha+1)\bar{\lambda}_\theta\hat{\Psi}=0\,,\\
&2\left(p^\mu+(\alpha+1)^2\bar{\varphi}\gamma^\mu\varphi\right)\gamma_\mu\lambda_\theta-(\alpha+1)\chi\hat{p}\varphi-\\
&-2(\alpha+1)\hat{\Psi}\lambda_\varphi=0\,.
\end{aligned}\qquad(19)$$

In general, it is difficult to analyze the relations (19), but the analysis simplifies considerably at $\alpha = -1$. In this case (18), (19) become

$$P_\varphi = 0 , \tag{20a}$$

$$\hat{P}\varphi = 0 , \tag{20b}$$

$$p^2 = 0 , \tag{20c}$$

$$p^\mu \psi_\mu = 0 , \tag{20d}$$

$$\bar{P}_\theta + \bar{\theta}\hat{P} = 0 . \tag{20e}$$

(The constraints $P_e = 0$, $P_\chi = 0$ are first-class and simply means that the wave function is e and χ -independent, and we won't discuss them further).

This system can be analyzed easily. φ and P_φ enter only to (20a,b), the remaining system (20c,d,e) is simply the joining of constraints of spinning particle and superparticle. Firstly, let's remove φ and P_φ . The matrix $\hat{P}$ due to (20c) is degenerated, namely half (i.e. 2) of its eigenvalues are equal to zero. Hence, (20b) actually contains only two independent constraints. Together with two constraints from (20a) they form the two pairs of second-class constraints, and we solve them explicitly, equalling the corresponding φ and

P_φ to zero (the Poisson bracket of all remaining coordinates and momenta remains unchanged). The remaining two constraints from (20a) are first-class. After quantization, one must impose them on the wave function, and these equations will mean that the wave function is independent also of the remaining components of φ . So the wave function is completely φ-independent, and any sign of φ and P_φ disappears from the theory.

The quantization of system with constraints (20c,d,e) is well known, since, as we have mentioned above, they are simply constraints of spinning particle and superparticle considered simultaneously.

The first-class constraints in (20c,d,e) are

$$p^2 = 0 , \tag{21a}$$

$$p^\mu \psi_\mu = 0 , \tag{21b}$$

$$\hat{p}\, P_\theta = 0 . \tag{21c}$$

Since it is impossible to pick out, in Lorentz-covariant way, second-class constraints from (20c,d,e), one may demand that only the average value of constraints would be zero, by imposing the condition:

$$\bar{d}\,\Psi = 0 \qquad \bar{d} = (\bar{P}_\theta + \bar{\theta}\hat{p})(1+\gamma^5) \tag{22}$$

So, the average value of (20e) is equal to zero. Thus, quantum theory is described by the wave function Ψ on which one must impose the constraints

$$p^2\Psi = 0\,,$$

$$p^\mu\psi_\mu\Psi = 0,$$

$$\bar{d}\Psi = 0\,,$$

where operators $x, p, \theta, p_\theta, \psi$ satisfy the (anti)commutation relations:

$$\{\psi^\mu\,\psi^\nu\} = g^{\mu\nu}, \tag{23a}$$

$$[x^\mu\,p^\nu] = ig^{\mu\nu}, \tag{23b}$$

$$\{\bar{\theta}\;p_\theta\} = i\;, \tag{23c}$$

all others equal to zero.

The spectrum of our theory is, evidently, the tensor product, at fixed momentum, of that for spinning particle and superparticle. At fixed momentum the spectrum of superparticle consists of the following set of representations of the remaining SO(2) invariance group (we describe them by noting the helicity λ): $\lambda = 0, 0, \pm 1/2$. For spinning particle, if we take the γ^μ-matrix representation for ψ^μ , the spectrum is

$\lambda = \pm 1/2, \pm 1/2$ (or only $\pm 1/2$ if the reality condition is imposed).

The tensor product means the summing of helicities in all possible ways, and the answer is: $\lambda = 0, 0, \pm 1, \pm 1/2, \pm 1/2$ (or twice of this).

After completion of our work we have learned about the work [11] , where the theory of spinning superparticle in the case $\alpha = -1$ is constructed, and about the work [10], where a similar locally supersymmetric (in one-dimensional sense) and globally supersymmetric (in d-dimensional sense) theory is constructed, but in the first-order formalism, and it is shown that it is equivalent to usual superparticle, but has an advantage to permit a covariant quantization. The Lagrangian of Ref.[10] is

$$S = \int d^2 z \, P_\mu \left(D X^\mu - \bar{\theta} \gamma^\mu D \theta \right),$$

where P^μ is new independent superfield.

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