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COCYCLES OF AREA PRESERVING DIFFEOMORPHISMS AND ANOMALIES IN THEORY OF RELATIVISTIC SURFACES

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Տ.Ա. ԱՌԱՔԵԼՅԱՆ, Գ.Կ. ՍԱՎՎԻԴԻ

ՄԱԿԵՐԵՍԸ ՊԱՀՊԱՆՈՂ ԴԻՖՖԵՈՄՈՐՖԻԶՄՆԵՐԻ ԿՈՑԻԿԼԵՐԸ ԵՎ ԱՆՈՄԱԼԻԱՆԵՐԸ ՌԵԼՅԱՏԻՎԻՍՏԱԿԱՆ ՄԱԿԵՐԵՎՈՒՅԹՆԵՐԻ ՏԵՍՈՒԹՅԱՆ ՄԵՋ

Ուսումնասիրված է տարբեր տեղաբանության ռելյատիվիստական բոզոնային թաղանթի տեսությունը: Լույսի կոնի տրամաչափում մնացորդային համաչափությունը հանդիսանում է թաղանթի երկչափ մակերևույթի մակերեսի դիֆֆեոմորֆիզմների խումբը, հաշված են նրա կառուցվածքային հաստատունները, առաջին և երկրորդ կոհոմոլոգիայի խմբերը $H^1(\hat{S},\hat{S})$ և $H^2(\hat{S},R)$, որոնք քվանտացման դեպքում որոշում են այդ խմբի աջ և կենտրոնական ընդարձակումները:

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Т.А.АРАКЕЛЯН,Г.К.САВВИДИ

КОЦИКЛЫ ГРУППЫ ДИФФЕОМОРФИЗМОВ ДВУХМЕРНОЙ СФЕРЫ И АНОМАЛИЙ В ТЕОРИИ РЕЛЯТИВИСТСКИХ ПОВЕРХНОСТЕЙ

Исследуется теория релятивистской бозонной мембраны. Показано, что остаточной группой симметрии в калибровке светового конуса является группа диффеоморфизмов двухмерной сферы, вычислены ее структурные константы, первая и вторая группы когомологий $H^1(\hat{S},\hat{S})$ и $H^2(\hat{S},R)$, которые определяют правое и центральное расширения этой группы при квантовании.

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T.A.ARAKELYAN, G.K.SAVVIDY

COCYCLES OF AREA PRESERVING DIFFEOMORPHISMS AND ANOMALIES IN THEORY OF RELATIVISTIC SURFACES

A theory of relativistic bosonic membrane of different topology is studied. In the lightcone gauge a residual symmetry consists of a group of area preserving diffeomorphisms of two-dimensional surface of membrane; its structure constants as well as the first and second groups of cohomologies $H^1(\hat{S},\hat{S})$ and $H^2(\hat{S},R)$ are calculated, which determine the right and central extensions of this group under quantization.

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Yerevan 1988

1. Introduction.

String theory is a promising candidate for unification of fundamental forces of Nature including gravitation [1] . Such possibility is connected with successful construction of quantum theory of nonlocal objects of a string type. Up to now it is unknown whether there exist selfconsistent theories of nonlocal extended objects of higher dimensions. The present paper is devoted to this problem.

A fundamental fact determining the existence of quantum theory of relativistic string is its invariance under infinite-dimensional group of conformal transformations. In order to construct a quantum theory of relativistic surfaces, it is necessary to understand more deeply its hidden symmetry.

In the lightcone gauge a residual symmetry consists of a group of area preserving diffeomorphisms of a two-dimensional surface M of different topology [2-5] . The role of this group is close to that of a group of conformal transformations in string theory. Its structure constants, the first and second cohomology groups $H^1(\hat{S},\hat{S})$ and $H^2(\hat{S},R)$ which determine the right and central extensions of its algebra in quantization are calculated.

2. Residual Symmetry.

Let us introduce necessary notations and lightcone gauge conditions in the theory of relativistic surfaces. The action invariant under reparametrization of three-dimensional volume swept out in Minkowski space by a two-dimensional surface M has a form [6] :

$$S = -T\int d\tau \int_M d\sigma_1 d\sigma_2 \, \mathrm{Det}^{1/2}(-g) = \int d\tau \int_M d\sigma_1 d\sigma_2 L \ , \tag{1}$$

where $g_{\alpha\beta} = x^\mu_\alpha x_{\mu\beta}$, $x^\mu_\alpha = \partial x^\mu / \partial \xi^\alpha$, $\xi^\alpha = (\tau, \sigma_1, \sigma_2)$; $x^\mu(\xi)$ is a parametric representation of the swept volume, $(\sigma_1,\sigma_2) \in M$, $\mu = 0, \dots D-1$. The constraints have a form:

$$\omega = P^\tau_\mu P^{\tau\mu} - T^2[x^2_{\sigma_1} x^2_{\sigma_2} - (x_{\sigma_1} x_{\sigma_2})^2] \ , \tag{2}$$

$$\omega_a = P^\tau_\mu x^\mu_{\sigma_a} \ , \quad a = 1,2 \ ,$$

where

$$P^\alpha_\mu = -\frac{\partial L}{\partial x^\mu_\alpha} = T\sqrt{-g}\, g^{\alpha\beta} x_{\mu\beta} \tag{3}$$

One can choose a gauge consisting of three conditions:

$$nx = \lambda\tau \ ; \quad x_\tau x_{\sigma_a} = 0 \ ; \quad x^2_\tau = (4\pi T)^{2/3}[x^2_{\sigma_1} x^2_{\sigma_2} - (x_{\sigma_1} x_{\sigma_2})^2] \ , \tag{4}$$

where $\lambda=(nP)/(4\pi T)^{2/3}$, P_μ is total four-momentum of surface M . If $n_\mu=(1,1,0,0)$, then constraints (2) and additional conditions (4) can be solved, and X^- and P_-^τ can be expressed through transverse variables X^i and P_i^τ , where $i=2,\dots D-1$. In this gauge a hamiltonian has a form:

$$H=\frac{4\pi}{(4\pi T)^{2/3}}\int\limits_M d\sigma_1 d\sigma_2\left[P_i^\tau P_i^\tau+T^2\left[(X^i_{\sigma_1})^2(X^j_{\sigma_2})^2-(X^i_{\sigma_1}X^i_{\sigma_2})^2\right]\right] \qquad (5)$$

and together with the Poisson bracket

$$\left[X^i(\sigma),P_j^\tau(\sigma')\right]=\delta_{ij}\,\delta^{(2)}(\sigma-\sigma') \qquad (6)$$

determines correct equations of motion. Not all variables X^i and P_i^τ are independent, since gauge conditions (4) fix parametrization incompletely. The Frobenius integrability condition for constraints ω_a has a form:

$$L(\sigma)=X^i_{\sigma_1}P^\tau_{i,\sigma_2}-X^i_{\sigma_2}P^i_{i,\sigma_1} \qquad (7)$$

Expression (7) serves as a generator of residual symmetry group in gauge (4) (i.e. area preserving diffeomorphism) and reduces the number of independent transverse degrees of freedom to $D-3$.

One may readily be convinced that the constraint L commutes with hamiltonian (5):

$$[H,L(\sigma)]=0 \qquad (8)$$

and that

$$[L(\sigma),L(\sigma')]=\partial_1\delta(\sigma-\sigma')\cdot\partial_2'L(\sigma')-\partial_2\delta(\sigma-\sigma')\cdot\partial_1'L(\sigma')\,, \qquad (9)$$

To arbitrary function $\epsilon(\sigma)$ on M we'll compare the quantity

$$L_\epsilon \equiv L\cdot\epsilon = \int_M L(\sigma)\,\epsilon(\sigma)\,d^2\sigma .$$

The action of this representation on variables X^i and P_i^τ is given by a formula

$$\begin{aligned} \delta_\epsilon X^i &= [X^i, L_\epsilon] = \epsilon_{\sigma_1} X^i_{\sigma_2} - \epsilon_{\sigma_2} X^i_{\sigma_1} , \\ \delta_\epsilon P_i^\tau &= [P_i^\tau, L_\epsilon] = \epsilon_{\sigma_1} P^\tau_{i,\sigma_2} - \epsilon_{\sigma_2} P^\tau_{i,\sigma_1} . \end{aligned} \tag{10}$$

If we introduce a new bracket

$$\{\epsilon,\eta\} = \partial_1\epsilon\cdot\partial_2\eta - \partial_2\epsilon\,\partial_1\eta \tag{11}$$

then the hamiltonian (5) and constraint (7) can be rewritten as

$$\begin{aligned} H &= \frac{4\pi}{(4\pi T)^{2/3}} \int_M d^2\sigma \left[P_i^\tau P_i^\tau + T^2 \{X^i, X^j\}^2 \right] , \\ L &= \{X^i, P_i^\tau\} \end{aligned} \tag{12a,b}$$

and the transformation (10) as

$$\delta_\epsilon X^i = \{\epsilon, X^i\} , \quad \delta_\epsilon P_i^\tau = \{\epsilon, P_i^\tau\} . \tag{13}$$

For arbitrary functions ϵ and η from (9) we'll obtain

$$[L_\epsilon, L_\eta] = L_{\{\epsilon,\eta\}} . \tag{14}$$

The bracket (11) defines the Lie algebra $\hat{S}$ of the group of area $(d\sigma_1 d\sigma_2)$ preserving diffeomorphisms of the surface M .

Let us introduce orthogonal basis of this Lie algebra $\hat{S}$ for different surfaces M with topology of sphere S^2 and torus T by means of the functions $Y_{\ell m}$:

$$Y_{\ell m}^{(S^2)} = C_{\ell m} e^{im\sigma_2} P_\ell^{|m|}(-\sigma_1); \quad C_{\ell m} = (-1)^m \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-|m|)!}{(\ell+|m|)!}}$$

$$P_\ell^{|m|}(-\sigma_1) = \frac{1}{2^\ell \ell!}(1-\sigma_1^2)^{|m|/2} d^{\ell+|m|}(\sigma_1^2-1)^\ell / d(-\sigma_1)^{\ell+|m|}, \quad m>0; \tag{15}$$

$$Y_{\ell m}^{(T)} = \frac{1}{2\pi} \exp i(\ell\sigma_1 + m\sigma_2), \quad \sigma_1 \sigma_2 \in [0, 2\pi],$$

where for the sphere $\sigma_1 = -\cos\theta$, $\sigma_2 = \varphi$, and invariant measure has a form: $d^2\sigma = \sin\theta \, d\theta \, d\varphi$. Structure constants of this group g

$$\{Y_{\ell_1 m_1} Y_{\ell_2 m_2}\} = -i g_{\ell_1 m_1 \ell_2 m_2}^{\ell_3 m_3} Y_{\ell_3 m_3}^{*}, \tag{16}$$

are calculated in the Appendix and have the form:

$$S^2: \left[\frac{4\pi}{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}\right]^{1/2} g_{\ell_1 m_1 \ell_2 m_2}^{\ell_3 m_3} =$$

$$m_2 \sum_{K_1=0}^{\left[\frac{\ell_1-|m_1|-1}{2}\right]} (2(\ell_1-2K_1-1)+1) \left(\frac{(\ell_1-|m_1|)\dots(\ell_1-|m_1|-2K_1}{(\ell_1+|m_1|)\dots(\ell_1+|m_1|-2K_1}\right)^{1/2} \begin{pmatrix} \ell_1-2K_1-1; \ell_2; \ell_3 \\ m_1; m_2; m_3 \end{pmatrix} \begin{pmatrix} \ell_1-2K_1-1, \ell_2, \ell_3 \\ 0, 0, 0 \end{pmatrix} - \tag{17}$$

$$-m_1\sum_{K_2=0}^{\left[\frac{\ell_2-|m_2|-1}{2}\right]}\left(2(\ell_2-2K_2-1)+1\right)\left(\frac{(\ell_2-|m_2|)\dots(\ell_2-|m_2|-2K_2)}{(\ell_2+|m_2|)\dots(\ell_2+|m_2|-2K_2)}\right)^{1/2}\cdot\begin{pmatrix}\ell_1;\ell_2-2K_2-1;\ell_3\\ m_1;m_2;m_3\end{pmatrix}\begin{pmatrix}\ell_1;\ell_2-2K_2-1;\ell_3\\ 0;\ 0;\ 0\end{pmatrix}$$

$$T:\ 2\pi i\, g^{\ell_3 m_3}_{\ell_1 m_1\, \ell_2 m_2} = (m_2\ell_1 - m_1\ell_2)\,\delta_{\ell_1+\ell_2+\ell_3}\,\delta_{m_1+m_2+m_3}\quad ,$$

where for sphere m_1 and m_2 have the same sign. When m_1 and m_2 have different signs, they can be calculated by means of the relation (see the Appendix):

$$g^{\ell_1 m_1}_{\ell_2 m_2 \ell_3 m_3} = g^{\ell_3 m_3}_{\ell_1 m_1 \ell_2 m_2} = g^{\ell_2 m_2}_{\ell_3 m_3 \ell_1 m_1} \tag{18}$$

First coefficients for S^2 are

$$g^{10}_{\ell_1 m_1 \ell_2 m_2} = (-1)^{m_1} m_1\sqrt{\frac{3}{4\pi}}\,\delta_{\ell_1\ell_2}\,\delta_{m_1+m_2}\ ,$$

$$g^{11}_{\ell_1 m_1 \ell_2 m_2} = (-1)^{m_1}\sqrt{(\ell_1-m_1)(\ell_1+m_1+1)}\sqrt{\frac{3}{8\pi}}\,\delta_{\ell_1\ell_2}\,\delta_{m_1+m_2+1} \tag{19}$$

$$g^{1-1}_{\ell_1 m_1 \ell_2 m_2} = -(-1)^{m_1}\sqrt{(\ell_1+m_1)(\ell_1-m_1+1)}\sqrt{\frac{3}{8\pi}}\,\delta_{\ell_1\ell_2}\,\delta_{m_1+m_2-1}$$

from which we can see that Y_{10} , Y_{11} and Y_{1-1} form a finite subalgebra SO(3):

$$\begin{aligned}\{Y_{11}\,,Y_{1-1}\} &= i\sqrt{\frac{3}{4\pi}}\,Y_{10}\ ,\\ \{Y_{11}\,,Y_{10}\} &= i\sqrt{\frac{3}{4\pi}}\,Y_{11}\ ,\end{aligned} \tag{20}$$

$$\{Y_{1-1}, Y_{10}\} = -i\sqrt{\frac{3}{4\pi}}\, Y_{1-1} ,$$

which has the following commutation relations with the other generators:

$$\{Y_{10}, Y_{\ell m}\} = -im\sqrt{\frac{3}{4\pi}}\, Y_{\ell m}$$

$$\{Y_{11}, Y_{\ell m}\} = i\sqrt{(\ell-m)(\ell+m+1)}\sqrt{\frac{3}{8\pi}}\cdot Y_{\ell m+1} , \qquad (21)$$

$$\{Y_{1-1}, Y_{\ell m}\} = i\sqrt{(\ell+m)(\ell-m+1)}\sqrt{\frac{3}{8\pi}}\, Y_{\ell m-1} .$$

3. Constraint Algebra.

Expanding fields ϵ , η and X over basis, from (8), (12) and (14) we'll obtain

$$L^{\ell m} = -i\, g^{\ell m}_{\ell_1 m_1 \ell_2 m_2}\, X^{i}_{\ell_1 m_1}\, P^{\tau}_{i,\ell_2 m_2} , \qquad (22)$$

$$[L_{\ell_1 m_1}, L_{\ell_2 m_2}] = -i\, g^{\ell_3 m_3}_{\ell_1 m_1 \ell_2 m_2}\, L^{*}_{\ell_3 m_3} , \qquad (23)$$

$$[H, L_{\ell m}] = 0 \qquad (24)$$

The role of the constraint algebra (23) in the theory of relativistic surfaces is close to the Virasoro algebra in the string theory.

In quantization the classical algebra may be modified by quantum anomalies. At present there exists a well-developed formalism which allows to describe possible quantum anomalies without perturbation theory and which is based on elements of cohomology theory [7-10] . This technique allows to calculate possible Schwinger terms in the commutator (14), (23). The presence of such terms is connected with the existence of nontrivial cocycles from H^1 and H^2 of the algebra $\hat{S}$ with the Lie bracket (11):

$$\{\epsilon,\eta\} = \epsilon_1\eta_2 - \epsilon_2\eta_1 ,$$

where $\partial_1\epsilon = \epsilon_1$ and so on. The central extensions of algebra are described by a modified Lie bracket:

$$\{(a,\epsilon);(b,\eta)\} = (N\cdot C^{(2)}(\epsilon,\eta),\ \{\epsilon,\eta\}) \qquad (25)$$

Here N is an arbitrary real number which is called a central charge. The Jacobi identity for this commutator is equivalent to the fact that $C^{(2)}$ is a 2-cocycle:

$$dC^{(2)} = C^{(2)}(\epsilon,\{\eta,\chi\}) + C^{(2)}(\eta,\{\chi\epsilon\}) + C^{(2)}(\chi,\{\epsilon\eta\}) \qquad (26)$$

where ϵ , η and χ are arbitrary functions on M .

The following expressions are nontrivial cocycles of algebra $\hat{S}$ on the S^2 - (27), (28) and T - (28), (29):

$$C^{(2)}(\epsilon,\eta) = N_1 \int_T (\epsilon_2 \eta - \epsilon \eta_2)\, d^2\sigma \ , \tag{27}$$

T:

$$C^{(2)}(\epsilon,\eta) = N_2 \int_{T;S^2} (\epsilon_1 \eta - \epsilon \eta_1)\, d^2\sigma \ , \tag{28}$$

S^2:

$$C^{(2)}(\epsilon,\eta) = N_3 \int_{S^2} (\epsilon_{11} \eta - \eta_{11} \epsilon)\, d^2\sigma \ ; \tag{29}$$

in which one may be convinced substituting them into (26) and using the rules of integration by parts on manifold with $S^2 \backslash \pm 1$ and T topology. Cocycles (27-28) may be written in a more invariant form as

$$C^{(2)}_{\xi}(\epsilon\eta) = \int \xi^a (\nabla_a \epsilon \cdot \eta - \nabla_a \eta \cdot \epsilon)\, d\mu \ , \quad if \quad \nabla_a \xi^a = 0,$$

$$\xi^a = E^{ab} \cdot \nabla_b \xi \ , \quad E^{ab} = - E^{ba}$$

Note that if ξ function and its derivatives are single-valued on M, then the cocycle is trivial and equals

$$C^{(2)}_{\xi}(\epsilon\eta) = \int_M [\{\epsilon,\xi\}\eta - \{\eta,\xi\}\epsilon]\, d\mu \nleq 2 \int_M \xi \{\epsilon,\eta\}\, d\mu^{*}$$

and if only derivatives of ξ^a are single-valued on M, then the cocycle is nontrivial. On everywhere dense subalgebra consisted of polynomial $\hat{S}^{Pol}$ cocycles (27-29) have the form:

$$C^{(2)}(Y_{\ell_1 m_1}, Y_{\ell_2 m_2}) = i N_1 (m_1 - m_2)\, \delta_{m_1+m_2,0}\, \delta_{\ell_1+\ell_2,0} \tag{30a}$$

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* $\{\epsilon,\eta\} = E^{ab} \nabla_a \epsilon \nabla_b \eta$

$$C^{(2)}(\mathcal{Y}_{\ell_1 m_1},\mathcal{Y}_{\ell_2 m_2}) = i N_2(\ell_1-\ell_2)\,\delta_{m_1+m_2,0}\,\delta_{\ell_1+\ell_2,0} \qquad (30b)$$

$$C^{(2)}(\mathcal{Y}_{\ell_1 m_1},\mathcal{Y}_{\ell_2 m_2}) = 4\pi N_2\cdot C_{\ell_1 m_1} C_{\ell_2 m_2}\cdot \delta_{m_1+m_2,0}\,\cdot$$

$$\left\{ \frac{(\ell_1+|m_1|)!}{(\ell_1-|m_1|)!}\,\delta_{\ell_1,\ell_2-2K_2-1} - \frac{(\ell_2+|m_2|)!}{(\ell_2-|m_2|)!}\,\delta_{\ell_1-2K_1-1,\ell_2} \right\}, \qquad (31)$$

$$0 \leqslant K_2 \leqslant \left[\frac{\ell_2-|m_2|-1}{2}\right]; \qquad 0 \leqslant K_1 \leqslant \left[\frac{\ell_1-|m_1|-1}{2}\right],$$

$$C^{(2)}(\mathcal{Y}_{\ell_1 m_1},\mathcal{Y}_{\ell_2 m_2}) = \frac{1}{2} N_3\cdot\sqrt{(2\ell_1+1)(2\ell_2+1)}\,\cdot \qquad (32)$$

$$\delta_{m_1,0}\,\delta_{m_2,0}\,\delta_{\ell_1+\ell_2,2K}\left\{\ell_1(\ell_1+1)-\ell_2(\ell_2+1)\right\}$$

Cocycles (30-32) determine central extension of algebra $\hat{S}^{pol}$, and, as a result, the modification of commutation relations (9), (14), (23):

$$\{L(\sigma),L(\sigma')\}_Q = \partial_1\delta(\sigma-\sigma')\partial_2' L(\sigma') - \partial_2\delta(\sigma-\sigma')\partial_1' L(\sigma') + \qquad (33)$$

$$\left(N_1(\partial_1-\partial_2')+N_2(\partial_1-\partial_1')+N_3(\partial_1^2-\partial_1'^{2})\right)\delta(\sigma-\sigma').$$

The $\hat{S}$ algebra also has one-dimensional right extension connected with space H^1. Cocycle $C^{(1)}$ from $H^{(1)}$ has a form:

$$C^{(1)}(\epsilon) = (\xi^a \partial a - 1)\epsilon \tag{34}$$

where ξ^a is a vector field on M with constant divergence

$$\partial a\, \xi^a = \lambda \tag{35}$$

which we'll choose for the S^2 as $\xi^a = (\sigma', \frac{3}{2} i)$.

The structure of extended algebra is determined by a modified bracket:

$$\{(\lambda,\epsilon);(\mu,\eta)\}_{aL} = \{\{\epsilon,\eta\}_L + \mu C^{(1)}(\epsilon) - \lambda C^{(1)}(\eta);\ 0\}. \tag{36}$$

For this commutator the Jacobi identity is equivalent to the condition

$$dC^{(1)} = C^{(1)}(\{\epsilon,\eta\}_L) - \{\epsilon, C^{(1)}(\eta)\}_L + \{\eta, C^{(1)}(\epsilon)\}_L, \tag{37}$$

which coincides with the definition of 1-cocycle. In subalgebra $\hat{S}^{pol}$ (34) has a form:

$$K^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2} = \mu \cdot C^{(1)}_{\ell_3 m_3}(y_{\ell_1 m_1}) - \lambda C^{(1)}_{\ell_3 m_3}(y_{\ell_2 m_2}) =$$

(38a,b)

$$= -\frac{3}{2}\left(\mu(m_1+1)\delta_{\ell_1\ell_3}\delta_{m_1 m_3} - \lambda \cdot (m_2+1)\delta_{\ell_2\ell_3}\delta_{m_2 m_3}\right)$$

$$-\frac{1}{2}\Big(\mu\sqrt{(2\ell_1+1)(2\ell_3+1)}\,\delta_{\ell_1+\ell_3,2n+1}\,\delta_{m_1 0}\,\delta_{m_3 0}\;-$$

$$-\lambda\sqrt{(2\ell_2+1)(2\ell_3+1)}\,\delta_{\ell_2+\ell_3,2m+1}\,\delta_{m_2 0}\,\delta_{m_3 0}\Big).$$

One-dimensional right extension (36) can be presented as the following change of commutation relations for constraint $L(\sigma)$:

$$\{L_\epsilon, L_\eta\}_Q = \left(\{\epsilon,\eta\}_L + \mu C^{(1)}(\epsilon) - \lambda C^{(1)}(\eta)\right)L\,, \tag{39}$$

and a complete modification due to right and central extensions as

$$\begin{aligned}\{L_\epsilon, L_\eta\}_Q &= L_{\{\epsilon,\eta\}_L} + \mu C^{1}(\epsilon) - \lambda C^{1}(\eta) \;+ \\ &+ N_1 C_1^{(2)}(\epsilon,\eta) + N_2 C_2^{(2)}(\epsilon,\eta) + N_3 C_3^{(2)}(\epsilon,\eta)\,.\end{aligned} \tag{40}$$

The work devoted to the operator realization of this algebra in quantization will be published elsewhere.

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APPENDIX

Let us calculate structure constants $g^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2}$ for the S^2. From the definition (16) one can see that

$$g^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2} = i \int_{S^2} y_{\ell_3 m_3} \{ y_{\ell_1 m_1}, y_{\ell_2 m_2} \}_L \, d\mu \qquad \text{(A 1)}$$

Using the rules of integration by parts for functions given on sphere, these rules are

$$\int_{S^2} \epsilon \partial_2 \eta \, d\mu = - \int_{S^2} \eta \partial_2 \epsilon \, d\mu \, , \qquad \text{(A 2)}$$

$$\int_{S^2} \epsilon \partial_1 \eta \, d\mu = \int \epsilon \eta \Big|_{-1}^{+1} d\sigma_2 - \int_{S^2} \partial_1 \epsilon \cdot \eta \, d\mu \qquad \text{(A 3)}$$

Note that if ϵ or η contain derivatives in σ_2 , then the first term in (A 3) is zero; so we obtain

$$\int_{S^2} \epsilon \{ \eta, \chi \} d\mu = \int_{S^2} \chi \{ \epsilon, \eta \} d\mu = \int_{S^2} \eta \{ \chi, \epsilon \} d\mu$$

$$g^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2} = g^{\ell_2 m_2}_{\ell_3 m_3, \ell_1 m_1} = g^{\ell_1 m_1}_{\ell_2 m_2, \ell_3 m_3} \qquad \text{(A 4)}$$

Structure constants g are real, and since $y^*_{\ell m} = (-1)^m y_{\ell, -m}$, then

$$g^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2} = - g^{\ell_3, -m_3}_{\ell_1, -m_1, \ell_3, -m_3} \qquad \text{(A 5)}$$

Further from $y_{\ell m}(\pi - \theta, \varphi + \pi) = (-1)^\ell y_{\ell m}$ we have

$$g^{\ell_3 m_3}_{\ell_1 m_1, \ell_2 m_2} = 0 \qquad \text{(A 6)}$$

if $\ell_1+\ell_2+\ell_3$ is even number, and, finally, since $g^{\ell_3 m_3}_{\ell_1 m_1,\ell_2 m_2}$ is proportional to $\int \exp i(m_1+m_2+m_3)\sigma_2 \, d\sigma_2$, then

$$g^{\ell_3 m_3}_{\ell_1 m_1,\ell_2 m_2}=0\ , \qquad m_1+m_2+m_3\neq 0 \tag{A 7}$$

Calculate the Lie bracket (11) in the definition of structure constants (16); for that we shall use the formula

$$\partial_1 P_\ell^{|m|}=-(1-\sigma_1^2)^{-1/2}P_\ell^{|m|+1}-\sigma_1(1-\sigma_1^2)^{-1}|m|\cdot P_\ell^{|m|},$$

$$\partial_2 y_{\ell m}=im\,y_{\ell m} \tag{A 8}$$

Then

$$\{y_{\ell_1 m_1},y_{\ell_2 m_2}\}_L=i\,C_{\ell_1 m_1}C_{\ell_2 m_2}\,e^{i(m_1+m_2)\sigma_2}\,.$$

$$\Big[(1-\sigma_1^2)^{-1/2}\big(m_1P_{\ell_1}^{|m_1|}P_{\ell_2}^{|m_2|+1}-m_2P_{\ell_1}^{|m_1|+1}P_{\ell_2}^{|m_2|}\big)+ \tag{A 9}$$

$$+\sigma_1(1-\sigma_1^2)^{-1}\big(m_1|m_2|-m_2|m_1|\big)P_{\ell_1}^{|m_1|}\cdot P_{\ell_2}^{|m_2|}\Big].$$

In order to reduce the upper index $|m|+1$ in Legendre polynomial, we'll use a recursion relation:

$$(1-\sigma_1^2)^{-1/2}P_\ell^{|m|+1}=(2\ell-1)P_{\ell-1}^{|m|}+(1-\sigma_1^2)^{-1/2}P_{\ell-2}^{|m|+1}=$$

$$=\sum_{K=0}^{[\ell-|m|-1/2]}\big[2(\ell-2K-1)+1\big]P_{\ell-2K-1}^{|m|}\ , \tag{A 10}$$

Substituting (A 10) into (A 9) we obtain

$$i\{y_{\ell_1 m_1},\, y_{\ell_2 m_2}\}_L =$$

$$m_2\sum_{K_1}\sqrt{(2(\ell_1-2K_1-1)+1)(2\ell_1+1)}\sqrt{\frac{(\ell_1-|m_1|)\ldots(\ell_1-|m_1|-2K_1)}{(\ell_1+|m_1|)\ldots(\ell_1+|m_1|-2K_1)}}\, y_{\ell_1-2K_1-1,m}\, y_{\ell_2 m_2} \qquad \text{(A 11)}$$

$$-m_1\sum_{K_2}\sqrt{(2(\ell_2-2K_2-1)+1)(2\ell_2+1)}\sqrt{\frac{(\ell_2-|m_2|)\ldots(\ell_2-|m_2|-2K_2)}{(\ell_2+|m_2|)\ldots(\ell_2+|m_2|-2K_2)}}\, y_{\ell_1 m_1}\cdot y_{\ell_2-2K_2-1,m_2}\,.$$

The integral in (A 1) now can be calculated via the Wigner formula:

$$\int y_{\ell_1 m_1} y_{\ell_2 m_2} y_{\ell_3 m_3}\, d\mu = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ 0 & 0 & 0\end{pmatrix} \qquad \text{(A 12)}$$

where $3j$ -symbol is nonzero if ℓ_1 , ℓ_2 and ℓ_3 produce a vector triangle, i.e. none of ℓ_i is larger than a sum of two others; $3j$ -symbol turns to zero if m_i is larger than ℓ_i . Finally we'll obtain that at $m_1 \geqslant 0,\, m_2 \geqslant 0$ or $m_1 \leqslant 0,\, m_2 \leqslant 0$ the expression for structure constants has a form

$$\left(\frac{4\pi}{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}\right)^{1/2} g^{\ell_3 m_3}_{\ell_1 m_1 \ell_2 m_2} =$$

$$m_2\sum_{K_1}(2(\ell_1-2K_1-1)+1)\sqrt{\frac{(\ell_1-|m_1|)\ldots(\ell_1-|m_1|-2K_1)}{(\ell_1+|m_1|)\ldots(\ell_1+|m_1|-2K_1)}}\begin{pmatrix}\ell_1-2K_1-1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\begin{pmatrix}\ell_1-2K_1-1 & \ell_2 & \ell_3\\ 0 & 0 & 0\end{pmatrix} - \qquad \text{(A 13)}$$

$$-m_1\sum_{K_2}(2(\ell_2-2K_2-1)+1)\sqrt{\frac{(\ell_2-|m_2|)\ldots(\ell_2-|m_2|-2K_2)}{(\ell_2+|m_2|)\ldots(\ell_2+|m_2|-2K_2)}}\begin{pmatrix}\ell_1 & \ell_2-2K_2-1 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\begin{pmatrix}\ell_1 & \ell_2-2K_2-1 & \ell_3\\ 0 & 0 & 0\end{pmatrix}$$

When m_1 and m_2 have different signs, $g^{\ell m}_{\ell_1 m_1, \ell_2 m_2}$ can be calculated by means of (A 4).

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**КОЦИКЛЫ ГРУППЫ ДИФФЕОМОРФИЗМОВ ДВУХМЕРНОЙ СФЕРЫ И АНОМАЛИИ
В ТЕОРИИ РЕЛЯТИВИСТСКИХ ПОВЕРХНОСТЕЙ**

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