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PARTON STRUCTURE OF HEAVY MESONS FROM QCD SUM RULES

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ԾԱՆՐ ՄԵԶՈՆՆԵՐԻ ՊԱՐՏՈՆԱՅԻՆ ԿԱՌՈՒՑՎԱԾՔԸ՝ ՍՏԱՑՎԱԾ
ՔՔԴ ԳՈՒՄԱՐՆԵՐԻ ԿԱՆՈՆՆԵՐԻՑ

ՔՔԴ գումարների կանոնների եղանակով հաշվարկված են ծանր քվարկ-պարտոն (c, b) բաշխման ֆունկցիայի մոմենտները ծանր մեզոնում (D, B)։ Արդյունքները վերարտադրում են ծանր քվարկների առաջնորդող լինելու երևույթը և հնարավորություն են տալիս մոդելից դուրս ստանալու ծանր քվարկների՝ ծանր մեզոնների բեկորանալու մոմենտները։

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G.L.BALAYAN, A.Yu.KHODJAMIRIAN, A.G.OGANESSIAN

PARTON STRUCTURE OF HEAVY MESONS
FROM QCD SUM RULES

The moments of heavy quark-parton (c,b) distribution function in heavy meson (D,B) are calculated by means of QCD sum rules. The results reproduce heavy quark leading effect and allow to obtain the moments of fragmentation function of heavy quark into a heavy meson in a model independent way.

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ПАРТОННАЯ СТРУКТУРА ТЯЖЕЛЫХ МЕЗОНОВ ИЗ ПРАВИЛ СУММ КХД

Методом правил сумм КХД вычислены моменты функции распределения тяжелого кварка-партона (c , b) в тяжелом мезоне (D , B). Результаты воспроизводят эффект лидирования тяжелого кварка и дают возможность внемодельным образом получить функцию фрагментации тяжелого кварка в тяжелый мезон.

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1. Introduction

The method of QCD sum rules (SR) [1] which is traditionally applied to calculation of hadron masses and widths, turned out to be useful in attempts to reproduce detailed characteristics of hadrons such as formfactors and parton structure. We recall the first successful calculations [2,3] of π-meson formfactor in intermediate Q^2 region. Among later results there is $D \to K e \nu$ width [4]. At last, recently quark parton distributions in nucleon were reproduced [5,6] by means of SR.

In this paper we shall try to obtain the moments of heavy valence quark (c,b) distribution function in a heavy meson (D,B) applying QCD SR for four-quark correlator. The correlator is chosen in such a form that the lowest resonance contribution to its physical part contains an amplitude of forward elastic scattering of virtual photon on a given heavy meson. As it is demonstrated below, taking a few terms of operator product expansion for this correlator we are able to reproduce both the behaviour and normalization of $c(x,Q^2)$ distribution in D-meson (or $b(x,Q^2)$ in B-meson) in the intermediate Q^2 region.

This distribution itself is of purely heuristic interest since for example D-meson structure function cannot be mea -

sured. But at large Q^2 this function is related [7] with the inclusive $e^+e^- \to D + \ldots$ annihilation structure function. The last function has been already measured [8] (up to the separation of D* contribution).

The peculiar feature of heavy meson parton structure have been predicted long ago [9 - 11] on almost kinematical grounds. The prediction is that heavy quark carries the main share of meson momentum:

$$\langle x \rangle_c = \int x \cdot c(x)\, dx \sim 1 - \mu/m_c$$

where $\mu \sim (\langle q\bar q \rangle)^{1/3} \sim 200$ - 300 MeV is a characteristic hadronic scale. In e^+e^--annihilation this leading effect reveals in the dominance of $C \to D$ fragmentation function at

$$z = 2P_D/\sqrt{s} \sim 1 - \mu/m_c$$

The other important observation based on perturbative QCD [12] is that in e^+e^- - annihilation the heavy quark leading is provided already at the stage of hard gluon bremsstrahlung.

The further hadronization almost does not influence on the quark-parton momentum distributions in heavy hadrons (the so called "soft decoloration ").

In results obtained below confirm in a model independent way both the heavy quark leading effect and the validity of "soft decoloration " for heavy mesons. Finally the c(x) distribution is of certain interest for soft interaction quark-gluon models [13 - 15]. In these models parton distributions are related with parameters of large distance hadron physics. In particular applying the obtained moments of c(x) we fix the unknown value of $c\bar c$ Regge trajectory intercept $\alpha_\psi(0)$

The rest of this paper is organized as follows.

In section 2 necessary calculations of QCD diagrams corresponding to the four-current correlator in Euclidean region are carried out. In section 3 SR for the moments of c-quark distribution in D-meson are obtained. Numerical analysis of these SR is done in section 4. In section 5 the c(x) distribution is discussed from the viewpoint of soft processes. In section 6 the moments of $c \to D$ fragmentation function are reproduced by means of standard QCD evolution. The results are compared with available e^+e^--data. Finally in Section 7 the analogous results for B-meson parton structure are presented.

In this paper we restrict ourselves by pseudoscalar heavy mesons. The calculations for vector D*,B*-mesons can be carried out in the same way, although technically they are more complicated. We plan to present these calculations in future.

2. Calculation of Four-Current Correlator in QCD

Consider the following four-current correlator:

$$\Box_{\mu\nu}(P_1,P_2,q_1,q_2) = \int d^4x\, d^4y\, d^4z\, e^{iq_1x - iq_2y - iP_2z} \times \tag{1}$$

$$\times \langle 0|T\{\bar c\gamma_5 u(0), \bar c\gamma_\mu c(x), \bar c\gamma_\nu c(y), \bar u\gamma_5 c(z)\}|0\rangle$$

The current $\bar c\gamma_5 u$ corresponds to the creation of charmed pseudoscalar meson D^0. For definiteness we take u as a light quark. The replacement $u \to d,s$ ($D^0 \to D^+$,Ds) does not influence on the results obtained below. The $\bar c\gamma_\mu c$ current corresponds to the emission or absorption of a photon by the heavy c-quark.

At fixed external 4-momenta squared P_i^2, q_i^2 (i=1,2) the correlator (1) depends on two independent kinematical invariants $s=(p_1+q_1)^2$ and $t=(q_2-q_1)^2$. Consider the amplitude (1) at $t=0$ in unphysical region $p_i^2, q_i^2, s<0$. If $|p_i^2|, |q_i^2| \gg \mu^2$ (in fact $|p_i^2|, |q_i^2| \sim m_c^2$ is enough for it) this amplitude corresponds to a deep virtual fluctuation and may be calculated in QCD by means of operator product expansion.

It is essential that our choice $t = 0$ is not dangerous since physical $c\bar{c}$-states in t-channel are located for enough (at $t \gtrsim 4m_c^2$). Otherwise we would be forced at once to take the imaginary part of (1) over S, as it was done in [5,6]. In this case the possibility to develop SR over two independent variables p_1^2, p_2^2 would be already lost.

The main contribution to the operator product expansion of the correlator (1) at small distances comes from the unit operator which is simply square quark loop of Fig. 1a. The main power corrections come from quark condensate $\langle\bar{\psi}\psi\rangle$ (dimension d=3) as well as from quark-gluon condensate $g_s\langle\bar{\psi}G_{\mu\nu}\sigma_{\mu\nu}\psi\rangle$ (d=5) (Fig.1b,c). These condensates are essential since their coefficients are proportional to heavy quark mass m_c. As for gluon condensate $\langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle$ (with d=4), we shall not take it into account, since estimates indicate that the corresponding contribution is small. The reason is straightforward. The gluon condensate contribution is determined by loop diagrams which are suppressed as compared with tree diagrams of Fig. 1 b,c. The four-quark condensate with d=6 is inessential since its coefficient is not proportional to m_c.

The zeroth on α_s order of the correlator (1) determined by

the square diagram of Fig. 1a has the following form of standard Feynman integral:

$$\Box^0_{\mu\nu} = 3i\int\frac{d^4 l}{(2\pi)^4}\, Sp\left\{\gamma_5 \frac{1}{\hat{l}-\hat{p}_1}\gamma_5 \frac{1}{\hat{l}+\hat{q}_1-\hat{q}_2-m_c}\gamma_\nu \times \frac{1}{\hat{l}+\hat{q}_1-m_c}\gamma_\mu \frac{1}{\hat{l}-m_c}\right\} \qquad (2)$$

The current mass of light quark is neglected. Hereafter we shall be interested in the invariant amplitude in front of the kinematical structure $g_{\mu\nu}$: $\Box_{\mu\nu} = \Box\cdot g_{\mu\nu}+\ldots$. Let us express this amplitude in terms of Mandelstam double dispersion representation over p_1^2 and p_2^2 variables which are the squared 4-momenta of pseudoscalar $\bar{c}\gamma_5 u$ vertices.

$$\Box^0(p_1^2,p_2^2,q_1^2,q_2^2,s) = \frac{1}{\pi^2}\int\frac{ds_2}{s_2-p_2^2}\int\frac{ds_1}{s_1-p_1^2}\,\rho^0(s_1,s_2,q_1^2,q_2^2,s) \qquad (3)$$

The spectral density $\rho^0(s_1,s_2,q_1^2,q_2^2,s) \equiv \mathrm{Im}_{s_2}\mathrm{Im}_{s_1}\Box^0(s_1,s_2,q_1^2,q_2^2,s)$ is calculated by means of standard Cutkosky rule. Afterwards it is convenient to put $q_1^2=q_2^2=-Q^2$ at once. Then ρ^0 has the form

$$\rho^0 = \delta(s_1-s_2)\,\tilde{\rho}^0(s_1,Q^2,s)\,\theta(s_1-m_c^2)\,\theta(s_2-m_c^2) \qquad (4)$$

This expression reflects the symmetry of square diagram over p_1^2, p_2^2 at $q_1^2=q_2^2$ and $t=0$.

The most suitable expression for the function $\tilde{\rho}^0(s_1,Q^2,s)$ is in the form of dispersion representation over s, differentiated n times:

$$\frac{1}{n!}\frac{\partial^n}{\partial s^n}\tilde{\rho}^0(s_1,Q^2,s) = \int_{s_-}^{s_+}\frac{d\tilde{s}}{(\tilde{s}-s)^{n+1}}\cdot f^0(s_1,Q^2,\tilde{s}) \qquad (5)$$

where $\rho^{\circ}=\frac{3}{8}(s_1-m_c^2)\left\{\left(1+\frac{\tilde{s}}{Q^2}+\frac{s_1+2m_c^2}{Q^2}\right)\left[\left(1+\frac{\tilde{s}}{Q^2}-\frac{s_1}{Q^2}\right)^2+\frac{4s_1}{Q^2}\right]^{-\frac{1}{2}}-\right.$

$$\left.-\frac{4m_c^2}{Q^2}\left(1+\frac{4m_c^2}{Q^2}\right)^{-\frac{1}{2}}\right\},$$

$$S_{\pm}=Q^2 z_{\pm}$$

$$z_{\pm}=s_1\left(\frac{1}{Q^2}+\frac{1}{2m_c^2}\right)-\frac{1}{2}\pm\frac{1}{2}\left(1+\frac{4m_c^2}{Q^2}\right)^{\frac{1}{2}}\left(\frac{s_1}{m_c^2}-1\right).$$

The asymptotics of ρ° at $\tilde{s}\to\infty$ demands that $n\geq 1$. The representation (3) allows double Borel transform in P_1^2 and P_2^2.

$$\square_B^{0}(M_1^2,M_2^2,Q^2,s)=\hat{B}_1\hat{B}_2\square^{0}(P_1^2,P_2^2,Q^2,s)=\frac{1}{\pi^2}\int\frac{ds_1}{M_1^2}\frac{ds_2}{M_2^2}\rho^{\circ}e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}$$

where e.g.

$$\hat{B}_1\equiv\lim_{P_1^2\to-\infty,\,k\to\infty,\,|P_1^2|/k=M_1^2}\left\{\frac{1}{(k-1)!}|P_1^2|^k\left(-\frac{\partial}{\partial P_1^2}\right)^k\right\}$$

Now it is convenient to put $M_1^2=M_2^2=M^2$ taking into account that both channels P_1^2 and P_2^2 are saturated by the same physical states with $\mathcal{D}$ -meson quantum number. Not that the effective Borel parameter is $M^2/2$.

Substituting ρ° from (4), (5), we obtain the final answer for square quark loop contribution at t = 0 as a function of Borel parameter M^2 and kinematical variables Q^2 and S, dif - ferentiated n times over S:

$$\frac{1}{n!}\frac{\partial^n}{\partial s^n}\left(\square_B^{\circ}\right)=\frac{1}{\pi^2}\int_{m_c^2}^{\infty}ds_1\cdot M^{-4}e^{-2s_1/M^2}\times$$
$$\times\int_{S_-}^{S_+}\frac{d\tilde{s}}{(\tilde{s}-s)^{n+1}}\rho^{\circ}(s_1,Q^2,\tilde{s}) \qquad (6)$$

To obtain the total QCD part of SR we need analogous expressions for quark and quark-gluon contributions to amplitude (1). The simplest way to calculate these contributions is to use the familiar Fock-Schwinger gauge. In this gauge

the light quark field expansion has the form:

$$\psi(z)=\psi(0)+z_\rho\mathcal{D}_\rho\psi(0)+\frac{1}{2}z_\rho z_\alpha\mathcal{D}_\rho\mathcal{D}_\alpha\psi(0)+\dots \qquad (7)$$

($\mathcal{D}_\rho$ is covariant derivative). The quark condensate contribution is determined by the Fig.1b diagram, i.e. by the first term of the expansion (7). The quark-gluon condensate $g_s\langle\bar{\psi}\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}\psi\rangle$ contribution consists of two parts. The first part emerges from the third term of (7) (Fig.1b diagram). The second part corresponds to the first term of (7) when simultaneously the heavy quark interaction with gluon condensate is taken in the first order. (Fig.1c diagram). The last interaction is determined by heavy quark propagator expansion:

$$S(x,y)=S^{\circ}(x-y)-g_s\int d^4\xi\, S^{\circ}(x-\xi)\hat{A}(\xi)\,S^{\circ}(\xi-y)+\dots$$

where $A_\mu(x)=\frac{1}{2}x_\rho G_{\rho\mu}(0)$ is the vacuum gluon field in the fixed point gauge.

We obtain the following expressions for the power corrections after Borel transform and differentiation over S:

a)quark condensate contribution:

$$\frac{1}{n!}\frac{\partial^n}{\partial s^n}\left(\square_B^{\langle\bar{\psi}\psi\rangle}\right)=-\frac{m_c\langle\bar{\psi}\psi\rangle L Q^2}{M^4(m_c^2-s)^{n+1}}e^{-2m_c^2/M^2} \qquad (8)$$

where the factor $L=\left\{\ln(m_c/\Lambda)/\ln(\tilde{\mu}/\Lambda)\right\}^{4/9}$ accounts for the anomalous dimension of operator $\langle\bar{\psi}\psi\rangle$ normalized in point $\tilde{\mu}$;

b) quark-gluon condensate contribution:

$$\frac{1}{n!}\frac{\partial^n}{\partial s^n}\left(\square_B^{\langle\bar{\psi}G\psi\rangle}\right)=\frac{m_c g_s\langle\bar{\psi}\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}\psi\rangle Q^2}{2M^4(m_c^2-s)^{n+1}}\times$$

(9)

$$\times\left\{\frac{2m_c^2}{M^4}-\frac{4}{M^2}+\frac{(Q^2+2m_c^2)(n+1)}{(m_c^2-s)M^2}-\frac{1}{Q^2}-\right.$$

$$\left.-\frac{2(n+1)}{3(m_c^2-s)}+\frac{m_c^2(n+1)(n+2)}{(m_c^2-s)^2}\right\}e^{-2m_c^2/M^2}.$$

3. Sum Rules for The Moments of c-Quark Distribution in D-Meson

Let us discuss the physical states contributing to the correlator (1). Represent this correlator in the form of double dispersion integral (3) over variables P_1^2, P_2^2. Instead of ρ^0 there stands physical double spectral function:

$$\rho_{\mu\nu}(s_1,s_2,q_1^2,q_2^2,s)\equiv \mathrm{Im}_{s_2}\mathrm{Im}_{s_1}\square_{\mu\nu}(s_1,s_2,q_1^2,q_2^2,s)$$

The lowest resonance contribution to $\rho_{\mu\nu}$ in both $\bar c\gamma_5 u$ -channels is given by D^0-meson:

$$\rho_{\mu\nu}^{\mathcal{D}}=-\pi^2 g_{\mathcal{D}}^2 m_{\mathcal{D}}^4\,\delta(s_1-m_{\mathcal{D}}^2)\delta(s_2-m_{\mathcal{D}}^2)\,T_{\mu\nu}^c(q,P) \qquad (10)$$

where $g_{\mathcal{D}} m_{\mathcal{D}}^2\equiv\langle 0|i\bar c\gamma_5 u|\mathcal{D}\rangle$ is the matrix element determining $\mathcal{D}\to\mu\nu$ decay. The amplitude

$$T_{\mu\nu}^c(q,P)=\int d^4x\cdot e^{iqx}\langle\mathcal{D}|T\{\bar c\gamma_\mu c(x),\bar c\gamma_\nu c(0)\}|\mathcal{D}\rangle=g_{\mu\nu}T_1^c(Q^2,s)+\ldots \qquad (11)$$

corresponds at t=0, $q_1^2=q_2^2=-Q^2$, $P_1^2=P_2^2=m_{\mathcal{D}}^2$ (in this case $q_1=q_2=q$, $P_1=P_2=P$) to the forward scattering of virtual photon on the c-quark inside D-meson.

Following to the standard assumption of SR method [1].we replace the total contribution of higher states in both chan - nels S_1, S_2 by the contribution of square quark loop from the

region $S_1\geq S_{10}$, $S_2\geq S_{20}$ in dispersion integral (3). It simply means that D-meson contribution is equal to the integral (3) over region $m_c^2\leq S_1\leq S_{10}$, $m_c^2\leq S_2\leq S_{20}$. In is naturally to take the same continuum thresholds in both channels: $S_{10}=S_{20}$

The first impression is that amplitude (11) is not connected with real physics since : 1) we are in unphysical region, 2)Only c-quark currents figure in (11) i.e. the virtual photon scattering on the light quark of D-meson is not taken into account. Nevertheless, the discontinuity of this amplitude in S nonvanishing at positive $S\geq m_{\mathcal{D}}^2$ has a definite physical sense. Indeed, in terms of standard deep inelastic scattering variables $S=m_{\mathcal{D}}^2+Q^2(1/x-1)$ where $x=Q^2/2\nu$, $\nu=q\cdot P$. At large enough $S\sim\nu\sim Q^2$ the parton model of D-meson works. Due to the additivity of virtual photon scattering cross section on separate partons we may relate $\mathrm{Im}T^c$ with c-quark distribution function. Namely, if we define

$$W_{\mu\nu}^c\equiv\frac{1}{4}\sum_X\langle\mathcal{D}|\bar c\gamma_\mu c|X\rangle\langle X|\bar c\gamma_\nu c|\mathcal{D}\rangle(2\pi)^4\,\delta^{(4)}(P+q-P_X)=$$

$$=-\pi\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)W_1^c(\nu,Q^2)+\frac{1}{m_{\mathcal{D}}^2}\left(P_\mu-\frac{\nu q_\mu}{q^2}\right)\left(P_\nu-\frac{\nu q_\nu}{q^2}\right)W_2^c(\nu,Q^2)$$

then at large Q^2, and ν

$$2xW_1^c=W_2^c=x\,c(x,Q^2). \qquad (12)$$

From the other hand applying the familiar relation between discontinuity of the virtual photon forward scattering amplitude and structure functions we obtain

$$\mathrm{Im}_s T_1^c=-2\pi W_1^c(\nu,q^2) \qquad (13)$$

(see e.g. [16]). The other way of arguments leading to (12) and (13) is to imagine from the beginning some fictious photon which interacts only with c-quark.

Making double Borel transform of D-meson contribution in P_1^2, P_2^2 and using (12), (13) we finally obtain at $M_1^2 = M_2^2 = M^2$

$$\frac{1}{n!}\frac{\partial^n}{\partial s^n}\left(\Box_B^D\right) = -\frac{g_D^2 m_D^4}{M^4} e^{-2m_D^2/M^2} \cdot \frac{1}{\pi}\int_{m_D^2}^{\infty} \frac{\mathrm{Im}_s T_1^c(Q^2,\zeta)}{(\zeta - s)^{n+1}} d\zeta =$$
$$= \frac{g_D^2 m_D^4}{M^4} e^{-\frac{2m_D^2}{M^2}} \int_0^1 \frac{dx}{x^2} c(x,Q^2)\, Q^2 \left(m_D^2 - Q^2 + \frac{Q^2}{x} - s\right)^{-n-1} \tag{14}$$

If we now choose $s = m_D^2 - Q^2$ the r.h.s. of (14) immediately transforms into the n-th moment of c-quark distribution with Q^{-2n} weight.

The D-meson contribution to correlator (1) now can be related with the corresponding QCD representation :

$$\frac{\partial^n}{\partial s^n}\left(\Box_B^D\right)\Big|_{s=m_D^2-Q^2} = \frac{\partial^n}{\partial s^n}\left[\Box_B^0 + \Box_B^{\langle\bar\Psi\Psi\rangle} + \Box_B^{\langle\bar\Psi G\Psi\rangle}\right]\Big|_{s=m_D^2-Q^2}$$

Using expressions (14) and (6), (8), (9) for respectively l.h.s, and r.h.s. of this relation we finally obtain SR for the moments of c-quark distribution:

$$M_n^c \equiv \int_0^1 dx \cdot x^{n-1} c(x,Q^2) =$$
$$= g_D^{-2} m_D^{-4} \Big\{ \frac{1}{\pi^2} \int_{m_c^2}^{s_{10}} ds_1 e^{-\frac{2(s_1 - m_D^2)}{M^2}} \int_{z_-}^{z_+} dz\, f^0(s_1, Q^2, Q^2 z) \times$$
$$\times \left(1 + z - m_D^2/Q^2\right)^{-n-1} - m_c\langle\bar\Psi\Psi\rangle \Big\{ L + \frac{m_0^2}{2M^2}\Big[4 - \frac{2m_c^2}{M^2} - (n+1)\times \tag{15}$$
$$\times\left(1 + \frac{2m_c^2}{Q^2}\right)\left(1 - \frac{m_D^2 - m_c^2}{Q^2}\right)^{-1} + \frac{M^2}{Q^2}\Big[1 + \frac{2}{3}(n+1)\left(1 - \frac{m_D^2 - m_c^2}{Q^2}\right)^{-1} -$$
$$- \frac{m_c^2}{Q^2}(n+1)(n+2)\left(1 - \frac{m_D^2 - m_c^2}{Q^2}\right)^{-2}\Big]\Big]\Big\} \times$$

$$\times\left(1 - \frac{m_D^2 - m_c^2}{Q^2}\right)^{-n-1} e^{-2(m_c^2 - m_D^2)/M^2}\Big\}.$$

Recall that we take in (6) an upper limit s_{10} in the integral over s_1 to account for continuum contribution. In (15) the following parametrization was used:

$$g_s \langle\bar\Psi \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} \Psi\rangle = m_0^2 \langle\bar\Psi\Psi\rangle$$

The expressions for f^0, $z_\pm$ were given in (5).

Looking at SR (15), we first notice that the region $Q^2 \sim m_D^2 - m_c^2$ is inaccessible since power corrections blow up there (in fact we enter the physical region $s \sim m_c^2$).

The Bjorken scaling limit $Q^2 \gg m_D^2, m_c^2, M^2$ is well defined:

$$M_n^c = g_D^{-2} m_D^{-4} \Big\{ \frac{3}{8\pi^2 n} \int_{m_c^2}^{s_{10}} ds_1 \left[1 - (m_c^2/s_1)^n\right](s_1 - m_c^2) e^{-\frac{2(s_1 - m_D^2)}{M^2}} -$$
$$- m_c\langle\bar\Psi\Psi\rangle \Big\{ L + \frac{m_0^2}{M^2}\Big[\frac{3}{2} - \frac{n}{2} - \frac{m_c^2}{M^2}\Big]\Big\} e^{-\frac{2(m_c^2 - m_D^2)}{M^2}} \Big\} \tag{16}$$

In this limit quark loop contribution dominates. Of course, we cannot use limit (16) since at $\alpha_s \ln\left[(Q^2 + m_c^2)/\Lambda^2\right] \sim 1$ the leading logarithms of QCD perturbation theory become essential (see below). Nevertheless we argue that there exists an intermediate region where approximate scaling and parton model are valid and simultaneously SR(15) work.

4. The Analysis of Sum Rules and Results for $c(x,Q^2)$ Distribution

We have done the numerical analysis of SR(15) in the region $Q^2 \gg m_D^2 - m_c^2$.

The important circumstance is that in r.h.s. of (15) there are no new parameters. Indeed, both $g_D = f_D/m_c$ in our aefinition and the continuum threshold S_{10} are fixed from SR for two-current [17] and three-current [4] correlators containing $\bar{c}\gamma_5 u$ current: $g_D = 0.13(f_D = 170 \text{MeV})$, $S_{10} = 6 \text{ GeV}^2$. The remaining parameters are universal and have been corrected many times: $m_c = 1.35$ GeV $(p^2 = +m_c^2)$; $\langle\bar{\psi}\psi\rangle = -(240 \text{ MeV})^3$ [1], $m_0^2 = 0.8 \pm 0.2 \text{ GeV}^2$ [18], $\bar{\mu} = 0.5$ GeV, $\Lambda = 100$ MeV

The SR(15) are applicable if a stability region on M_n^c exists with respect to the variation of unphysical Borel parameter M^2. Simultaneously in this region: a) the hierarchy of power corrections is to be valid i.e. the contributions of higher dimension operators are small; b) the continuum contribution is to be suppressed.

Dependence of the first moment $M_1^c = \int c(x,Q^2)dx$ on M^2 following from (15) is shown in Fig.3 at $Q^2 = 20 \text{ GeV}^2$. In the region $2.5 \leq M^2 \leq 3.5 \text{ GeV}^2$ stability is evident and both conditions a) and b) are valid. The contribution of quark-gluon condensate practically vanishes and simultaneously the continuum contribution is less than 35%. The most important result is that in this region $M_1^c \simeq 1$ i.e. the correct absolute normalization of distribution function is reproduced (one c-quark in a D-meson).

The total accuracy of SR determined by the contributions which have not been taken into account (gluon and higher condensates, perturbative α_s-corrections) plus the uncertainty of continuum saturation may be estimated by deviation $1 - M_1^c \sim 0(1\%)$.

The sensitivity of M_1^c to the variation of Q^2 up to $Q^2 = 10 \text{ GeV}^2$ does not exceed 10%. At lower Q^2 as it was noted above scaling violation increases while the power corrections grow up. It is interesting to observe that the approximate scaling at $Q^2 \sim 10 \text{ GeV}^2$ is provided by some compensation of nonscaling terms from quark loop and power corrections.

In the same region of M^2 SR(15) predict a stable value of the second moment which is the mean momentum of c-quark in D-meson: $M_2^c = \langle x \rangle_c = 0.85$. Unfortunately at $n \geqslant 3$ the contribution of power terms grow up. Nevertheless we are able to proceed taking the ratios of neighbouring moments $\tau_n = M_{n+1}^c / M_n^c$. Note that predictions of SR for these ratios are independent of g_D value and are less sensitive to SR parameters.

Numerical analysis in the approximate scaling region $Q^2 \sim 10 \text{ GeV}^2$ shows that up to n=4 the τ_n values are stable at $2 \lesssim M^2 \lesssim 5 \text{ GeV}^2$. In this region τ_n are mainly (up to 80-90%) determined by the quark loop. Quark(quark-gluon) condensate contribution is no more than 10 - 20% (15%) due to cancellations in ratios. The continuum contribution is <15%. In Fig.4 dependence of τ_n on Borel parameter is shown at $Q^2 = 10$ and 20GeV^2. The resulting mean values of moments obtained from τ_n at $M_n^c = 1$ are presented in Table 1. Note that M_2^c coincides with the value derived directly from (15) in absolute normalization.

The large (~ 1) values of M_n^c obtained from QCD SR confirm the c-quark leading effect. The origin of this effect in our approach is in fact due to an approximate duality of D-meson to the quark loop in the region $S_1 \sim M^2 \sim m_D^2$. Indeed, if a free quark pair $c\bar{u}$ has an invariant mass squared $\sim m_D^2 \sim (m_c + \mu)^2$

the energy momentum conservation immediately dictates that in the infinite momentum frame

$$(m_c+\mu)^2 \sim \frac{m_c^2}{x} + \frac{\mu^2}{1-x}$$

where x is the part of momentum carried by c-quark. From this relation $\langle x\rangle_c \sim 1-\mu/m_c$. If we restrict by the quark loop contribution to SR given by (5), (6), it is easy to reproduce the c(x) distribution itself. This distribution is determined by the discontinuity of Fig. 1a diagram in s after Borel transform and continuum subtraction. Up to now we used the integral of this discontinuity. The resulting distribution in the limit $Q^2 \gg m_c^2$ has the following simple form:

$$c(x) \simeq \frac{3M^4}{32\pi^2 g_D^2 m_D^4} e^{2(m_D^2 - \frac{m_c^2}{x})/M^2} \left[1 + \frac{2m_c^2}{M^2}\frac{(1-x)}{x}\right] \theta\left(x - \frac{m_c^2}{S_{10}}\right) \qquad (17)$$

This distribution is shown in Fig.5 at $M^2 = m_D^2$ and $Q^2 = 20 GeV^2$. The boundary at small x is purely kinematical and corresponds to the upper limit $S_1 = S_{10}$ of the $c\bar{u}$-mass. In fact the $x \sim 0$ region is saturated by higher terms of operator product expansion including gluon condensate contribution. Recall that our result for the integral $\int c(x)dx$ indicates that these omitted contributions are inessential. The distribution (17) is inapplicable at $x \leq 1 - \frac{\mu}{m_c}$ since at $x \to 1$ the contribution of quark condensates substantially increases. Really, the interaction of $c\bar{u}$-pair with quark condensate takes place in the limit when all the external momentum is carried by heavy quark line. It is important that the quark condensate $\langle\bar{\psi}\psi\rangle$ which gives the next contribution after quark loop enhances the c-quark leading effect in D-meson.

Let us summarise the results. SR (15) provide approximate scaling in the region $Q^2 \sim 10$ GeV^2 and reliably predict a few moments of distribution function c(x). In the region $m_c^2/S_{10} < x < 1-\mu/m_c$ this distribution is well described by quark loop contribution.

5. The c-Quark Distribution in D-Meson and Soft Processes

Unfortunately we cannot compare obtained results with experiment since deep inelastic scattering on D-meson is impossible. Nevertheless the information about c-quark distribution in D-meson is useful from the point of view of soft processes with charmed hadrons. Two following remarks illustrate this conjecture.

1. The c-quark leading effect means that a share of light quarks and gluons in D-meson is no more than 10 - 15% of total momentum. Therefore the ladders of light partons inside fast D-meson are much less developed as compared with light mesons. Since charmed hadron interaction with nucleons are mainly due to these light partons, we come to conclusion that the mean inelasticity of this interaction is at same level of 10 - 15% (see also [19]). This circumstance is important e.g. for the analysis of anomalously long flying cascades in matter which at high energies might be generated by heavy hadrons.

2. One of the universal models of soft processes formulated in terms of partons is the quark-gluon string model [13,14]. In this model c(x) distribution in D-meson is parametrized as [13]

$$c(x)=\left(\frac{x}{1-x}\right)^{-\alpha_\psi(0)}\theta\left(\tfrac{1}{2}-x\right)+\left(\frac{1-x}{x}\right)^{-\alpha_\rho(0)}\theta\left(x-\tfrac{1}{2}\right) \qquad (20)$$

Using the obtained values of c(x) moments we are able to ex - tract the Regge trajectory intersepts:

$\alpha_\rho(0)=0.2 \div 0.5$; $\alpha_\psi(0)=\div(2.0-3.0)$. The value of $\alpha_\psi(0)$ is very important since there is no independent information on this parameter from soft processes. Recall that $\alpha_\psi(0)$ determines the inclusive spectra of charmed hadrons in the framework of quark-gluon string model [20]. Of course we must keep in mind that the distribution (20) is not normalised at high Q^2 but rather at some soft process scale. Nevertheless taking into account the "poorness" of light quark-gluon sea in D-meson we may argue that difference between c-quark-parton and "structure" c-quark is inessential.

6. QCD- Evolution and the Moments of Fragmentation Function of c-Quark into D-Meson

The c-quark distribution moments obtained from SR are inapplicable at large Q^2 since we have not taken into account gluon radiation and absorption in deep inelastic scattering process. In terms of initial correlator (1) this radiation corresponds to virtual gluon insertions into the quark loop. These diagrams are calculable in the framework of QCD perturbation theory.

Here we must distinguish between two essential effects. First is the effect of $O(\alpha_s)$ corrections to correlator (1) at

moderate Q^2. The calculation of these corrections would im - prove the accuracy of SR by a few percents. This problem is however out of the scape of this paper.

The second effect is essential at large Q^2 when the leading logarithms $\sim\alpha_s \ln(m_c^2+Q^2)$ are summed up. In fact the problem is easily solved using standard QCD evolution equations and taking as an initial conditions the moments M_n^c calculated from SR. The evolution equations can be written in the form

$$Q^2\frac{d}{dQ^2}\begin{pmatrix}M_n^c\\ M_n^q\\ M_n^G\end{pmatrix}=-\frac{\alpha_s(Q^2)}{4\pi}\begin{pmatrix}\gamma_{qq}^n & 0 & \gamma_{qG}^n\\ 0 & \gamma_{qq}^n & 7\gamma_{qG}^n\\ \gamma_{Gq}^n & \gamma_{Gq}^n & \gamma_{GG}^n\end{pmatrix}\begin{pmatrix}M_n^c\\ M_n^q\\ M_n^G\end{pmatrix} \qquad (21)$$

$n\geq 2$ (see e.g. [21]). Here $M_n^G(M_n^q)$ is the moment of gluon distribution (sum of the moments of all quark and antiquark distributions besides c) in D-meson. γ_{qq}, γ_{qG}, γ_{Gq}, γ_{GG} are standard anomalous dimensions. In leading logarithm approximation we neglect the difference between splitting functions of heavy and light quarks. From the results obtained above it follows that at $Q^2\sim 10$ GeV^2 $M_2^G+M_2^q=1-M_2^c\simeq 10-15\%$. Hence the initial moments $M_n^G(Q_0^2), M_n^q(Q_0^2)$ may be neglected in the solution of (21) as compared with $M_n^c(Q_0^2)$. Resulting values of $M_n^c(Q^2)$ at different Q^2 are presented in Table 2. The dependence on the initial point of evolution taken in the region $Q^2\sim 10-20$ GeV^2 is inessential. Note that even at $Q^2\simeq 10^4$ GeV^2 the part of momentum carried by c-quark is still $\gtrsim 0.5$.

We now use the familiar relation [7] between structure function of deep inelastic scattering and structure function of inclusive annihilation. In the scaling limit this relation allows to equate $c(x,Q^2)$ moments to the moments of fragmentation function of c-quark into a D-meson:

$$\int_0^1 c(x,Q^2)\, x^{n-1} dx = \int_0^1 D^c(z,Q^2)\, z^{n-1} dz , \qquad (22)$$

where

$$D^c(z,Q^2) = \frac{1}{\sigma^c(e^+e^- \to D+\ldots)} \frac{d\sigma^c}{dz}(e^+e^- \to D+\ldots) \qquad (23)$$

$z = P_D/E$ is the part of c-quark energy in e^+e^- annihilation $(E=\sqrt{Q^2}/2)$ carried by D-meson. Subscript c in (23) denotes the main part of inclusive cross section of D-meson production in $e^+e^- \to c\bar{c}$ process. Note that (22) is in fact a more general condition than direct relation [7] between structure functions.

We conclude that the values presented in Table 2 are in fact the values of moments of fragmentation function (23). The second moment $M_2^c = \langle z \rangle_D$ reflects the c-quark leading effect in D-meson production. In particular, the value $\langle z \rangle_D = 0.71$ at $Q^2 = 10^3 \mathrm{GeV}^2$ can be compared with the experimental result $\langle z \rangle_D = 0.6 \pm 0.1$ (see e.g. [8]) measured at E = 15 GeV. Note that the last number corresponds to the sum of D-meson spectra from $e^+e^- \to D + \ldots$ and $e^+e^- \to D^* + \ldots$ Therefore complete comparison with experiment will be possible after calculation of D* fragmentation function moments. Detailed experimental data are needed as well.

Using the arguments presented above one may independently be convinced that the $e^+e^- \to c\bar{c}$ process is the main source of leading D-mesons with $z \sim 1$. As it was realized above the mean

part of D^0-momentum carried by light $\bar{u}$-quark $\langle x \rangle_u \leq 1 - \langle x \rangle_c \leq$ 0.10 - 0.15. After evolution this value may only decrease. In analogy with (22) we may argue that $\langle x \rangle_u$ is equal to the mean part of $\bar{u}$-quark energy carried by D^0-meson generated in $e^+e^- \to u\bar{u}$ process. The last argument proofs the statement.

Having at our disposal a few first moments of $c \to D$ fragmentation function we may fix the parameters determining this function in a given hadronization model e.g. in the familiar model suggested in [15].

$$D^c(z) \sim \left(\frac{\varepsilon_c}{1-z} + \frac{1}{z} - 1\right)^{-2} \qquad (24)$$

The value $\langle z \rangle_D = 0.71$ (at 2E = 32 GeV) obtained from SR corresponds to $\varepsilon_c = 0.035$ or $\mu = 250$ MeV (parametrizing $\varepsilon_c = (\mu/m_c)^2$). The higher moments (24) at this choice of ε_c are: $M_3^c = 0.54$, $M_4^c = 0.42$. These values are 5 - 10% lower than corresponding higher moments obtained from SR(see Table 2).

Recall that the correct treatment of heavy quark fragmentation is to be done in two stages [12]:

$$D^c(z) = \int_z^1 \frac{dz_c}{z_c} \tilde{D}^c(z_c) W_D\left(\frac{z}{z_c}\right) \qquad (25)$$

where $\tilde{D}^c(z)$ accounts for hard gluon bremsstrahlung by c-quark and is calculated in leading logarithm approximation of QCD perturbation theory. The second stage is soft hadronization which is described by W_D.

Using SR we moove as if in opposite direction. We start from D-meson itself and fix its parton structure. Then we consider evolution of this structure caused by gluon radiation up to to the scale Q^2. Therefore it is interesting to compare the results of two approaches. For this purpose we use the factorization of moments following from (25):

$$M_n^c = \left[\int_0^1 \tilde{D}^c(z_c)\, z_c^{n-1} dz_c \right] M_n^D$$

where

$$M_n^D = \int_0^1 W_D(x)\, x^{n-1} dx$$

Taking the expression for $\tilde{D}^c(z,Q^2)$ given in [12] as well as the M_n^c values from Table 2 we obtain that moments M_n^D are almost independent of Q^2. It means that according to expectations the energy dependence of fragmentation is due to hard gluon radiation. The values of soft hadronisation function moments turn out to be $M_2^D = 0.92$, $M_3^D = 0.85$, $M_4^D = 0.79$. These values unequivocally confirm the "soft decoloration hypothesis suggested by authors of [12]. According to this hypothesis the influence of soft hadronization on the heavy meson spectrum is almost inessential. If we now parametrize W_D in the form (24) and account for $\sim 10\%$ uncertainty from evolution, then $\varepsilon_D = 6.10^{-4} \div 5.\ 10^{-3}$ or $\mu = 30 \div 100$ MeV at $\varepsilon_D = (\mu/m_c)^2$.

Note that in two stage picture of fragmentation (bremsstrahlung plus hadronization) there is always some uncertainty of where the gluon radiation becomes hard. Our approach is able to avoid this problem. In fact we obtain the moments of total fragmentation function $D^c(z)$ directly comparable with experiment.

7. The Parton Structure of B-Meson

In all expressions presented above we can replace $c \to b$ and immediately obtain analogous results for b-quark parton distribution in pseudoscalar B-meson.

Following [4; 17] we choose $m_b(p^2 = +m_b^2) = 4.8$ GeV, $g_B = 0.027$ (f_B = 130 MeV), $S_{10} = 33\,GeV^2$. The approximate scaling region $Q^2 \gg m_B^2 - m_b^2$ as well as the stability region over Borel mass naturally displace above ($6 \le M^2 \le 10$ GeV^2, $Q^2 = 30 - 70$ GeV^2). The values of power corrections in the b-quark case are less. Quark-gluon condensate contribution to the ratios of b(x) moments doesn't exceed 5%. In Table 1 the moments M_n^b are presented at $Q^2 = 70$ GeV^2. Note that SR predict that moments approach to unit while quark mass increases. Simultaneously the following relation is valid.

$$(1 - \langle x_c \rangle) m_c \simeq (1 - \langle x_b \rangle) m_b \simeq \mu \simeq 150 \div 250 \text{ MeV}$$

The evolution of b-quark distribution moments leads to the moments of b→B fragmentation function presented in Table 2. Uncertainty emerging when light quark and gluon contributions to evolution initial conditions are neglected is no more than 5%. The value $\langle z \rangle_B = 0.85$ at $Q^2 = 10^3$ GeV^2 may be compared with experiment: $\langle z \rangle_B = 0.8 \pm 0.03$ at $E = 15$ GeV [8]. $\langle z \rangle_B = 0.85$ corresponds to $\varepsilon_b = 3.5.10^{-3}$ (μ = 280 MeV) for fragmentation function in the form (24). For this choice of ε_b : $M_3^b = 0.75$ $M_4^b = 0.66$ in good agreement with corresponding moments presented in Table 2. Using two-stage pattern of fragmentation [12] as in the case of c-quark we may extract energy independent moments of soft hadronization function:

$$M_2^B = 0.94, \quad M_3^B = 0.90, \quad M_4^B = 0.86$$

We conclude that "soft decoloration" is more pronounced for heavier quark. These three moments are well described by function (24) at $\varepsilon_\ell = 3.10^{-4}$ (μ = 80 MeV at $\varepsilon_\ell = (\mu/m_\ell)^2$).

Conclusion

The further development is possible in two directions. First, it is important to calculate the contributions of higher terms of operator product expansion including perturbative (nonlogarithmic) corrections. This calculation would substantially improve our results. Second, it is possible to use the suggested method for other heavy hadrons (D* , Λ_c, B*, Λ_b, ...) and to feel the difference between parton structures of various hadrons containing the same heavy quark. Up to now the models similar to [15] were not sensitive to this difference. For detailed comparison with experiment data on separate reactions $e^+e^- \to D + \ldots$, $D^* + \ldots$ are needed at different energies up to $E \approx 50$ GeV (SLC, LEP).

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Table 1. The moments of c(b)-quark distribution in D(B)-meson

	M_n^c		M_n^b
n	Q^2=10GeV^2	Q^2=20GeV^2	Q^2=70GeV^2
2	0,895	0,85	0,94
3	0,83	0,75	0.89
4	0.78	0.675	0.85
5	0.74	0.6I	0.8I

Table 2. The moments of c(b)-quark fragmentation function into D(B)-meson.

Q^2	M_n^c (Q_0^2 = 10GeV^2(20GeV^2))			M_n^b (Q_0^2 = 70GeV^2)		
(10^3GeV^2)	n=2	n=3	n=4	n=2	n=3	n=4
0.5	0.72(0.72)	0.60(0.58)	0.52(0.49)	0.86	0.77	0.70
I.0	0.7I(0.7I)	0.58(0.56)	0.49(0.46)	0.85	0.74	0.67
3.0	0.68(0.68)	0.54(0.53)	0.45(0.43)	0.80	0.70	0.62
5.0	0.67(0.66)	0.53(0.5I)	0.44(0.4I)	0.78	0.68	0.60
T0.0	0.65(0,65)	0.5I(0.49)	0.42(0.40)	0.76	0.66	0.58

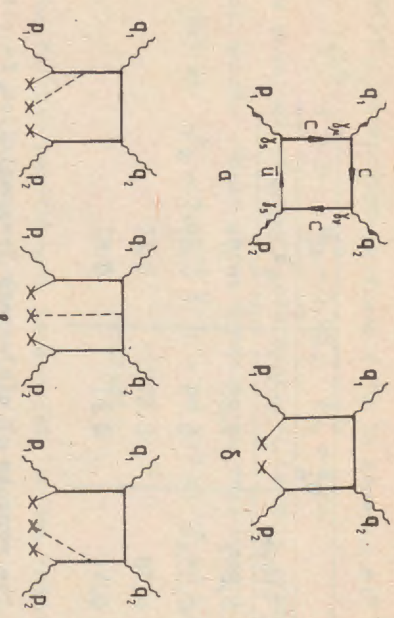

Fig.1.

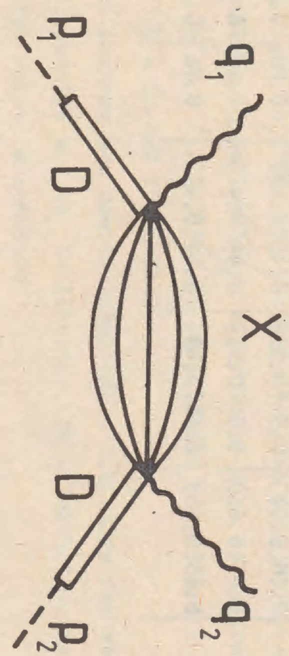

Fig.2.

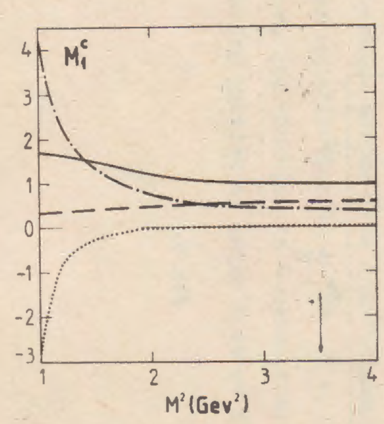

Fig.3.

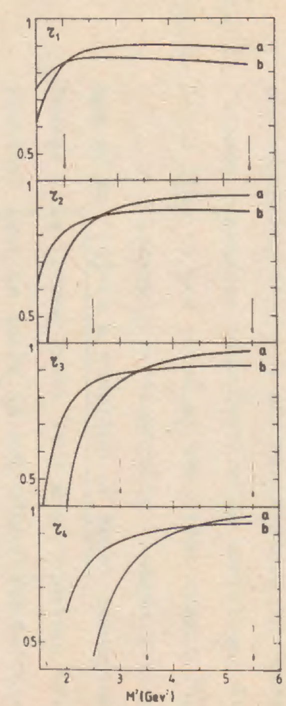

Fig.4.

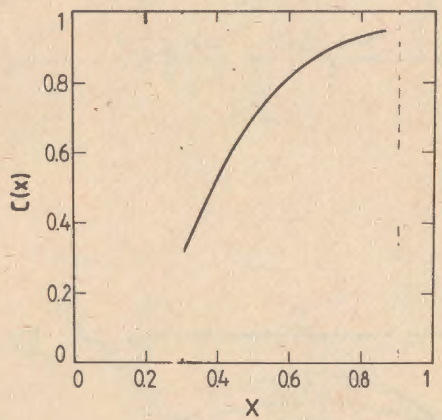

Fig.5.

Figure Captions

Fig.1. QCD diagrams corresponding to correlator (1). Thick (thin) lines denote heavy (light) quarks, dashed lines are gluons. a) diagram of quark loop; b)c)-diagrams corresponding to quark and quark-gluon condensates.

Fig.2. D-meson resonance contribution to correlator (1).

Fig.3. Solid line is the value of the first moment $M_1^c=\int c(x,Q^2)dx$ of c-quark distribution in D-meson following from sum rules (15) as a function of Borel parameter M^2. Dashed line is the quark loop contribution; dash-dotted (dotted) line is quark (quark-gluon) condensate contribution. In the region leftward of the arrow the continum contribution is < 35%.

Fig.4. The ratios of moments of c-quark distribution in D-meson $r_n=M_{n+1}^c/M_n^c$ as functions of Borel parameter M^2. The curves a (b) correspond to $Q^2=10(20)$ GeV^2. In the interval between arrows both quark-gluon condensate and continuum contribution is < 15%.

Fig.5. c-quark distribution in D-meson following from SR in the approximation of quark loop at $Q^2 = 20$ GeV^2, $M^2=m_D^2$.

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