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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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A NUMERICAL STUDY OF A d2 ISING MODEL WITH p-ADIC COUPLING CONSTANT

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Դ.Բ.ՍԱՀԱԿՅԱՆ

ԻԶԻՆԳԻ d 2 ՄՈԴԵԼԻ ԹՎԱՅԻՆ ՃԱՌԱԳԱՅԹՈՒՄԸ p -ԱԴԻԿ ԿԱՊԻ ՀԱՍՏԱՏՈՒՆՈՎ

Հաշվարկված է մոդելի վիճակագրական գումարի արժեքը՝ L , N տարբեր չափերով ցանցի վրա։ Կրիտիկական կետում վիճակագրական գումարի արժեքը ընկած է p -ադիկ թվերի քառակուսային ընդլայնված դաշտում։ Քննվում է այն սահմանը, երբ ցանցի չափերը ձգտում են անվերջության։

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A NUMERICAL STUDY OF A d2 ISING MODEL WITH p-ADIC COUPLING CONSTANT

A value of the statsum of the lattice model with different sizes L, N is calculated. At a critical point the value of the statsum lies in a quadratically extended field of p-adic numbers. The limit when the lattice dimensions tend to infinity is discussed.

Yerevan Physics Institute

Yerevan 1989

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Д.Б.СААКЯН

ЧИСЛЕННОЕ ИЗУЧЕНИЕ d 2 МОДЕЛИ ИЗИНГА С P -АДИЧЕСКОЙ КОНСТАНТОЙ СВЯЗИ

Вычисляется значение статсуммы модели на решетке с разными размерами L , N . В критической точке значение статсуммы лежит в квадратично расширенном поле P -адических чисел. Обсуждается предел, когда размеры решетки стремятся к бесконечности.

Ереванский физический институт
Ереван 1989

In the recent years attempts have been made as to use p-adic numbers to describe theoretical physics models [1,2].

According to Volovich, it makes no difference to the Nature what field of numbers we use to describe its phenomena.

Anyhow, it seems reasonable to study with p-adic numbers the critical phenomena where universality is present and rational critical indices are expected. When attempting to construct p-adic quantum models we encounter ideological complications. Say, it is obscure in what field the wave function should be defined.

In Ref. [3] Green functions are constructed for a free particle and an oscillator which take values in a field of ordinary complex numbers. There is an attractive possibility [4] to use p-adic complex numbers in order to solve conformal field theories at $d \geqslant 3$, since a complex number in p-adics may consist not only of two (as in a real case) but of any amount of usual numbers.

Consider now a statsum (factorized) of a d2 Ising model:

$$Z = \sum_{\sigma} \prod_{(ij)} (1 + x\sigma_i \sigma_j) \qquad (1)$$

At a transition point $x = \sqrt{2} - 1$, so the value of there lies in the quadratic extension of p-adic integer numbers (in the field of numbers of the type $m + n\sqrt{2}$). We'll be interested in a function $Z(L,N)$. In calculations we'll restrict ourselves to a ring of numbers $0, 1, \ldots, p^{\alpha} - 1$. For the case $\alpha = 1$ we have a Galois field, for $\alpha \to \infty$ we come to p-adic numbers.

Calculation (1) in a ring of p^{α} numbers is carried out similarly as in the case of real numbers [5] . Here during calculations we should take α such that $p^{\alpha} > N, L$. In case we need a value of Z for small values of α , we must factorize numbers only in a final answer (i.e. pass from a large value of modulus p^{β} to the required value of p^{α}).

In the case of real values we have [5] :

$$Z = (Z_1 + Z_2 + Z_3)/2$$

$$Z_i^2 = 2^{2LN} \sum_{m,n} \left[(1+x^2)^2 - 2x(1-x^2)(\cos P_m + \cos P_n)\right] \qquad (2)$$

P_m, P_n are momenta determined from the condition:

$$\exp(iLP_m) = \pm 1, \quad \exp(iNP_n) = \pm 1 \qquad (3)$$

where ± 1 is taken versus the boundary condition. At a critical point the amplitude Z_4 corresponding to the periodic boundary condition vanishes. If we throw out the zero eigenvalue, we'll

obtain Z_4 as a statsum of a free scalar particle.

In the case of the ring of numbers p^{α} the similar calculations give

$$Z_i^2 = 2^{2LN} \prod_{m,n} \left[(1+x^2)^2 - x(1-x^2)(u^m + u^{-m} + v^n + v^{-n})\right], \qquad (4)$$

where

$$u^L = \pm 1, \quad v^N = \pm 1 \qquad (5)$$

Calculating with a computer the values of $Z(L,N)$ by formula (4) we come to the answer in a form:

$$n = n_0 + n_1 q + n_2 q^2 + \ldots n_{M-1} q^{M-1}, \qquad (6)$$

where $M/2$ is a minimum number having L, N as divisors.

Expression (6) should be transformed such that only n_0 is left, so one should subtract from (6) the zero vectors of the type:

$$1 + q^{\beta} + q^{2\beta} + \ldots q^{M-\beta} = \frac{1 - q^M}{1 - q} \qquad (7)$$

This can be achieved using the following algorithm.

We find all divisors of the number M . Arrange them in a series according to growth. Take out of (6) the number

$$n_{m_1}(1 + q^{m_1} + q^{2m_1} + \ldots) \qquad (8)$$

We obtain a new expression for n . Now we repeat the procedure for the second divisor, m_2 , and so on. Finally we obtain an expression for the number n where only n_0 is nonzero.

We calculated $Z_i(L,N)$ for lattices with $2 \leqslant L \leqslant 20$, $2 \leqslant N \leqslant 70$. For each L there existed a period T such that

$$Z_i(L,N) = Z_i(L,N+T). \qquad (9)$$

The periods T increased with growing L,N. Presumably for the case of Galois field there are no such periods T_1 and T_2 that for all

$$Z(L,N) = Z_i(L,N+T_1) \qquad (10)$$

$$Z(L,N) = Z_i(L+T_2,N).$$

This points out the absence of limit (in a sense) for infinite lattice. However there is a possibility that this limit exists for the case of p-adic numbers. The numerical data favour that the function $Z_i(L,N)$ continuously (in the p-adic sense) depends on L,N. To elucidate the situation, one should carry out calculations with lattices of larger dimensions.

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