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HETEROTIC STRING BOSONIZATION IN A SPACE  
OF ARBITRARY DIMENSION



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Գրին-Շվարցի հետերոտիկ լարի համար դուրս է բերված ոչ-Աբելյան  
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1. The evaluation of Polyakov surface-integral quantization approach for the Green-Schwarz (GS) superstring is essential for investigation of strings in non-critical dimensions. A new development in quantization of the Liouville action has recently been performed by Polyakov in [1]. He reduced the problem of quantization of the Liouville action to the investigation of  $SL(2, \mathbb{R})$  2-dimensional conformal field theory.

What kind of two-dimensional conformal field theory can correspond to the GS heterotic superstrings? This is the question raised here. For this purpose we evaluate fermionic functional integral associated with GS heterotic superstring. In other words we carry out non-abelian bosonization of the GS heterotic superstring. The computations are the generalization of the technique developed in [2,3] which describes the evaluation of the fermionic determinants. This follows the line of thought indicated in [4,5], according to which two-dimensional fermion determinants are computable in closed form when there are as many anomalies as function parameters in the lagrangian.

The answer turns out to be expressible in terms of Liouville action, together with a Wess-Zumino-Novikov-Witten multivalued functional, whose integrand is the Chern-Simons density for the group  $SO(2) \times SO(d-2)$  evaluated at a connection of  $so(2) \times so(d-2)$ . So, we must develop  $\frac{SO(d)}{SO(2) \times SO(d-2)}$  conformal field theory constrained by Peterson-Codazzi consistency conditions of embedding two-dimensional surface into  $R^d$ .

2. The action of left-side moving part of heterotic GS superstring is

$$S(\vec{x}, \theta, g) = \frac{1}{\pi} \int d^2 \xi \left[ \sqrt{g} g^{\alpha\beta} (\partial_\alpha X^\mu - K_\alpha^\mu) (\partial_\beta X_\mu - K_{\beta\mu}) + i \epsilon^{\alpha\beta} \partial_\alpha X_\mu K_\beta^\mu \right] \quad (1)$$

where  $K_\alpha^\mu = i \bar{\theta} \gamma^\mu \partial_\alpha \theta$ ,  $\mu = 1, 2, \dots, d$ ;  $X^\mu(\xi)$  - world surface coordinates,  $\theta(\xi)$  -  $SO(d)$  spinors;  $g_{\alpha\beta}(\xi)$  - world-sheet internal metric parametrized by  $\xi = \{\xi^\alpha, \alpha = 1, 2\}$

Our aim is to carry out functional integration over in the case of genus  $G=0$ :

$$e^{W(g, \vec{x})} = \int D\theta \cdot e^{-S} \quad (2)$$

For exact integration we must remove four-fermionic term in  $S$  and have gaussian integral over  $\theta(\xi)$ .

For each point on the surface  $\Sigma$ , consider basic vectors  $\{\partial_\alpha X^\mu, n_i^\mu\}$ ,  $\alpha = 1, 2$ ,  $i = 1, \dots, d-2$ ;  $n^\mu \cdot \partial_\alpha X_\mu = 0$ ,  $n_\mu^i \cdot n^{j\mu} = \delta^{ij}$

Let's induce on the surface  $\Sigma$  Dirac matrices

$$\gamma_\alpha(\xi) = \partial_\alpha X^\mu \cdot \gamma_\mu; \quad \gamma^i(\xi) = n_\mu^i \cdot \gamma^\mu \quad (3)$$

where  $\gamma^\mu$  - flat Dirac matrices in  $R^d$ . Introduce a zweibein  $e_\alpha^a d\xi^\alpha$ ,  $a = 1, 2$  with

$$\{\gamma_\alpha, \gamma_\beta\} = 2 \partial_\alpha X^\mu \partial_\beta X_\mu = 2 e_\alpha^a e_\beta^a, \quad (4)$$

and orthonormal basic set of Dirac matrices

$$\gamma_\alpha(\xi) = e_\alpha^a \gamma_a(\xi), \quad \gamma_i(\xi). \quad (5)$$

Using local Siegel supersymmetry of  $S$  we can fix a gauge

$$\gamma^+(\xi) \theta(\xi) = 0, \quad (6)$$

where  $\gamma^+ = \gamma^1 + i\gamma^2$ .

So we have covariant chiral fermions.

The Gausse-Weingarten formulae

$$d\gamma^a = \Gamma^{ab} \gamma_b + h^{ai} \gamma_i, \quad (7)$$

$$d\gamma^i = -h^{ai} \gamma_a + H^{ij} \gamma_j,$$

where  $\Gamma^{ab}$  -  $SO(2)$  - tangent space connection and

$H^{ij}$  -  $SO(d-2)$  - normal space connection,

help us to rewrite the term

$$\frac{1}{2} g^{\alpha\beta} K_\alpha^\mu K_\beta^\mu = -\frac{1}{8} (\bar{\theta} \gamma^i \gamma^j \gamma^k \theta) h_\alpha^{j+} (\bar{\theta} \gamma^i \gamma^k \gamma^- \theta) h_\beta^{k+} = -\frac{g^{\alpha\beta}}{2} B_\alpha^i B_\beta^i. \quad (8)$$

Now using the identity

$$\exp(-1/2 \int \sqrt{g} d^2 \xi g^{\alpha\beta} B_\alpha^i B_\beta^i) = \int D A_\alpha^i \exp(-1/2 \int \sqrt{g} d^2 \xi (A_\alpha^i A_\beta^i - 2i A_\alpha^i B_\beta^i)) \quad (9)$$

we obtain a new action

$$S(A^i, X^\mu, \theta) = \frac{1}{\pi} \int \sqrt{g} d^2 \xi \left[ \frac{1}{2} (\partial_\alpha X^\mu \cdot \partial_\beta X^\mu + A_\alpha^i A_\beta^i) - i \bar{\theta} \not{D} \theta \right]. \quad (10)$$

Consider Spin(d) operation  $\Omega$  which transform  $\gamma^a(\xi)$ ,  $\gamma^i(\xi)$  into the flat Dirac matrices  $\gamma^\mu$

$$\Omega^{-1}(\xi) \gamma_\mu \Omega(\xi) = \gamma_{\alpha, i}(\xi)$$

One has

$$\Omega^{-1} d\Omega = \frac{1}{8} \left\{ \Gamma^{ab} [\gamma_\alpha, \gamma_\beta] + H^{ij} [\gamma_i, \gamma_j] + 2h^{ai} [\gamma_\alpha, \gamma_i] \right\}. \quad (11)$$

The Dirac operator  $\not{D}$  contains the fields  $\Gamma^{ab}$ ,  $H^{ij}$  and  $g^{\alpha\beta} A_\alpha^i h_\beta^{j+}$ . Let separate symmetric and antisymmetric part in

$$g^{\alpha\beta} A_\alpha^i h_\beta^{j+} \gamma_i \gamma_j = e_{\alpha+} \left( g^{\alpha\beta} - \frac{i\varepsilon^{\alpha\beta}}{\sqrt{g}} \right) (A_\beta^{ij} [\gamma_i, \gamma_j] + a_\beta). \quad (12)$$

After the fixing of the conformal gauge  $g^{\alpha\beta} = \rho \delta^{\alpha\beta}$  and notating

$$\rho e_{-+} = e^{2q}, \quad a_+ = \partial_+ a, \quad (A_+ - \frac{1}{4} H_+)^{ij} \gamma_{ij} = g^{-1} \partial_+ g \quad (13)$$

we obtain

$$\not{D} = (\Omega^{-1} g^{-1} e^{q-a}) \gamma_- \partial_+ (e^{q+a} g \Omega) \quad (14)$$

The determinant of  $\not{D}$  we compute following to Kavalov et al. 2 and Leutwyler 6 and the final effective action is

$$W(\bar{X}, g, A) = \frac{2^{d/2}}{24\pi} \int d^2 \xi \left( \partial_+ q \cdot \partial_- q + \mu^2 e^q + \right. \quad (15)$$

$$\left. + \frac{1}{2\pi} \text{tr} \left[ \frac{1}{2} \int_{\Sigma} d^2 \xi \partial_\alpha (\Omega^{-1} g^{-1} e^{-a}) \cdot \partial_\alpha (e^a g \Omega) + \frac{i}{3} \int_{\Sigma} \left\{ (\Omega^{-1} g^{-1} e^{-a}) d(e^a g \Omega) \right\}^3 \right]. \right.$$

In the gauge  $g=1$ ,  $e^a=1$  we have

$$\int_{\Sigma_t} (\Omega^{-1} d\Omega)^3 = \frac{2^{d/2}}{16} \int_{\Sigma_t} (\Gamma_{ab} d\Gamma^{ab} + H_{ij} dH^{ij} - \frac{2}{3} (H^{ij})^3). \quad (16)$$

In order to write down the last term in (15) in the general coordinate-invariant form we recall that the squared Dirac operator metric  $D_+ D_-$  or  $D_- D_+$  whose curvature  $R\sqrt{g}$  enters the expansion series of its kernel has a form:

$$(g^{\alpha\delta} - \frac{i\varepsilon^{\alpha\delta}}{\sqrt{g}}) e_{\delta+} (g^{\beta\gamma} + \frac{i\varepsilon^{\beta\gamma}}{\sqrt{g}}) e_{\gamma-} \partial_\alpha \partial_\beta = \lambda g^{\alpha\beta} \partial_\alpha \partial_\beta, \quad (17)$$

where we used the identity:

$$(g^{\alpha\delta} - \frac{i\varepsilon^{\alpha\delta}}{\sqrt{g}}) (g^{\beta\gamma} + \frac{i\varepsilon^{\beta\gamma}}{\sqrt{g}}) = (g^{\alpha\beta} - \frac{i\varepsilon^{\alpha\beta}}{\sqrt{g}}) (g^{\delta\gamma} + \frac{i\varepsilon^{\delta\gamma}}{\sqrt{g}}). \quad (18)$$

Insofar as at  $g^{\alpha\beta} \rightarrow \lambda g^{\alpha\beta}$ ,  $R\sqrt{g} \rightarrow R\sqrt{g} - \partial_\alpha (\sqrt{g} g^{\alpha\beta} \partial_\beta \log \lambda)$ , a covariant expression coinciding with the last term in (15) at the fixed gauge (13) will be

$$W_L = \frac{2^{d/2}}{192\pi} \int d^2 \xi \left[ R\sqrt{g} - \nabla^2 \log \lambda \right] \frac{1}{\nabla^2} \left[ R\sqrt{g} - \nabla^2 \log \lambda \right]. \quad (19)$$

Thus, instead of a usual Liouville action  $\int d^2 \xi R\sqrt{g} \frac{1}{\nabla^2} R\sqrt{g}$  we have obtained a modified one, which corresponds to a double set of fields in the bosonized theory of heterotic superstring

relative to the bosonized theory of Dirac action induced on two-dimensional surface. Indeed, along with the terms connected with the metric induced on the surface and with  $SO(2) \times SO(d-2)$  gauge fields stipulated by rotation of basic tangents and normals to the surface of vectors at its every point (which were obtained in 3), the effective action (15) contains a metric and  $SO(2) \times SO(d-2)$  gauge fields that are "internal" with respect to the surface.

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