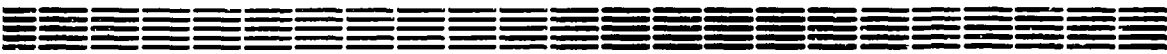


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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ  
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**MULTIPLICATIVE RENORMALIZATION OF  $N=1$   
SUPERSYMMETRIC YANG-MILLS THEORIES:  
THE  $SU(2)$  GROUP**

ЦНИИатоминформ  
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տրամաբանական լոգիկայի հիմունքները և դրանց կիրառությունները

686 է դրամ

տրամաբանական լոգիկայի հիմունքները

Երևանի պետական համալսարանի ֆիզիկա-մաթեմատիկական գիտությունների ֆակուլտետի մաթեմատիկական խումբի անդամները և Երևանի պետական համալսարանի ֆիզիկա-մաթեմատիկական գիտությունների ֆակուլտետի մաթեմատիկական խումբի անդամները (1-15) գիտական աշխատանքները  
, նախընտրելով տրամաբանական լոգիկայի հիմունքները, Երևանի պետական համալսարանի ֆիզիկա-մաթեմատիկական գիտությունների ֆակուլտետի մաթեմատիկական խումբի անդամները և Երևանի պետական համալսարանի ֆիզիկա-մաթեմատիկական գիտությունների ֆակուլտետի մաթեմատիկական խումբի անդամները

Երևան (Հ)ՈՏ Երևանի պետական համալսարան

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MULTIPLICATIVE RENORMALIZATION OF N=1  
SUPERSYMMETRIC YANG-MILLS THEORIES:  
THE SU(2) GROUP

In supersymmetric gauge in a specially chosen parametrization of a supergauge multiplet the multiplicativity of renormalization of general-class supergauge theories with nonexpanded (N=1) supersymmetry is proved.

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МУЛЬТИПЛИКАТИВНАЯ ПЕРЕНОМИРОВКА  $N = 1$   
СУПЕРСИММЕТРИЧНЫХ ЯНГ-МИЛЛОВСКИХ ТЕОРИЙ: ГРУППА  $SU(2)$

Доказана мультипликативность перенормировки суперкалибровочных теорий общего класса с нерасширенной ( $N = 1$ ) суперсимметрией в суперсимметричной калибровке в специально выбранной параметризации суперкалибровочного мультиплетта.

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1. This paper is devoted to prove multiplicativity of renormalization of general-class supergauge theories with nonexpanded ( $N=1$ ) supersymmetry [1,2] in supersymmetric gauge in a specially chosen parametrization of a supergauge multiplet. Many works are devoted to the problem of the renormalizability of supersymmetric theories. Limiting ourselves to theories with  $N=1$ , let us present the results of refs [3,4], where the renormalization structure was investigated for the general-class supergauge theories (including matter fields) in supersymmetric gauge as well as in the Wess-Zumino gauge [1]. It was in particular shown that in both gauges renormalization conserves the supergauge and supersymmetries. In the Wess-Zumino gauge the renormalization is a multiplicative one, but in the supersymmetric gauge the renormalization, which is multiplicative for all the parameters and fields except for the gauge supermultiplet and which for the gauge supermultiplet has the form of a nonlinear substitution of variables, does not contradict to the Ward's identities. Such situation is due to dimensionless gauge supermultiplet and is typical for all the theories with nonlinearly realized symmetry. In particular, renormalization of general - class two-dimensional chiral theories is also accompanied by nonlinear reparametrization [5]. Nevertheless, in some two-dimensional symmetric chiral models (e.g.,  $O(n)/O(n-1)$ [6],  $SU(n) \times SU(n)$  [7])

the renormalization multiplicativity has been proved. The proof was performed for special parametrization of theory using the Zinn-Justin method [8]. By modifying the scheme [7] it was later proved that a wide class of two-dimensional chiral theories could be multiplicatively renormalized [9].

The main problem arising is that of the choice of a new parametrization.

The above-mentioned considerations as well as similarity of the structures of renormalization of two-dimensional chiral and supergauge theories allow one to assume that in supergauge  $N=1$  theories too, by suitable choice of parametrization one can achieve an overall multiplicativity of renormalization in the supersymmetric gauge. The content of this paper just consists in the solution of this task. The form of the supergauge multiplet parametrization will be described below. The paper is planned as follows. In section 2 the modified action  $S$  of the theory is constructed, an equation for the producing  $\Gamma$ -functional resulting from the BRST-symmetry is derived and its consequences are studied to find the structure of theory divergences. In section 3 the Ward's identities resulting from global gauge symmetry are analyzed for the same purpose. In section 4, on the basis of the results from the sections 2 and 3, the multiplicativity of renormalization for the case with the gauge symmetry  $SU(2)$  group and a concrete parametrization is proved. We shall as far as possible use condensed notations. Derivatives with respect to the fields are the right hand ones, and those with respect to the sources are the left hand ones.

2. Thus, we shall consider a theory containing chiral matter superfields  $\Psi_{\pm}^i(x, \theta)$  as well as Hermitian fields of the supergauge multiplet  $V \equiv V^{ij}(x, \theta) = \Gamma_a^{ij} V^a(x, \theta)$ . The anti-Hermitian matrices  $\Gamma_a$  are generators of super gauge transformations of matter fields the parameters of which are the chiral superfields  $\Lambda_+ = \Gamma_a \Lambda_+^a(x, \theta)$ :

$$\Psi_+ \rightarrow \Psi'_+ = e^{\Lambda_+} \Psi_+; \quad \Psi_- \rightarrow \Psi'_- = e^{-\Lambda_+^\dagger} \Psi_- \quad (1)$$

The gauge supermultiplet is transformed as follows:

$$e^{iV} \rightarrow e^{iV'} = e^{-\Lambda_+^\dagger} e^{iV} e^{-\Lambda_+} \quad (2)$$

The supergauge- and supersymmetry-invariant part of the action has the form

$$S_{SS} = \frac{1}{g^2} S_V(V) + S_{V,\psi} + S_\psi,$$

where  $S_\psi$  is expressed as

$$S_\psi \sim m_+ \Psi_+^2 + f_+ \Psi_+^3 + m_- \Psi_-^2 + f_- \Psi_-^3 + h.c.$$

and  $S_V$  and  $S_{V,\psi}$  represent the self-interaction of the supergauge multiplet and its interaction with matter fields supermultiplets, respectively (explicit form of  $S_{SS}$  will not be henceforth necessary (see, e.g. [2]; the set of all the matter fields  $\Psi_{\pm}, \Psi_{\pm}^\dagger$  will be hereinafter denoted by  $\Phi$ ).

Not yet defining concretely the form of the supergauge multiplet new parametrization, let us generally represent transition to it by

$$(e^{i\tilde{\Gamma}_a V^a})^{ij} \equiv M^{ij}(x, \theta) = i\tilde{\Gamma}_a^{ij} \pi^a(V(x, \theta)) + T_n^{ij} \sigma^n(\pi(V)) \quad (3)$$

where  $\tilde{\Gamma}_a$  does not coincide with  $\Gamma_a$  in general;  $\tilde{\Gamma}_a$  and  $T_n$  represent the full set of matrices. The gauge transformation (2)

now will be written as  $(\Lambda_+ = \Lambda_+^a \tilde{\Gamma}_a)$

$$M^{ij}(\pi(x, \theta)) \rightarrow M'^{ij}(\pi(x, \theta)) = (e^{-\Lambda_+^\dagger} M(\pi(x, \theta)) e^{-\Lambda_+})^{ij}$$

For infinitesimal  $\Lambda_+(x, \theta)$  we have

$$\delta M(\pi) = -\Lambda_+^\dagger M - M \Lambda_+ = \tilde{\Gamma}_a \delta \pi^a + T_n \delta \mathcal{G}^n(\pi),$$

whence the law (linear) of transformation of  $\mathbb{H}$  and  $\mathcal{G}(\pi)$  one via the other is obtained at group transformations. Further we shall consider a theory where instead of  $V^a$  the fields  $\mathbb{H}^a$  are chosen as the main supergauge fields, and we shall introduce the set  $(\mathbb{H}^a, \mathcal{G}^n(\pi)) \equiv \omega^r(\mathbb{H})$ . In terms of the fields  $\omega^r$  the infinitesimal supergauge transformations have the form

$$\delta \omega^r = R_{+a}^r(\omega) \Lambda_+^a + R_{-a}^r(\omega) \Lambda_+^{+a} = R_A^r(\omega) \Lambda^A = R_{AS}^r \omega^S \Lambda^A, \quad (4)$$

where the following notations are introduced

$$\begin{aligned} R_{+a}^r + R_{-a}^r &= R_{(\alpha)\alpha}^r, & i(R_{+a}^r - R_{-a}^r) &= R_{(\beta)\alpha}^r, & (R_{(\alpha)\alpha}^r, R_{(\beta)\alpha}^r) &\equiv R_A^r \\ \Lambda_+^a + \Lambda_+^{+a} &= 2\Lambda_{(\alpha)}^a, & \Lambda_+^a - \Lambda_+^{+a} &= 2i\Lambda_{(\beta)}^a, & (\Lambda_{(\alpha)}^a, \Lambda_{(\beta)}^a) &\equiv \Lambda^A, \end{aligned} \quad (5)$$

and  $R_{AS}^r$  are already numerical matrices.

Let us now describe the modified actions. As a gauge function choose the function  $T^A = t_r^A \bar{D}D \omega^r$ , where  $t_r^A$  are some (gauge) parameters and the gauge-fixing term  $S_{gf}$  in the total action represent as

$$\begin{aligned} S_{gf} &= \int d^4x d^4\theta [B_{+a} B_{-a} + B_A t_r^A \bar{D}D \omega^r] = \\ &= \int d^4x d^4\theta [B_{+a} B_{-a} + (B_{+a} t_r^{-a} + B_{-a} t_r^{+a}) \bar{D}D \omega^r], \end{aligned} \quad (6)$$

$$\mathfrak{z}_A = \left( \frac{1}{2}(B_{+a} + B_{-a}), \frac{1}{2i}(B_{+a} - B_{-a}) \right), \quad t_r^A = (t_r^{+a} + t_r^{-a}, i(t_r^{-a} - t_r^{+a})).$$

Let us by general rules introduce the following action for

the ghost fields

$$S_{gh} = \int d^4x d^4\theta \mathcal{D}_A \bar{D} D t_r^A R_B^r C^B = \int d^4x d^4\theta \tilde{\mathcal{D}}_A t_r^A R_B^r C^B,$$

where notation  $\bar{D} D \mathcal{D}_A = \tilde{\mathcal{D}}_A$  is used, and the fields  $C^B$  and  $\mathcal{D}_A$  consist of chiral fields and fields Hermitianly-conjugated with them in analogy with the formulae (5).

Now we can write the modified action

$$S = S_{SS}(\pi, \Phi) + S_{gf} + S_{gh} + \left[ K_{\Phi, I} R_{\Phi, A}^I(\Phi) C^A + K_r R_A^r(\omega) C^A + \right. \\ \left. + \frac{1}{2} L_D f_{AB}^D C^A C^B + \lambda_r \omega^r + \chi_r^A \omega^r \tilde{\mathcal{D}}_A \right] \equiv \bar{S} + \left[ \lambda_r \omega^r + \chi_r^A \omega^r \tilde{\mathcal{D}}_A \right], \quad (7)$$

where  $\lambda_r$  is an external source, a general real superfield;  $\chi_r^A$  are constant Grassmanian numbers;  $K_{\Phi, I}$ ,  $K_r$  and  $L_D$  are external sources, the chiral properties of which are presented in ref. [3];  $f_{AB}^D$  are structure constants:

$$R_B^b \frac{\delta R_A^a}{\delta \pi^b} - R_A^b \frac{\delta R_B^a}{\delta \pi^b} = f_{BA}^D R_D^a. \quad (8)$$

The modified action  $\bar{S}$ , as a consequence of the BRST-symmetry, satisfies the following equation (this kind of equation for the pure Young-Mills theory was for the first time written by Zinn-Justin [8]).

$$\frac{\delta \bar{S}}{\delta \Phi^I} \frac{\delta \bar{S}}{\delta K_{\Phi, I}} + \frac{\delta \bar{S}}{\delta \pi^a} \frac{\delta \bar{S}}{\delta K_{r, a}} + \frac{\delta \bar{S}}{\delta C^A} \frac{\delta \bar{S}}{\delta L_A} + \frac{\delta \bar{S}}{\delta \tilde{\mathcal{D}}_A} \bar{D} D B_A = \\ = \lambda_r \frac{\delta \bar{S}}{\delta K_r} + \chi_r^A \frac{\delta \bar{S}}{\delta \chi_r^A}.$$

The last term in the right hand side of this equation clarifies the meaning of  $\chi_r^A$ :  $\chi_r^A$  is the BRST-variation of the parameter  $t_r^A$  [10]. Introducing in a standard way the producing functional  $\Gamma$  of vertex functions, we obtain that it satisfies

a similar, as S does, equation

$$\begin{aligned} \frac{\delta\Gamma}{\delta\Phi^I} \frac{\delta\Gamma}{\delta K_{\Phi,I}} + \frac{\delta\Gamma}{\delta\pi^a} \frac{\delta\Gamma}{\delta K_{\pi,a}} + \frac{\delta\Gamma}{\delta C^A} \frac{\delta\Gamma}{\delta L_A} + \frac{\delta\Gamma}{\delta\tilde{\mathcal{D}}_A} \bar{D}DB_A = \\ = \lambda_r \frac{\delta\Gamma}{\delta K_r} + \chi_r^A \frac{\delta\Gamma}{\delta t_r^A} . \end{aligned}$$

For the divergent part of the one-loop approximation of  $\Gamma$  (denote it by  $\Gamma_{1d}$ ) we obtain:

$$\frac{\delta S}{\delta\Phi^I} \frac{\delta\Gamma_{1d}}{\delta K_{\Phi,I}} + \frac{\delta\Gamma_{1d}}{\delta\Phi^I} \frac{\delta S}{\delta K_{\Phi,I}} + \frac{\delta S}{\delta\pi^a} \frac{\delta\Gamma_{1d}}{\delta K_{\pi,a}} + \frac{\delta\Gamma_{1d}}{\delta\pi^a} \frac{\delta S}{\delta K_{\pi,a}} + \quad (9)$$

$$\frac{\delta S}{\delta C^A} \frac{\delta\Gamma_{1d}}{\delta L_A} + \frac{\delta\Gamma_{1d}}{\delta C^A} \frac{\delta S}{\delta L_A} + \frac{\delta\Gamma_{1d}}{\delta\tilde{\mathcal{D}}_A} \bar{D}DB_A = \lambda_r \frac{\delta\Gamma_{1d}}{\delta K_r} + \chi_r^A \frac{\delta\Gamma_{1d}}{\delta t_r^A} .$$

Note that since the supersymmetry transformations are linear over the fields, the supersymmetry of S results in that of  $\Gamma$ . Consequently,  $\Gamma_{1d}$  is a supersymmetric functional. Let us now construct the general expression for  $\Gamma_{1d}$  issuing from the Feynman diagram divergence index. The latter (with account of all additional vertice) is equal to

$$\omega_G = 2 - N - 2N_\lambda \quad (10)$$

where N is the total number of the external to the diagram superfields:  $C^A, \tilde{\mathcal{D}}_A, \Phi^I, K_{\Phi,I}, L_A$  and  $\chi_r^A$  ( $\chi_r^A$  has a mass dimension and its ghost number is -1),  $N_\lambda$  denotes the number of vertices  $\lambda\omega$ . It is seen from (10) that with account of the ghost number conservation  $\Gamma_{1d}$  does not depend on  $L_A$  at all, the source  $K_r$  can be included in  $\Gamma_{1d}$  only linearly, in the form of production of  $KC$  and  $K\chi$ ;  $\chi$ , in its turn, can also be included only linearly, and besides  $K\chi$  - in the form of the production  $\chi\tilde{\mathcal{D}}$ . The matter fields  $\phi$  can be included in  $\Gamma_{1d}$

no more than quadratically. [There are no divergences having the structure of the production of the fields and sources with the same chirality [11,12]. Thus, the general structure of  $\Gamma_{1d}$  is

$$\begin{aligned} \Gamma_{1d} = & S_1(\Phi, \pi, t) + \int [B_{+a} f^{ab}(\pi, t) B_{-b} + B_{+a} g_{-}^a(\pi, t) + \\ & B_{-a} g_{+}^a(\pi, t) + \tilde{\omega}_D R_{1B}^D(\pi, t) C^B + K_r Q_{1B}^r(\pi, t) C^B + \lambda_r \omega_1^r(\pi, t) + \\ & + \chi_r^A \mathcal{X}_{1A}^r(\pi, t) \tilde{\omega}_B + \chi_r^A \xi_{1A}^{rs}(\pi, t) K_s], \end{aligned} \quad (11)$$

where  $S_1$  contains matter fields in the combination  $\Phi\Phi^+$ . The functional  $S_1$ , as well as the coefficient functions  $f^{ab}$ ,  $g_{\pm}^a$ ,  $R_{1B}^D$ ,  $Q_{1B}^r$ ,  $\omega_1^r$ ,  $\mathcal{X}_{1A}^{rB}$ ,  $\xi_{1A}^{rs}$  depend in general on  $\pi$  and  $t$ . Besides, it follows from the supersymmetry of  $\Gamma_{1d}$  that these coefficient functions are superfields and  $S_1$  is a supersymmetric functional. It particularly means, that all the coefficient functions are unambiguously restored from their values at  $\Theta = 0$ .

Substituting the expression (11) for  $\Gamma_{1d}$  into (9) and setting to zero the expressions before different field structures, we obtain the following set of equations:

$$\frac{\delta R_B^r}{\delta \pi^a} Q_{1A}^a + \frac{\delta Q_{1B}^r}{\delta \pi^a} R_A^a - (A \leftrightarrow B) = f_{AB}^D Q_{1D}^r, \quad (12)$$

$$\frac{\delta \omega^r}{\delta \pi^a} Q_{1A}^a + \frac{\delta \omega_1^r}{\delta \pi^a} R_A^a = Q_{1A}^r, \quad (13)$$

$$- \frac{\delta R_B^s}{\delta \pi^a} \xi_{1A}^{ra} + \frac{\delta \xi_{1A}^{rs}}{\delta \pi^a} R_B^a = \frac{\delta Q_{1B}^s}{\delta t_r^A}, \quad (14)$$

$$\frac{\delta \omega^s}{\delta \pi^a} \xi_{1A}^{ra} = \xi_{1A}^{rs} - \frac{\delta \omega_1^s}{\delta t_r^A}, \quad (15)$$

$$\frac{\delta \xi_{1B}^{st}}{\delta t_r^A} - \frac{\delta \xi_{1A}^{rt}}{\delta t_s^B} = 0, \quad (16)$$

$$\mathcal{Y}_\alpha(\pi, t) \Big|_{t_0=0} = T_\alpha^i \pi^i f(\pi^2). \quad (42)$$

Let us come back to the general case when  $t_0^a \neq 0$ . Let us take the opportunity that the functions  $\mathcal{Y}_0(\pi, t)$  and  $\mathcal{Y}_\alpha(\pi, t)$  are invariants of the Lorentz group, i.e.

$$\mathcal{Y}_0(\pi, t) = \mathcal{Y}_0(\pi_\Lambda, t_\Lambda), \quad \mathcal{Y}_\alpha(\pi, t) = \mathcal{Y}_\alpha(\pi_\Lambda, t_\Lambda),$$

where  $\pi_\Lambda$  and  $t_\Lambda$  are the values of  $\pi$  and  $t$  obtained as a result of Lorentz-transformation  $\Lambda$ . Transformation  $\Lambda$  (see (36)) choose from the condition when  $t_{\Lambda,0}^a = 0$ . Obviously this transformation is always in the vicinity of  $t_0^a = 0$ . Then, according to (41), (42) we have

$$\mathcal{Y}_0(\pi_\Lambda, t_\Lambda) = f_0(\pi_\Lambda^2), \quad \mathcal{Y}_\alpha(\pi_\Lambda, t_\Lambda) = T_{\Lambda,\alpha}^j \pi_\Lambda^j f(\pi_\Lambda^2).$$

Now consider the following Lorentz-invariants:

$$\begin{aligned} \rho^{ab} &= t_r^a \bar{t}^{b,r} = \rho_\Lambda^{ab} = t_{\Lambda,i}^a t_{\Lambda,i}^b, \\ L_{\alpha\beta} &= L_{\Lambda,\alpha\beta} = T_{\Lambda,\alpha}^i T_{\Lambda,\beta}^j L_{ij}, \quad \rho^{ab} L_{bc} = \delta_c^a, \\ \omega^r t_r^a &= \omega_\Lambda^r t_{\Lambda,r}^a = \pi_\Lambda^i t_{\Lambda,i}^a = \pi_\Lambda^i \bar{t}_\Lambda^{a,i}. \end{aligned}$$

Then we have

$$\begin{aligned} \pi_\Lambda^2 &\equiv X = \pi_\Lambda^i \bar{t}_\Lambda^{a,i} T_{\Lambda,\alpha}^j T_{\Lambda,\beta}^k \bar{t}_\Lambda^{b,k} \pi_\Lambda^k = \\ &= \omega_\Lambda^r t_{\Lambda,r}^a L_{\Lambda,\alpha\beta} t_{\Lambda,s}^b \omega_\Lambda^s = \omega^r t_r^a L_{\alpha\beta} t_s^b \omega^s, \end{aligned} \quad (43)$$

$$T_{\Lambda,\alpha}^j \pi_\Lambda^j = T_{\Lambda,\alpha}^k T_{\Lambda,\beta}^l \bar{t}_\Lambda^{b,l} \pi_\Lambda^l = L_{\Lambda,\alpha\beta} t_{\Lambda,r}^b \omega_\Lambda^r = L_{\alpha\beta} t_r^b \omega^r,$$

and from the formula (38) we obtain

Let us, at last, solve the set (12)-(18) taking (19) into consideration. From (16) we find the general solution for  $\xi_{1A}^{rt}$  :

$$\xi_{1A}^{rt}(\pi, t) = \frac{\delta \tilde{u}^t(\pi, t)}{\delta t_r^A} \quad (20)$$

where  $\tilde{u}^t(\pi, t)$  is some function. With regard to (20) one obtains from (15)

$$\omega_1^s(\pi, t) = \alpha^s(\pi) + \tilde{u}^s(\pi, t) - \frac{\delta \omega^s}{\delta \pi^a} \tilde{u}^a(\pi, t). \quad (21)$$

Taking into consideration (19), from which it follows that

$\alpha^a(\pi) = 0$  and introducing the new functions  $\tilde{\tilde{u}}^s(\pi, t)$  by

$$\tilde{\tilde{u}}^a(\pi, t) = \tilde{u}^a(\pi, t), \quad \tilde{\tilde{u}}^n(\pi, t) = \tilde{u}^n(\pi, t) + \alpha^n(\pi),$$

rewrite (21) as

$$\omega_1^s(\pi, t) = \tilde{\tilde{u}}^s(\pi, t) - \frac{\delta \omega^s}{\delta \pi^a} \tilde{\tilde{u}}^a(\pi, t). \quad (22)$$

At the same time, rewrite (20) via  $\tilde{\tilde{u}}$  :

$$\xi_{1A}^{rt} = \frac{\delta \tilde{\tilde{u}}^t(\pi, t)}{\delta t_r^A}. \quad (23)$$

The equation (12) for  $\gamma = \alpha$  was solved in ref. [3].

$$Q_{1A}^a = \alpha^b(\pi, t) \frac{\delta R_A^a}{\delta \pi^b} - R_A^b \frac{\delta \alpha^a(\pi, t)}{\delta \pi^b}, \quad (24)$$

where  $\alpha^b(\pi, t)$  are some functions. With account of eq.(13) eq.(12)

is identically satisfied for  $r=n$ .

Now let us find the solution of the equation (18) (see ref. [3]):

$$S_1 = \alpha^\beta(\pi, t) \frac{\delta S_{cc}}{\delta \pi^\beta} + \frac{2\delta Z_1}{g^2} S_\pi(\pi) - 2\delta Z_2 S_{\pi, \phi}.$$

And the eq. (17) leads to

$$\frac{\delta}{\delta t_r^A} \delta Z_i = 0, \quad i=1,2; \quad \frac{\delta}{\delta t_r^A} (\alpha^a(\pi, t) + \tilde{u}^a(\pi, t)) = 0.$$

Now consider the functions  $u_0^a(\pi) = \alpha^a(\pi, t) + \tilde{u}^a(\pi, t)$ , then the functions

$$u^r = \tilde{u}^r - \frac{\delta \omega^r}{\delta \pi^\beta} u_0^\beta$$

Then one can write (see eqs. (22), (23))

$$\omega_1^r = u^r - \frac{\delta \omega^r}{\delta \pi^a} u^a, \quad \xi_{1A}^{rt} = \frac{\delta u^t(\pi, t)}{\delta t_r^A}, \quad \alpha^a = -u^a. \quad (25)$$

From eq.(13) one finds

$$Q_{1A}^r = -u^\beta \frac{\delta R^r}{\delta \pi^\beta} + R_A^\beta \frac{\delta u^r}{\delta \pi^\beta}, \quad (26)$$

(compare with (24)); the eq.(14) is satisfied identically. This finishes the solution of the set (12)-(18). The obtained solutions together with the relations (19) allow one all the coefficient functions in (11) to express via the functions  $u^r(\pi, t)$ . Our task is to define the form of all these functions concretely.

3. In this section the Ward's identities as a consequence of global gauge symmetry will be derived and analyzed. Infinite-

simil transformations of this symmetry can be obtained from (4) if independent on  $\chi$  and  $\theta$   $\Lambda_+^a$  parameters are chosen:

$$\Lambda_+^a = (\alpha + i\beta)^a \text{ where } \alpha \text{ and } \beta \text{ are real numbers. So,}$$

$$\delta_g \omega^r = R_a^r(\omega) \xi^a = R_{AS}^r \omega^S \xi^A, \quad \xi_{(\alpha)}^a \equiv \alpha^a, \quad \xi_{(\beta)}^a \equiv \beta^a, \quad (27)$$

where  $\delta_g$  denotes global variation. The fields  $C^A$  are transformed as

$$\delta_g C^D = f_{AB}^D C^A \xi^B,$$

and the matter fields  $\Phi$  are transformed according to (1) with constant  $\Lambda_+^a$ . Making global transformations of the fields in (7), we come to an identity for the modified action  $\mathcal{S}$ . Then, by usual procedure we obtain the same identity for the  $\Gamma$ -functional:

$$\begin{aligned} & \frac{\delta \Gamma}{\delta \pi^a} R_{AS}^a \frac{\delta \Gamma}{\delta \lambda_S} + \frac{\delta \Gamma}{\delta \phi^I} R_{\Phi, AJ}^I \Phi^J + \frac{\delta \Gamma}{\delta C^D} f_{BA}^D C^B = \text{tr} R_{AS}^r \frac{\delta \Gamma}{\delta t_S^r} + \\ & + K_{\Phi, I} R_{\Phi, AJ}^I \frac{\delta \Gamma}{\delta K_{\Phi, J}} + K_r R_{AS}^r \frac{\delta \Gamma}{\delta K_S} + L_c f_{BA}^c \frac{\delta \Gamma}{\delta L_B} + \\ & + \lambda_r R_{AS}^r \frac{\delta \Gamma}{\delta \lambda_S} + \chi_r^B R_{AS}^r \frac{\delta \Gamma}{\delta \chi_S^B}. \end{aligned} \quad (28)$$

In future it is sound practice to exclude from consideration the value  $\delta \Gamma / \delta t_S^D$  present in (28). It can be done by means of eq.(9), differentiating it with respect to  $\chi$  beforehand. As a result we obtain the following identity for  $\Gamma_{1d}$ :

$$\frac{\delta \Gamma_{1d}}{\delta \pi^a} R_{AS}^a \frac{\delta \mathcal{S}}{\delta \lambda_S} + \frac{\delta \mathcal{S}}{\delta \pi^a} (R_{AS}^a \frac{\delta \Gamma_{1d}}{\delta \lambda_S} + t_r^F R_{At}^r \xi_{IF}^{ta}) + \frac{\delta \Gamma_{1d}}{\delta \phi^I} R_{\Phi, AJ}^I \Phi^J +$$

$$\begin{aligned}
\frac{\delta \Gamma_{1d}}{\delta C^D} f_{BA}^D C^B &= K_{\phi, I} R_{\phi, AJ}^I \frac{\delta \Gamma_{1d}}{\delta K_{\phi, J}} + K_r R_{AS}^r \frac{\delta \Gamma_{1d}}{\delta K_S} + L_c f_{BA}^c \frac{\delta \Gamma_{1d}}{\delta L_B} + \\
&+ \lambda_r R_{AS}^r \frac{\delta \Gamma_{1d}}{\delta \lambda_S} + \chi_r^B R_{AS}^r \frac{\delta \Gamma_{1d}}{\delta \chi_S^B} + t_r^F R_{At}^r \left[ \frac{\delta \omega^t}{\delta \pi^a} \tilde{\varrho}_F Q_{1B}^a C^B - \right. \\
&- \frac{\delta \omega^t}{\delta \pi^a} \tilde{\varrho}_F \chi_r^B \xi_{1B}^{ra} + \frac{\delta \varpi_{1F}^{tB}}{\delta \pi^a} \tilde{\varrho}_B R_D^a C^D + \frac{\delta \xi_{1F}^{tS}}{\delta \pi^a} K_S R_B^a C^B + \\
&\left. + \varpi_{1F}^{tB} \tilde{D} D B_B + \lambda_r \xi_{1F}^{tr} + \chi_r^D \frac{\delta \varpi_{1F}^{tB}}{\delta t_r^D} \tilde{\varrho}_B + \chi_r^B \frac{\delta \xi_{1F}^{tS}}{\delta t_r^B} K_S \right].
\end{aligned}$$

Substituting expression (11) for  $\Gamma_{1d}$  into this, one obtains the following equations:

$$\frac{\delta \omega^r}{\delta \pi^a} \left( R_{AS}^a \omega_1^S + t_S^F R_{At}^S \xi_{1F}^{ta} \right) + \frac{\delta \omega_1^r}{\delta \pi^a} R_{AS}^a \omega^S = t_S^F R_{At}^S \xi_{1F}^{tr} + R_{AS}^r \omega_1^S \quad (29)$$

$$\frac{\delta S_{cc}}{\delta \pi^a} \left( R_{AS}^a \omega_1^S + t_S^F R_{At}^S \xi_{1F}^{ta} \right) + \frac{\delta S_1}{\delta \pi^a} R_{AS}^a \omega^S + \frac{\delta S_1}{\delta \phi^I} R_{\phi, AJ}^I \phi^J = 0 \quad (30)$$

(the remaining equations are satisfied identically that is why they are not written).

Comparing (30) with (18) one obtains

$$Q_{1A}^a = R_{AS}^a \omega_1^S + t_S^F R_{At}^S \xi_{1F}^{ta}$$

and, with account of this relation, the comparison of (29) with (12) yields

$$Q_{1A}^r = R_{AS}^r \omega_1^S + t_S^F R_{At}^S \xi_{1F}^{tr}, \quad (31)$$

which is the generalization of the former expression.

Substituting instead of  $Q_{1A}^r$ ,  $\omega_1^s$  and  $\xi_{1F}^{tr}$  their expressions via  $u(\pi, t)$  into (31) (see (25), (26)) one obtains

$$\hat{Q}_A u^r = R_{AS}^r u^s, \quad \hat{Q}_A \equiv R_A^i \frac{\delta}{\delta \pi^i} - t_S^F R_{AU}^S \frac{\delta}{\delta t_U^F} \quad (32)$$

The relation (32) testifies, that under the action of the operators  $\hat{Q}^A$  the values  $u^r$  are transformed as vectors. As to the operators themselves, it is not difficult to check that they form an algebra which is isomorphous to the algebra of the operators  $\hat{R}_A = R_A^i \delta / \delta \pi^i$ :

$$[\hat{Q}_A, \hat{Q}_B] = f_{BA}^D \hat{Q}_D$$

(cf. (8)). Also note, that under the action of the operator  $\hat{Q}^A$ ,  $\omega^S$  and  $t_S^B$  are also transformed as vectors:

$$\hat{Q}_A \omega^S = R_{At}^S \omega^t, \quad \hat{Q}_A t_S^B = -R_{AS}^r t_r^B \quad (33)$$

4. All our further considerations will apply to the theory with the SU(2) group and a concrete parametrization which will be now discussed. As  $i\tilde{\Gamma}_a$  (see (3)) consider the doublet-representation generators  $\tilde{\tau}_i$ , where  $\tau_i$  are the Pauli's matrices:

$$e^{\tau_i v^i} = chv + \tau_i \frac{v^i}{V} shv \equiv M(x, \theta) = \tau_i \pi^i + \sigma(\pi),$$

whence

$$\pi^i(v) = \frac{v^i}{V} shv, \quad \sigma(\pi) = chv, \quad \sigma^2 - \pi^2 = 1, \quad v = \sqrt{(v^i)^2} \quad (34)$$

Thus, the field  $\omega^r$  is a four-component one:  $\omega^r = (\pi^i, \omega^0) \equiv (\pi^i, \sigma)$  in this section in the indices like  $r, s, t, \dots$ , instead of  $\alpha, \beta, \gamma, \dots$  we shall put  $i, j, k, \dots$ :  $s=(1,0)$ ). From (34)

it is seen, that the value of  $\sigma^2 - \pi^2$  is an invariant, and the global gauge transformations

$$M(x, \theta) \rightarrow M'(x, \theta) = e^{i\tau_\kappa(\alpha - i\beta)^\kappa} M(x, \theta) e^{-i\tau_\epsilon(\alpha + i\beta)^\epsilon} \quad (35)$$

are transformations from the group  $SL(2, \mathbb{C})$  which is isomorphous to the Lorentz group. The four-vector  $q^\mu$  transformation law at Lorentz transformations is ( $\alpha = 0$ )

$$q_\mu^0 = q_\mu^0 \operatorname{ch} 2\beta + q_i \frac{\beta^i}{\beta} \operatorname{sh} 2\beta, \quad (36)$$

$$q_\mu^i = q_\mu^i + q_\mu^0 \frac{\beta^i}{\beta} \operatorname{sh} 2\beta + 2q_\mu^\kappa \frac{\beta^i \beta^\kappa}{\beta^2} \operatorname{sh}^2 \beta, \quad \beta = \sqrt{(\beta^i)^2}$$

The transformation laws of the fields  $\pi^i$  and  $\sigma$  at the transformations (35) for infinitesimal  $\alpha$  and  $\beta$  have the form of (27), where

$$R_{(\alpha)jk}^i = 2\epsilon_{ikj}, \quad R_{(\alpha)jo}^i = 0, \quad R_{(\alpha)jk}^0 = 0, \quad R_{(\alpha)jo}^0 = 0,$$

$$R_{(\beta)jk}^i = 0, \quad R_{(\beta)jo}^i = 2\delta_{ij}, \quad R_{(\beta)jk}^0 = 2\delta_{jk}, \quad R_{(\beta)jo}^0 = 0. \quad (37)$$

In accordance with the noticing made in the preceding section (see (33)), the vectors  $\omega^s$  and  $t_s^a = (t_s^{+a} + t_s^{-a}, i(t_s^{-a} - t_s^{+a}))$  are in the theory. We shall consider the case of  $t_s^{+a} = t_s^{-a} = t_s^a$ . Introduce the vectors  $\bar{t}^{a,s}$  via

$$\bar{t}^{a,s} = \eta^{sr} t_r^a, \quad \eta^{sr} = \begin{pmatrix} +1 & & & \\ & +1 & & \\ & & +1 & \\ 0 & & & -1 \end{pmatrix}$$

and present  $U^s(\pi, t)$  as an expansion

$$U^s(\pi, t) = \omega^s(\pi) \varphi_0(\pi, t) + \bar{t}^{a,s} \varphi_a(\pi, t). \quad (38)$$

Substituting this expansion into eq.(32), one obtains

$$\hat{Q}_R \mathcal{Y}_0(\pi, t) = \hat{Q}_R \mathcal{Y}_a(\pi, t) = 0, \quad (39)$$

i.e. the coefficient functions  $\mathcal{Y}_0$  and  $\mathcal{Y}_a$  are the Lorentz group invariants.

Next will be considered the so-called singular gauge, when  $S_{gf} = \int B_R T^R = \int (B_{+a} t_r^{-a} + B_a t_r^{+a}) \bar{D}D\omega^r$  (see(6)). Formally one can pass to this gauge by the substitutions  $t_r^R \rightarrow \alpha t_r^R$ ,  $B_R \rightarrow (1/\alpha) B_R$  with following tending of  $\alpha$  to  $\infty$ . This gauge is noteworthy due to the following observation. In the modified action  $S$  there are two summands which are proportional to the parameters  $t_r^R$ :  $S_{gf}$  and  $S_{gh}$ . At  $t_i^{+a} = t_i^{-a} = t_i^a$  one can change the variables in  $S$

$$B'_{\pm i} = B_{\pm a} t_i^a, \quad \tilde{\mathcal{D}}'_{\pm i} = \tilde{\mathcal{D}}_{\pm a} t_i^a, \quad \chi_r'^{\pm i} = \chi_r^{\pm a} T_a^i,$$

$$\tilde{\mathcal{D}}_{-a} = \tilde{\mathcal{D}}_+^{+a}, \quad t_i^a T_B^i = \bar{t}^{a,i} T_B^i = \delta_B^a, \quad \bar{t}^{a,i} T_a^i = \delta^{ij}, \quad (40)$$

passing in the functional integral to integration over the primed fields, after which, at  $t_0^a = 0$ , the dependence on  $t_r^R$  vanishes in the theory. This observation is equivalent to the statement that in the singular gauge, at  $t_0^a = 0$ , the theory is independent of the parameters  $t_r^R$ . Let us first consider the case when  $t_0^a = 0$ . In this case  $U^S$  does not depend on  $t$ . Considering the expansion (38) for  $S = 0$ , we obtain, that at  $t_0^a = 0$ ,  $\mathcal{Y}_0$  is independent of  $t$ . And from (39) it follows that  $\mathcal{Y}_0$  is a function of  $\pi^2$  only:

$$\mathcal{Y}_0(\pi, t) \Big|_{t_0=0} = f_0(\pi^2). \quad (41)$$

Likewise, for  $S=i$  from (38) we obtain:

$$-\frac{\delta S_{cc}}{\delta \pi^a} \xi^{ra} = \frac{\delta S_1}{\delta t_r^a}, \quad (17)$$

$$\frac{\delta S_1}{\delta \phi^I} R_{\phi, A}^I + \frac{\delta S_{cc}}{\delta \pi^a} Q_{1A}^a + \frac{\delta S_1}{\delta \pi^a} R_A^a = 0. \quad (18)$$

The equations being identically satisfied with account of the eqs (12-18) and a series of additional relations are not written here. These relations will be derived below. One can come to these relations issuing from the equalities for the modified action  $S$  which follow from its expression (7):

$$\frac{\delta S}{\delta \bar{\omega}_A} = -E_K t_r^A \frac{\delta S}{\delta K_r} + E_K \chi_r^A \frac{\delta S}{\delta \lambda_r}$$

$$\frac{\delta S}{\delta B_{\pm a}} = B_{\mp a} + E_{\mp} \bar{D} D t_r^{\mp a} \frac{\delta S}{\delta \lambda_r},$$

$$\frac{\delta S}{\delta \lambda^a} = \pi^a$$

where  $E_K = E_+ + E_-$ ,  $E_+$ ,  $E_-$  are projectors on the chiral superfields. The functional  $\Gamma$  also satisfies these equations.

Directly write equations for  $\Gamma_{1d}$ :

$$\frac{\delta \Gamma_{1d}}{\delta \bar{\omega}_A} = -E_K t_r^A \frac{\delta \Gamma_{1d}}{\delta K_r} + E_K \chi_r^A \frac{\delta \Gamma_{1d}}{\delta \lambda_r},$$

$$\frac{\delta \Gamma_{1d}}{\delta B_{\pm a}} = E_{\mp} \bar{D} D t_r^{\mp a} \frac{\delta \Gamma_{1d}}{\delta \lambda_r},$$

$$\frac{\delta \Gamma_{1d}}{\delta \lambda^a} = 0.$$

Substituting the expression for  $\Gamma_{1d}$  (11) into this, one obtains

$$\omega_1^a(\pi, t) = 0, \quad f^{ab}(\pi, t) = 0, \quad R_{1B}^D = t_r^D Q_{1B}^r, \quad (19)$$

$$Q_{\pm}^a = E_{\pm} \bar{D} D t_r^{\pm a} \omega_1^r, \quad \alpha_{1B}^{rA} = t_s^A \xi_{1B}^{rs} + \delta_B^A \omega_1^r.$$

$$u^r(\pi, t) = \omega^r f_0(x) + \bar{t}^{a,r} L_{a\beta} t_\beta^b \omega^s f(x). \quad (44)$$

Next, let us present  $\hat{\ell}^{a\beta}$ ,  $L_{a\beta}$  in a more convenient form:

$$\begin{aligned} \hat{\ell}^{a\beta} &= t_i^a \hat{\ell}^{ij} t_j^\beta, & \hat{\ell}^{ij} &= \delta^{ij} - \gamma^i \gamma^j, & \gamma^i &= T_a^i t_0^a, \\ L_{a\beta} &= T_a^i \hat{L}^{ij} T_\beta^j, & \hat{L}^{ij} \hat{\ell}^{jk} &= \delta^{ik}. \end{aligned} \quad (45)$$

rewrite the relations (43), (44) in terms of  $\gamma^i$ :

$$\begin{aligned} x &= (\pi^k + \sigma \gamma^k) \hat{L}^{ke} (\pi^e + \sigma \gamma^e), \\ u^i(\pi, t) &= \pi^i \tilde{f}_0(x) + [\hat{L}^{ij} (\pi^j + \sigma \gamma^j) - \pi^i] f(x), \\ u^0(\pi, t) &= \sigma \tilde{f}_0(x) - [\gamma^i \hat{L}^{ij} (\pi^j + \sigma \gamma^j) + \sigma] f(x), \quad \tilde{f} \equiv f_0 + f. \end{aligned}$$

The advantage of writing in terms of the values introduced in (45) is in the fact that there the whole dependence of the theory on the parameters  $t_r^a$  is expressed in terms of  $\gamma^i$ . Indeed, if in the functionals  $S_{gf}$  and  $S_{gh}$  the substitution (40) is accomplished, then  $S_{gf}$  and  $S_{gh}$  will have the form

$$S_{gf} = (B'_{+i} + B'_{-i}) \bar{D} D (\pi^i + \gamma^i \sigma), \quad S_{gh} = (\tilde{\mathcal{D}}'_{+i} + \tilde{\mathcal{D}}'_{-i}) [R_B^i C^B + \gamma^i R_B^0 C^B].$$

Hence it is seen that  $\gamma^i$  is included in the interaction vertices only. It means, that any quantum field value,  $u^r$  among them, in any order of the perturbation theory must be a finite  $\gamma^i$  polynomial. Consider the consequences of this fact for the functions  $\tilde{f}_0(x)$  and  $f(x)$ .

Choose  $\gamma^i$  without restriction to generality in the form  $\gamma^i = \gamma \delta^{i1}$ . Then we have:

$$\hat{\ell}^{ij} = \delta^{ij} - \gamma^2 \delta^{i1} \delta^{j1} = \begin{pmatrix} 1-\gamma^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \hat{L}^{ij} = \begin{pmatrix} \frac{1}{1-\gamma^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X = \frac{(\pi^1 + \gamma\sigma)^2}{1 - \gamma^2} + (\pi^2)^2 + (\pi^3)^2, \quad U^0(\pi, t) = \sigma \tilde{f}_0(x) - \frac{\pi^1 \gamma + \sigma}{1 - \gamma^2} f(x),$$

$$U^1(\pi, t) = \pi^1 \tilde{f}_0(x) + \frac{\gamma(\pi^1 \gamma + \sigma)}{1 - \gamma^2} f(x), \quad U^2(\pi, t) = \pi^2 \tilde{f}_0(x), \quad U^3(\pi, t) = \pi^3 \tilde{f}_0(x)$$

For simplicity take  $\pi^2 = \pi^3 = 0$  :

$$X = \frac{(\pi^1 + \gamma\sigma)^2}{1 - \gamma^2}; \quad \sigma = \sqrt{1 + (\pi^1)^2} \quad (46)$$

According to the aforesaid  $\tilde{f}_0(x)$  and  $f_1(\pi, \gamma) \equiv f(x)(\pi^1 \gamma + \sigma) / 1 - \gamma^2$  must be polynomials over  $\gamma$ . Consider  $X$  at  $\pi^1 = 0$ :

$$X|_{\pi^1=0} = \gamma^2 / 1 - \gamma^2. \quad \text{Then } (\gamma^2 / 1 - \gamma^2) \tilde{f}_0 = P_0(\gamma^2),$$

( $P_0$ -polynomial) or  $\tilde{f}_0(x) = P_0(x/1+x)$ . Substituting the value of  $X$  from (46) into the right hand side of the last relation, we obtain

$$\tilde{f}_0(x) = P_0 \left[ \frac{(z + \gamma)^2}{(1 + \gamma z)^2} \right], \quad z = \frac{\pi^1}{\sigma}, \quad (47)$$

The right hand side must have no singularities. The expression (47) is symmetric over  $\gamma$  and  $z$ , hence it is a final-order polynomial both over  $\gamma$  and  $z$  and, consequently, has no peculiarities over  $\gamma$  and  $z$ . Hence it follows that the polynomial  $P_0$  in (47) is a constant, i.e.  $\tilde{f}_0(x) = C$ . And for  $f_1(\pi_0, \gamma)$

we have:

$$f_1|_{\pi^1=0} = \frac{1}{1 - \gamma^2} f\left(\frac{\gamma^2}{1 - \gamma^2}\right) = P_1(\gamma^2)$$

whence

$$f(x) = \frac{1}{1+x} P_1\left(\frac{x}{1+x}\right)$$

Substituting the value of  $X$  from (46) we obtain

$$f_1(\pi, \gamma) = \frac{1}{\sigma} \frac{1}{1 + z\gamma} P_1 \left[ \frac{(z + \gamma)^2}{(1 + z\gamma)^2} \right],$$

where, as apposed to  $\tilde{f}_0(x)$ , there is always a pole at  $z = 1/\gamma$ .

That is why  $P_1(\gamma) \equiv 0$ .

Thus, we finally obtain the expansion of  $U^\Gamma(\pi, t)$ :

$$U^\Gamma(\pi, t) = \delta Z_\pi \omega^\Gamma(\pi), \text{ where } \delta Z_\pi \equiv C \text{ does not depend on } \pi \text{ and } t.$$

Now it is seen, that the one-loop renormalized action  $S - \eta \Gamma_{1d}$  is obtained by expanding  $S_{1R}$  with an accuracy to the terms having the first order over  $\eta$ , which is constructed by the initial modified action  $S$  by the substitution:

$$\begin{aligned} \pi &\rightarrow Z_\pi \pi, & \phi &\rightarrow Z_\phi^{1/2} \phi, & C &\rightarrow C, & \tilde{\mathcal{D}} &\rightarrow \tilde{\mathcal{D}}, \\ K &\rightarrow Z_\pi^{-1} K, & K_\phi &\rightarrow Z_\phi^{-1} K_\phi, & L &\rightarrow L, & \lambda &\rightarrow Z_\pi^{-1} \lambda, & \chi &\rightarrow Z_\pi^{-1} \chi, \\ t &\rightarrow Z_\pi^{-1} t, & g &\rightarrow Z_g g, & f &\rightarrow Z_\phi^{-3} f, & m &\rightarrow Z_\phi^{-2} m, \\ Z_\pi &= 1 + \eta \delta Z_\pi, & Z_g &= 1 + \eta \delta Z_1, & Z_\phi &= 1 + \eta \delta Z_2. \end{aligned}$$

Then, considering the theory based on  $S_{1R}$  we find that  $S_{1R}$  satisfies all the equations satisfied by the action  $S$ . Consequently, the producing  $\Gamma$ -functional of the theory developed over  $S_{1R}$  will satisfy all the identities satisfied by the  $\Gamma$ -functional of the theory with the action  $S$ , etc. Therefore, in the highest approximation one can use the method of mathematical induction.

Thus, finally we obtain, that in the singular gauge the theory is renormalized multiplicatively.

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МУЛЬТИПЛИКАТИВНАЯ ПЕРЕНОРМИРОВКА  $N = 1$  СУПЕРСИММЕТРИЧНЫХ  
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**ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ**