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A Z_2 GAUGE MODEL WITH MATTER FIELDS ON BETHE LATTICE OF PLAQUETTES

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Z_2-КАЛИБРОВОЧНАЯ МОДЕЛЬ С ПОЛЯМИ МАТЕРИИ НА РЕШЕТКЕ БЕТЕ ИЗ ПЛАКЕТОВ

Z_2-калибровочная модель с полями материи точно решена на специальной бесконечномерной решетке, являющейся двухмерным обобщением решетки Бете. В области малых значений β_g получена линия фазовых переходов I рода, заканчивающаяся критической точкой фазового перехода II рода. Критические индексы, вычисленные в этой точке имеют классические значения. Приведено также выражение свободной энергии модели.

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1. Introduction

The lattice gauge theories have received great interest in the recent years due to the possibility to obtain qualitative and quantitative results beyond the scope of the perturbation theory. This especially concerns the theories that contain not only gauge fields but also matter fields. Precisely in the framework of this approach it has become possible to describe such phenomena of the field theory as confinement, Higgs mechanism, etc. Besides, the matter fields introduced allow one to obtain the second order phase transition point near which the continuous field theory corresponds to the lattice models [1] .

Nevertheless, an analytical study of such models offers great difficulties, which is due to nontrivial topological structure of real three- and four-dimensional lattices [2] . However one may introduce a particular lattice containing no closed surfaces. Such a lattice - a two-dimensional generalization of Cayley tree - compared to the standard lattices is nothing but some topological abstraction; however, under certain conditions the exact results obtained on it correspond to approximated results for usual lattices. Here, as distinct from other approximations, we obtain analytical expressions for quantities of interest.

A model possessing Z_2 gauge symmetry - a simplest one among the possible models - has nevertheless a nontrivial critical behavior [3-5] . Its phase diagram is schematically shown in Fig.1 where the constant β_g describes a self-action of gauge fields, and β_m is an interaction of the gauge fields with the matter fields. It should be noted some peculiar features of this model on standard lattices:

a) at $\beta_m = 0$ we have a pure gauge model with a confinement-deconfinement first order transition;

b) at $\beta_g \to \infty$ we have (in a corresponding gauge) a spin Ising model with usual order-disorder transition;

c) the line of phase transitions, AOC, ends at a critical point C, therefore, in a region of small β_g and large β_m the confinement and Higgs phases are continuously connected [6] .

We have considered the case $\beta_m = 0$ on our lattice in the previous paper [7] and also established the presence of the phase transition of the first order. The aim of this work is to study a critical behavior of the model at $\beta_m \neq 0$. Unfortunately, for the reasons mentioned below, we so far have to restrict ourselves to a region of not very large β_g .

In Sect. 2 the lattice is defined and its main features are pointed out. In Sect. 3 we give a formulation of the model, derive the equation of state and the expression for free energy. Section 4 is devoted to the study of critical properties of the model.

2. Lattice

The lattice on which we are studying the model is constructed by successive glueing of shells (Fig.2). The zero shell is the central plaquette, and the subsequent shells come out by glueing γ new plaquettes to each free link of the previous shell. The resultant lattice is characterized by the coordinating number (the number of plaquettes coming out of one link) $\gamma + 1$ and has infinite Hausdorff dimension.

Two main peculiarities of this lattice should be noted. First, the number of the boundary plaquettes is always of the same order as the total number of the lattice plaquettes; therefore, the boundary effect does not vanish even in the thermodynamic limit $n \to \infty$ (n is the number of shells). As a result, the plaquettes belonging to different shells turn out to be nonequivalent to each other. This problem also arises on the usual Cayley tree of links and ways to overcome it are known [8]. One of the ways is that one considers only local properties of the plaquettes deep inside the lattice (infinitely far from the boundary at $n \to \infty$) and being therefore entirely equivalent Effectively, this corresponds to omitting all boundary terms in the partition function. Precisely in this case the obtained results can be regarded as some approximation for standard lattices.

Second, each site of the lattice in the limit $n \to \infty$ turns out connected with the infinite number of the neighbour sites. This leads in particular to invalidity of the Elitzur theorem [9] . This theorem reads that if the model action is gauge-invariant, then the mean value of any gauge-invariant quantity is always zero (in other words, gauge invariance cannot be spontaneously broken). However, as shown in Ref.[10], this does not concern infinite-dimensional models, since an essential condition in the proof of the Elitzur theorem was a finite number of degrees of freedom affected by gauge transformation in the lattice site.

Because of the infinite number of "neighbours" around each site, we have to restrict ourselves in the present work to the analysis of a region of not very large values of the gauge constant β_g. As mentioned, when $\beta_g \to \infty$, the gauge fields become ordered, and the direct spin-spin interactions begin to play a significant role. Here we encounter additional difficulties which we hope to overcome in our further works.

3. Model

The Z_2-gauge model with matter fields is given by the action [11]:

$$S(\beta_g;\beta_m) = \beta_g \sum_{p\ell} U_{p\ell} + \beta_m \sum_{\langle ij\rangle} \sigma_i U_{ij} \sigma_j \,, \tag{3.1}$$

where: 1) $U_{ij} = \pm 1$ are gauge variables defined on the links;

2) $\sigma_i = \pm 1$ are spin variables defined in sites i;

3) $U_{p\ell} = U_{ij} U_{jk} U_{k\ell} U_{\ell i}$ is a product of gauge variables round the plaquette;

4) the first sum is taken over all plaquettes of the lattice, the second sum - over all links.

The model partition function is found as a sum over all possible configurations $\{U, \sigma\}$ of the field variables:

$$Z(\beta_g;\beta_m) = \sum_{\{U,\sigma\}} \exp S \tag{3.2}$$

and the free energy per one link is

$$f(\beta_g;\beta_m) = -\frac{1}{N_\ell} \ln Z(\beta_g;\beta_m) \tag{3.3}$$

where N_ℓ is the number of lattice links.

The action (3.1) is unvariant under the local (proceeding in the site) Z_2 transformation: $s = \pm 1$ such that

$$\sigma_i \to s_i \sigma_i \quad \text{and} \quad U_{ij} \to \sigma_i U_{ij} \sigma_j \,. \tag{3.4}$$

Using these, one may choose a convenient gauge which simplifies calculations significantly. In particular, choosing $s_i = \sigma_i$ for each configuration, all spin variables can be set equal to unity. Then the action takes a form:

$$S^{\Phi}(\beta_g;\beta_m) = \beta_g \sum_{p\ell} U_{p\ell} + \beta_m \sum_{\langle ij\rangle} U_{ij} \tag{3.5}$$

whence one can readily see a similarity in the consideration of the matter fields in gauge theories and of the external field in spin theories.

After gauge fixing, from the partition function there comes out a factor 2^{N_s} (N_s is the number of sites) corresponding to a so much decrease in the number of possible configurations:

$$Z(\beta_g;\beta_m) = 2^{N_s} Z^{\Phi}(\beta_g;\beta_m),$$

where (3.6)

$$Z^{\Phi}(\beta_g;\beta_m) = \sum_{\{u\}} \exp\left\{\beta_g \sum_{p\ell} u_{p\ell} + \beta_m \sum_{\langle ij\rangle} u_{ij}\right\}.$$

Because of the lattice specific nature we cannot, as mentioned, consider global correlation functions such as Wilson loop, Polyakov loop, etc. As the main local parameter we shal: use the mean:

$$L = \langle u \rangle = Z^{-1} \sum_{\{u\}} u_{ij} \exp S. \qquad (3.7)$$

which in fact is the gauge field magnetization.

Suppose our lattice contains n shells. Then the partition function (3.6) can be rewritten as follows:

$$Z^{\Phi} = \sum_{u^{\circ}} \exp(\beta_g u^{\circ}_{p\ell} + \beta_m \sum_{\langle ij\rangle^{\circ}} u^{\circ}_{ij})[g_n(u^{\circ}_{ij})]^{\gamma}[g_n(u^{\circ}_{jk})]^{\gamma}[g_n(u^{\circ}_{k\ell})]^{\gamma}[g_n(u^{\circ}_{\ell i})]^{\gamma},$$

where the index o denotes the belonging to the central plaquette, and

$$g_n(u^{\circ}_{ij}) = \sum_{\substack{\{u\} \\ \text{except } u^{\circ}}} \exp\left\{\beta_g \sum_{p\ell} u_{p\ell} + \beta_m \sum_{\langle ij\rangle} u_{ij}\right\}$$

in fact is a partition function of one of the lattice branches coming out of the link u°_{ij} and containing n shells. The

quantities $g_n(u^{\circ})$ can be readily expressed through $g_{n-1}(u^{1})$:

$$g_n(u^{\circ}) = \sum_{u^{1}} \exp(\beta_g u^{1}_{p\ell} + \beta_m \sum_{\langle ij\rangle^{1}} u^{1}_{ij})[g_{n-1}(u^{1}_{jk})]^{\gamma}[g_{n-1}(u^{1}_{k\ell})]^{\gamma}[g_{n-1}(u^{1}_{\ell i})]^{\gamma} \qquad (3.8)$$

We introduce a notation

$$x_n = \frac{g_n(u=-1)}{g_n(u=+1)} \qquad (3.9)$$

and having summed the expression (3.8) over u^{1} we obtain for x_n a recursion formula:

$$x_n = f(x_{n-1})$$

where (3.10)

$$f(x,\beta_g,\beta_m) = \frac{e^{\beta_g-3\beta_m}x^{3\gamma} + 3e^{-\beta_g-\beta_m}x^{2\gamma} + 3e^{\beta_g+\beta_m}x^{\gamma} + e^{-\beta_g+3\beta_m}}{e^{-\beta_g-3\beta_m}x^{3\gamma} + 3e^{\beta_g-\beta_m}x^{2\gamma} + 3e^{-\beta_g+\beta_m}x^{\gamma} + e^{\beta_g+3\beta_m}}.$$

In the thermodynamic limit $n \to \infty$ the recursion sequence: $\{x_n\} \to x^*$, where x^* is a stable solution of the equation:

$$f(x,\beta_g,\beta_m) = x. \qquad (3.11)$$

Hence, this is the basic equation which defines physical state of the system. The presence of more than one stable solution in it points out the existence of phase transition (for more details see Section 4).

The quantity x^*, although determines the system state at given β_g and β_m , has no direct physical meaning. To simplify the further derivations, we introduce new variables:

$$\theta = \coth \beta_g \qquad \text{and} \qquad y^3 = 0\,\frac{1-x}{1+x} \qquad (3.12)$$

Within the new notations one can rewrite equation (3.10) in the form:

$$\beta_m = \frac{1}{2}\ln\left[\left(\frac{\theta-y^3}{\theta+y^3}\right)^{\gamma}\,\frac{1+y}{1-y}\right]. \qquad (3.13)$$

Insofar as the gauge constant β_g is considered positive, i.e. $\theta > 1$, then the values of y are to lie in the interval: $-1 < y < +1$.

The gauge field magnetization (3.7) also can be expressed through x or y . For that we present it in the form:

$$L = \frac{\sum\limits_{U^0} U^0_{ij}\exp(\beta_m U^0_{ij})[g_n(U^0_{ij})]^{\gamma}}{\sum\limits_{U^0}\exp(\beta_m U^0_{ij})[g_n(U^0_{ij})]^{\gamma}}$$

whence with account of notations (3.9) and (3.12) we obtain:

$$L = \frac{e^{\beta_m}}{e^{\beta_m} + x^{\gamma}} = y\,\frac{\theta + y^2}{\theta + y^4} \qquad (3.14)$$

The quantity U_{ij} is not gauge-invariant. However its mean value is not identically zero even for invariant hamiltonian ($\beta_m = 0$), which points out the invalidity of the Elitzur theorem (see Sect.2).

Using quantity L we can calculate the model free energy.

For that we use a formula:

$$-\frac{\partial f}{\partial \beta_m} = L \qquad (3.15)$$

which can be readily obtained from (3.3) and (3.6) with account of the equivalence of all internal links of the lattice. The free energy is determined by integration:

$$f = -\int L\,d\beta_m + C = -\int L(y)\,\frac{d\beta_m}{dy}\,dy + C$$

Substituting $L(y)$ from (3.14) and $d\beta_m/dy$ from (3.13), we obtain an integral to be calculated up to the end. Finally, the free energy per one link of the lattice, with an accuracy up to a constant, is:

$$f = \frac{\gamma}{2}\ln(\theta^2 - y^6) - \frac{3\gamma-4}{4}\ln(\theta + y^4) - \frac{1}{2}\ln(1-y^2) \qquad (3.16)$$

4. The Model Critical Properties

Now we turn back to the equation of state written in the form of (3.13). First of all, we can see that β_m is an odd function of y for any θ (i.e. β_g). Therefore, to investigate critical properties, it is enough to consider the region $\beta_m > 0$. In this region, depending on the values of θ , three cases are possible in the behaviour of function $\beta_m(y)$.

1) For $\theta > \theta_c$ (small β_g) the function $\beta_m(y)$ monotonously increases. This means that Eq. (3.11) has only one solution, and for any only one state of the system is possible.

Thus, phase transitions are absent in this region of values for θ .

2) For $\theta < \theta_c$ (large β_g) a decreasing part appears on the plot of $\beta_m(y)$. Accordingly, there is a region of β_m values where the equation of state has three solutions: y_I , y_o and y_{II} . The medium of them, y_o , as can be readily shown, is unstable (is not an attractor for the recursion succession), and two others, y_I and y_{II} , are stable and represent two phases of the system. At some β_m the system jumps from one phase to another, which means the first-order phase transition.

The transition parameters are determined from the equality of free energies of both phases:

$$f(\theta, y_I) = f(\theta, y_{II})$$

considered together with the equation:

$$\beta_m(\theta, y_I) = \beta_m(\theta, y_{II})$$

Fig.4 shows lines of the first order phase transitions obtained for different γ .

3) At $\theta = \theta_c$ we have a boundary situation. Function has an inflection, and the inflection point is the end point C of the line of the first order phase transitions. All quantities at this point change continuously, so here we have second order phase transition. In order to find out coordinates of point C, we should solve a system of equations:

$$\frac{d\beta_m}{dy} = 0 \qquad \text{and} \qquad \frac{d^2\beta_m}{dy^2} = 0$$

Hence we obtain:

$$y_c^2 = \frac{5\gamma^2 - 1}{4\gamma^2} - \sqrt{\left(\frac{5\gamma^2 - 1}{4\gamma^2}\right)^2 - 1}$$

$$\theta_c = \frac{y_c^3}{2\sqrt{2}} \left(3\gamma\sqrt{\gamma^2 - 1} + \sqrt{9\gamma^2 - 1}\right. \tag{4.1}$$

$$\beta_m^c = \frac{\gamma}{2} \ln \frac{3\sqrt{\gamma^2-1}}{\sqrt{9\gamma^2-1} + 2\sqrt{2}} + \ln \frac{\sqrt{\gamma^2-1}}{\sqrt{9\gamma^2-1} - 2\sqrt{2}\gamma}$$

Finally, we can calculate critical exponents at this point. Since we do not use the usual order parameters, we should respectively re-define the relations of interest. Thus, exponent δ is calculated on curve $\beta_m(L)$ at $\theta = \theta_c$ by a formula:

$$\beta_m - \beta_m^c \sim (L - L_c)^\delta \tag{4.2}$$

exponent β - on the curve of phase co-existence, by a formula:

$$L - L_c \sim (\theta - \theta_c)^\beta \tag{4.3}$$

and exponent α - from the difference:

$$\beta_m^+ - \beta_m^- \sim (\theta - \theta_c)^{2-\alpha} \tag{4.4}$$

where the values of β_m^+ are taken on the curve $\beta_m(L_c, \theta)$, and β_m^- - on the co-existence curve.

As can be readily found, thus determined critical exponents have classical values:

$$\delta = 3, \quad \beta = 1/2, \quad \alpha = 0, \tag{4.5}$$

which just should be expected on the infinite-dimensional lattice.

In conclusion, we'd like to express our sincere gratitude to S.G. Matinyan, N.S. Izmailyan and S.A. Chatrchyan for their help in the work and useful discussions.

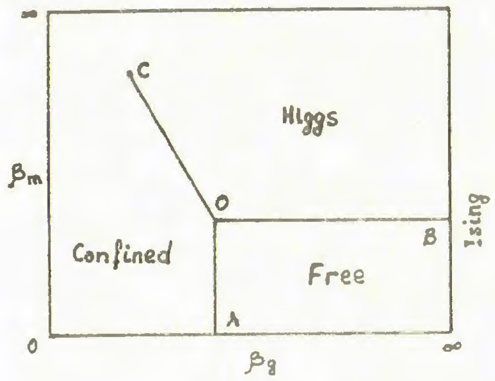

Fig.1

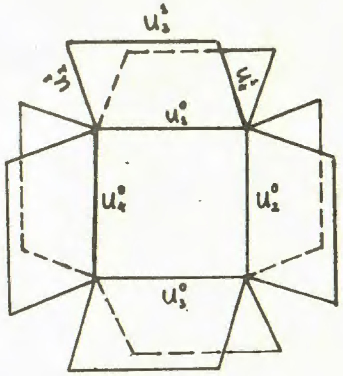

Fig.2

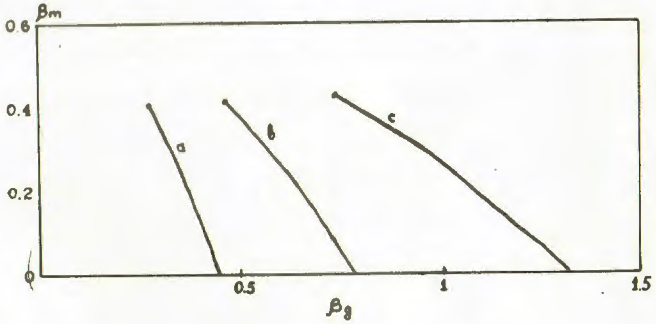

Fig.3

Figure Captions

Fig.1. A schematic phase diagram of the Z_2 gauge model with matter fields, obtained on standard lattices.

Fig.2. Bethe lattice of plaquettes with $\gamma = 2$ which contains n=2 shells.

Fig.3. Lines of the first order phase transitions for different γ : a - at γ=5; b - at γ=3; c - at γ=2.

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