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YEREVAN PHYSICS INSTITUTE

H.R.AVAKIAN, A.S.ARUTYUNIAN, C.YANG

THE RADIATION SPECTRUM OF CHANNELED ELECTRONS IN A THICK CRYSTAL

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ՀԱՍՏ ԲՅՈՒՐԵՂՈՒՄ ԿԱՆԱԼԱՑՎԱԾ ԷԼԵԿՏՐՈՆՆԵՐԻ ՃԱՌԱԳԱՅԹՄԱՆ ԵՐԱՆԳԱՆԻՆ

Տրված է կանալացված մասնիկների ճառագայթման հաճախության ի ,,դիտելի,, երանգանու հաշվարկման մի եղանակ՝ հաշվի առնելով այն հանգամանքը, որ երկու կամ ավել ֆոտոններ միաժամանակ հարվածելով դետեկտորին՝ գրանցվում են որպես մեկը՝ գումարային էներգիայով: CdS բյուրեղի (100) հարթություններով կանալացված 4,5 ԳէՎ էներգիայով էլեկտրոնների օրինակով հաշվարկված են ինչպես ,,դիտելի,, երանգանին, այնպես էլ՝ տեսականը, որը հաշվի չի առնում վերոբերյալ հանգամանքը: Մասնիկների ոչ կոհերենտ բազմակի ցրումը բյուրեղում հաշվի է առնվում երկու դեպքում էլ: Հաշվարկման արդյունքները համեմատված են փորձի հետ:

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Երևան 1989

1. Introduction

Recently [1] the radiation spectra of channeled electrons in CdS, quartz and lithium niobate crystals with thicknesses of 1-6 mm were investigated. The observed spectra were noticeably more hard but less intense (in the maximum region) than expected according to usual theoretical estimations. The crystals investigated in [1] were sufficiently thick so that the mean number of formed photons was greater than unity, i.e. there was certain probability of formation of two or more photons. It is natural to ask whether the spectra observed in the experiment [1] included the events where two or more photons were simultaneously caught by the detector and these photons were registered as a single photon with summary energy? In order to elucidate this problem we calculate both the theoretical spectrum without regard for the above-mentioned events and the "observed" spectrum where these events take place on the example of CdS crystal. In both cases the incoherent multiple scattering of particles in the crystal is taken into account.

2. Radiation Spectrum Averaged Over Beam Particles

We shall calculate the radiation frequency spectrum, $dW/L_c d\omega$, averaged over all beam particles from a crystal of thickness L_c according to the formula [2-4] (see also [5]):

$$\frac{dW}{L_c d\omega} = \int_0^\infty \left(\frac{dW}{L_c d\omega}\right)_{\varepsilon_\perp} \bar{f}(\varepsilon_\perp, L_c)\, d\varepsilon_\perp \, , \tag{1}$$

where $(dW/L_c d\omega)_{\varepsilon_\perp}$ is the unit path radiation frequency spectrum of a particle with a given transverse energy $\varepsilon_\perp$; $\bar{f}(\varepsilon_\perp, L_c)$ is the thickness L_c averaged distribution function of particles over the transverse energy.

The function $\bar{f}(\varepsilon_\perp, L_c)$ may be found by the formula

$$\bar{f}(\varepsilon_\perp, L_c) = L_c^{-1} \int_0^{L_c} d\ell \int d\theta\, P(\varepsilon_\perp | \theta)\, \Psi(\theta, \ell) \, , \tag{2}$$

where $P(\varepsilon_\perp | \theta)$ is the distribution function of particles over the transverse energy provided that the particles enter the planar channel at an angle θ; $\Psi(\theta, \ell)$ is the distribution function of particles over the angle θ at a given depth ℓ of the crystal (with account of incoherent scattering).

The function $P(\varepsilon_\perp, \theta)$ and consequently the function

$\bar{f}(\varepsilon_\perp, L_c)$ depend upon the shape of the averaged plane potential $U(x)$ which may be calculated on the basis of the potentials of the crystal atoms and its structure.

In the case of electron planar channeling and quasi-channeling an "overturned parabola" model is quite good for the potential well:

$$U(x) = \frac{4|x|}{d_x}\left(1 - \frac{|x|}{d_x}\right) U_0 \, , \qquad -\frac{d_x}{2} \leqslant x \leqslant \frac{d_x}{2} \, , \tag{3}$$

where d_x and U_0 are potential well parameters.

In this model with the assumption of uniform distribution of particles over x it is easy to obtain

$$P(\varepsilon_\perp | \theta) = \frac{\Theta(\varepsilon_\perp)\, \Theta(1 + \theta^2/\theta_L^2 - \varepsilon_\perp/U_0)}{2U_0 (1 + \theta^2/\theta_L^2 - \varepsilon_\perp/U_0)^{1/2}} \, , \tag{4}$$

where $\theta_L = (2U_0/E)^{1/2}$ is the Lindhard critical angle, E is the electron energy, $\Theta(x) = 0$ at $x < 0$ and 1 at $x > 0$.

Usually it is accepted that the function $\Psi(\theta, \ell)$ has a Gauss distribution form with the dispersion

$$\sigma^2 = \frac{E_s^2\, \ell}{E^2 L_R} + \sigma_0^2 \tag{5}$$

($E_s = 14.1$ MeV, L_R is the radiation length, σ_0 corresponds to the angle divergence of particle beam). For simplicity we approximate the Gauss distribution by means of a "table":

$$\Psi(\theta, \ell) = (2\sigma_1)^{-1}\, \Theta\left(\sigma_1^2 - (\theta - \theta_0)^2\right) , \quad \sigma_1^2 = \frac{\pi \sigma^2}{2} \tag{6}$$

where θ_0 is the mean incidence angle of beam particles. Substituting (4) and (6) into (2) we obtain (at $\theta_0 = 0$, $\sigma_0 = 0$)

$$\bar{f}(\varepsilon_\perp, L_c) = \left[\frac{1}{\eta^{1/2}} \ln \frac{A}{|1-\xi|^{1/2}} - \frac{B}{\eta} + \frac{(1-\xi)^{1/2}}{\eta}\Theta(1-\xi)\right]\Theta(1-\xi+\eta), \quad (7)$$

where $\xi = \varepsilon_\perp / U_0 > 0, \qquad \eta = \pi E_s^2 L_c / 2E^2 L_R \theta_L^2$.

$$A = (1+\xi^{1/2})\Theta(\eta-\xi) + [\eta^{1/2} + (1-\xi+\eta)^{1/2}]\Theta(\xi-\eta),$$

$$B = \Theta(\eta-\xi) + (1-\xi+\eta)^{1/2}\Theta(\xi-\eta).$$

(Fig.1).

It is not difficult to generalize formula (7) to the cases where the planar channel consists of two or larger number of potential wells.

The radiation frequency spectrum $(dW/L_c d\omega)_{\varepsilon_\perp}$ of an electron with a given transverse energy $\varepsilon_\perp$ moving in potential (3) may be calculated in a usual way [6] (Fig.2). On the basis of these results and formula (7) we obtain the radiation spectrum $dW/L_c d\omega$ averaged over all beam particles (Fig.3, curve 1) by formula (1).

3. The "Observed" Radiation Spectrum with Possible Simultaneous Registration of Photons

We assume that the probability P_n of emission of n quanta by a given charged particle is determined by the Poisson distribution [7] :

$$P_n = \frac{\langle N \rangle^n \exp(-\langle N \rangle)}{n!}, \quad (8)$$

where $\langle N \rangle$ is the mean number of radiation quanta emitted.

If the detector registers several photons emitted simultaneously as a single photon with the summary energy, the "observed" spectral intensity distribution $(dW/L_c d\omega)_{\varepsilon_\perp}^{obs}$ of radiation emitted by a particle with a given transverse energy $\varepsilon_\perp$ is determined by the formula

$$\left(\frac{dW}{L_c d\omega}\right)_{\varepsilon_\perp}^{obs} = \sum_{n=1,2,\dots} \left(\frac{dW}{L_c\, d\omega}\right)_{\varepsilon_\perp, n, \omega} P_n, \quad (9)$$

where $(dW/L_c d\omega)_{\varepsilon_\perp n,\omega}$ is the spectral intensity distribution of radiation when n photons are emitted with a summary energy ω $(\hbar = 1)$.

The probability $F_n(\omega)d\omega$ that the summary energy of n photons emitted is within the interval (ω, $\omega + d\omega$) is determined by the formula

$$F_n(\omega)d\omega = \int_0^\omega F_1(\omega_1) E_{n-1}(\omega-\omega_1)\, d\omega_1 d\omega, \quad (10)$$

where the integration is carried out over ω_1 from 0 to ω .

The probability $F_1(\omega)d\omega$ that the photon emitted from unit pathlength has energy within the interval (ω, $\omega + d\omega$) is determined by the radiation spectrum

$$F_1(\omega)d\omega = \left(\frac{dW}{L_c d\omega}\right)_{\varepsilon_\perp} \frac{d\omega}{C}, \quad (11)$$

where C is the total radiation energy from unit pathlength

$$C = \int \left(\frac{dW}{L_c d\omega}\right)_{\varepsilon_\perp} d\omega. \quad (12)$$

For concrete calculation of the observed spectrum it is quite convenient to approximate the function $(dW/d\omega)_{\varepsilon_\perp}$ by a gamma-distribution (see, for example [8]):

$$\Phi_1(\omega) = \frac{A\beta_o^{\alpha}}{\Gamma(\alpha)}\,\omega^{\alpha-1}\exp(-\beta_o\omega)\,, \tag{13}$$

where $\Gamma(\alpha)$ is the gamma-function.

By this we shall select the parameters A, α, β_o in such a way that the mean frequency, the mean square frequency deviation and the mean number of quanta calculated by means of the distribution $\Phi_1(\omega)$ coincide with the corresponding values $\langle\omega\rangle$, $\langle\Delta\omega^2\rangle$, $\langle N\rangle$ calculated from (11). It leads to the following equations

$$\alpha = \frac{\langle\omega\rangle^2}{\langle\Delta\omega^2\rangle}\,, \qquad \beta_o = \frac{\langle\omega\rangle}{\langle\Delta\omega^2\rangle}\,, \qquad A = (\alpha-1)\langle N\rangle\cdot\beta_o^{-1} \tag{14}$$

With the help of $\Phi_1(\omega)$ it is easy to find the expression $\Phi_n(\omega)$ for an arbitrary n according to (10):

$$\Phi_n(\omega) = \frac{A\beta_o^{n\alpha}}{\Gamma(n\alpha)}\,\omega^{n\alpha-1}\exp(-\beta_o\omega)\,. \tag{15}$$

Thus, for "observed" spectrum of radiation emitted by a particle with a given $\varepsilon_\perp$ we have

$$\left(\frac{dW}{L_c\,d\omega}\right)^{obs}_{\varepsilon_\perp} = C\sum_{n=1,2,\dots} F_n(\omega)P_n\,, \tag{16}$$

where as $C F_n(\omega)L_c$ we shall take (15) with regard for (14).

For the observed radiation spectrum averaged over beam particles we have according to (1)

$$\left(\frac{dW}{L_c\,d\omega}\right)^{obs} = \int_0^{\infty}\left(\frac{dW}{L_c\,d\omega}\right)^{obs}_{\varepsilon_\perp} f(\varepsilon_\perp, L_c)\,d\varepsilon_\perp\,. \tag{17}$$

In Fig.3 both the spectrum without regard for the simultaneous registration of several photons (curve 1) and the "observed" spectrum (curve 2) are presented. In the case under consideration the "observed" spectrum is much harder and wider than the first one. In this figure the experimental points are also marked. As it can be seen, the observed spectrum agrees much better with the experimental results.

4. Discussion

The theoretical spectra presented in Fig.3 are calculated in a model of one "overturned parabola" (3) with corresponding values of parameters. A concrete calculation shows that the averaged potential of planes (100) of CdS crystal consists of two potential wells (Fig.4) with depths of 34.2 and 9.57 eV due to planes consisting of Cd atoms only and S atoms only. Electrons moving in the deeper potential well bring the main contribution into the radiation intensity. Motion in the second smaller well gives noticeably weaker radiation, since the field is much smaller. The estimation shows that the radiation energy of electrons channeled in the second well is of about 10–15 % of the total radiation energy. The comparison of the calculation with the experiment (Fig.3) does not claim on a better accuracy.

Because of the model character of calculation it is not necessary to take into account the fact that the incoherent scattering of particles in a crystal proceeds otherwise than in an equivalent amorphous matter [9].

Besides, as it was pointed out earlier [10], formula (1) in general is correct only if the particle transverse energy $\varepsilon_\perp$ changes little due to incoherent multiple scattering during several particle transverse oscillations. It is more correct to calculate the radiation spectrum by the general formula of electrodynamics using particle trajectories obtained from numerical simulation at solution of the corresponding stochastic differential equations [10].

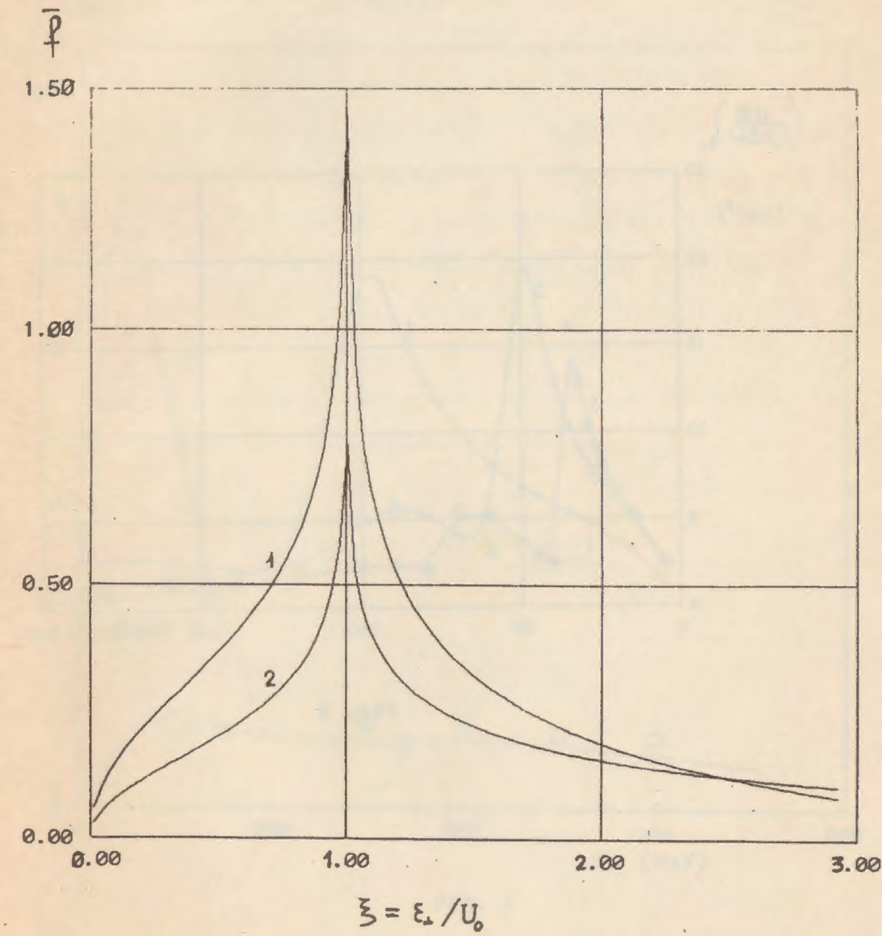
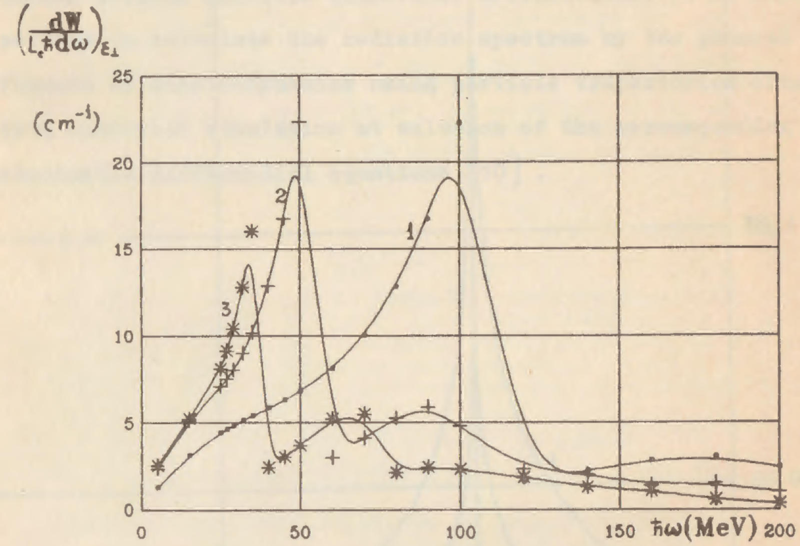


Fig. 1


(dW / L_c ħdω)_E⊥
(cm⁻¹)
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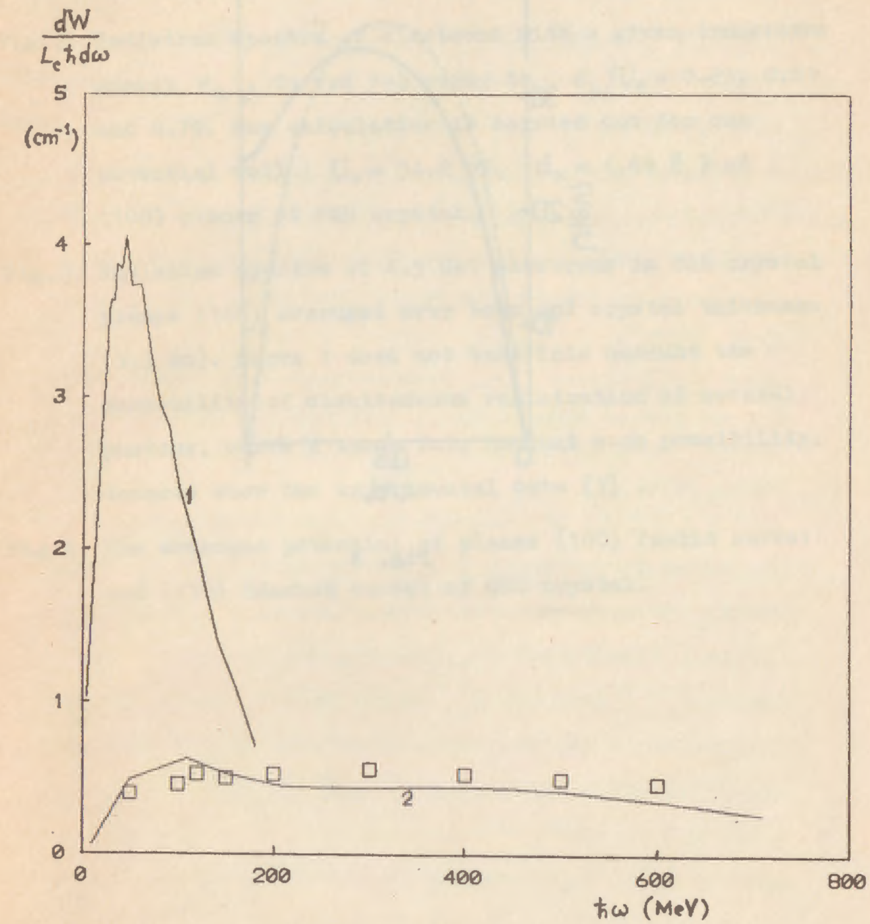
15
10
5
0
0
50
100
150
ħω(MeV)
200
1
2
3

Fig. 2


dW / L_c ħdω
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2

Fig. 3

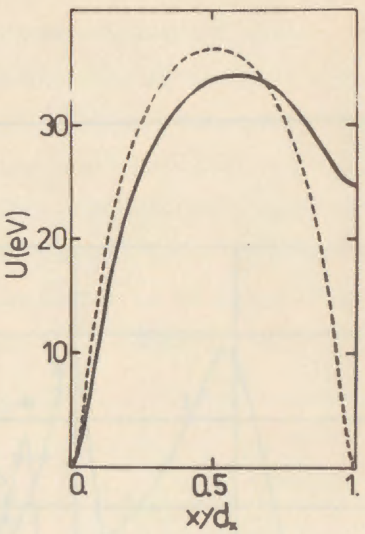

Fig. 4

Figure Captions

Fig.1. The dependence of distribution function $\bar{f}(\varepsilon_\perp, L_c)$ upon the transverse energy $\varepsilon_\perp/U_0$ calculated by formula (7). Curves 1 and 2 refer to $\eta^{1/2} = 2$ and 4.

Fig.2. Radiation spectra of electrons with a given transverse energy $\varepsilon_\perp$. Curves 1-3 refer to $\varepsilon_\perp/U_0 = 0.23$; 0.61 and 0.79. The calculation is carried out for one potential well ($U_0 = 34.2$ eV, $d_x = 1.69$ Å) of (100) planes of CdS crystal.

Fig.3. Radiation spectra of 4.5 GeV electrons in CdS crystal planes (100) averaged over beam and crystal thickness (3.5 mm). Curve 1 does not take into account the possibility of simultaneous registration of several photons, curve 2 takes into account such possibility. Squares show the experimental data [1] .

Fig.4. The averaged potential of planes (100) (solid curve) and (110) (dashed curve) of CdS crystal.

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А.Р.АВАКЯН, А.С.АРУТЮНЯН, ЯН ШИ

СПЕКТР ИЗЛУЧЕНИЯ КАНАЛИРОВАННЫХ ЭЛЕКТРОНОВ В ТОЛСТОМ КРИСТАЛЛЕ

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Ереван 36, ул.Братьев Алиханян,2

The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR