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TOWARDS THE COMPLETE ACTION OF THE FIELD THEORY OF INTERACTING UNIVERSES

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Ռ.Լ.ՄԿՐՏՉՅԱՆ

ՓՈԽԱԶԴՈՂ ՏԻԵԶԵՐՔՆԵՐԻ ԴԱՇՏԻ ՏԵՍՈՒԹՅԱՆ ԼՐԻՎ ԳՈՐԾՈՂՈՒԹՅԱՆ ՄԱՍԻՆ

Քննված է փոխազդող տիեզերքների դաշտի տեսության գործողությունը։ Ելնելով այն ենթադրությունից, թե Ֆունկցիոնալ ինտեգրալի մեջ մտնում է բոլոր տոպոլոգիաների գումարը, պատճառաբանվում է փոխազդեցության քառակուսային և խորանարդային անդամների առկայությունը, և այն, որ տոպոլոգիորեն այդ անդամները ստացվում են ստանդարտ վերափոխությամբ։

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1. Introduction

The aim of this paper is to construct in the general form the complete action of the field theory of the interacting universes. This theory is implied to be the second-quantization of the theory of the single universe. As usual, the second quantization means that we consider a Fock space which is a direct sum of spaces each of which is a tensor product of N spaces for a single universe (N = 0,1,2,...). (Here a question arises about the statistics, i.e. whether the creation operators of universes commute or anticommute (or, maybe, something else?). Answer is unknown, but it doesn't affect the considerations of the present paper.) So, in this theory one may consider the interaction between different universes. It is worthwhile to define more concretely what we mean under the single universe. Under the single universe we mean the d-dimensional compact connected manifold V without boundary with the different fields on it. These fields include gravitational one plus scalar, spinor, etc.: presumably these are the fields of of the string theory, although it is completely inessential for the present considerations. For example, for our real world a possible choice may be d = 3 and field content as follows: gravitation, quarks, gluons, etc. Lagrangian of this theory may be thought to be a sum of curvature term plus Lagrangian of quantum chromodynamics plus...

Refs [1-10] are recent works (and related ones) on field theory of universes (theory of "wormholes"). Our aim here is to establish some general properties of the action of this theory (assuming its existence...). Our guideline will be the analogy with the string field theory (see, e.g. Refs [11,12]). To exploit this analogy, one has to resolve one apparent difference between string theory and theory of universe: the string (we consider for simplicity the bosonic string) is one-dimensional manifold (circle in the case of closed strings), which is imbedded (mapped) in some d-dimensional external space, and universe apparently is imbedded nowhere. But let's look at the imbedding of string from the other point of view - the imbedding means that one defines d scalar functions on the abstract one-dimensional manifold. The whole information about the position of string in the external d-dimensional space is contained in these functions. Now, if we compare this situation with the case of universe with a number of fields in it, we can see that string theory is the particular case of the theory of universe when the dimensionality of the latter is taken to be 1, and the set of fields consists of d scalar fields. It is clear from this discussion that the theory of p-branes also may be considered as a particular case of the theory of universes,

with definite choice of dimensionality and a set of fields in the universe. We note also that our considerations have more analogy with the string field theory in the light-cone gauge [11] rather than with the covariant approaches, which include ghosts [12] .

In Sect. 2 we discuss the action of the field theory of universes, in Sect. 3 we give arguments that it is minimal action which after quantization gives the sum over all manifolds in the first-quantized representation of propagator. In the final Section 4 we also generalize the Hartly-Hawking's wave function of Universe [13] on the field theory of universes.

2. Action of the Field Theory of Universes

From the first quantization of the theory of single universe one obtains the Schrodinger equation of the general form:

$$H\psi = 0, \tag{1}$$

where Ψ is the wave function of the single universe, H is some operator. (1) is called the Wheeler-DeWitt equation. From (1) one can read-off in a symbolic form the action for the second-quantized free theory:

$$S_0 = \psi(V)\, H\psi(V). \tag{2}$$

The integration (summation) over repeated arguments here and below is implied (it means taking the scalar product in the space of wave functions of first-quantized theory). Note that here and below V in the argument means not only the compact connected manifold V but also the entire set of coordinates defining the state of the universe V, i.e. the configuration of the fields, etc. So, in (2) summation (integration) goes over all compact connected manifolds V and all the configurations of matter fields on them. We shall denote generally the whole set of fields on V by φ .

The complete action S differs from this one by the terms, governing the interaction between different universes. It may be written in the general form:

$$S = S_0 + \mathcal{Y}(V_1, V_2)\Psi(V_1)\Psi(V_2) + \mathcal{Y}(V_1, V_2, V_3)\Psi(V_1)\Psi(V_2)\Psi(V_3) + \dots \quad (3)$$

where again the summation (integration) over the repeated arguments V_1 , V_2 ,... is implied.

The first term in (3) describes the free propagation of the single universe. The second term describes a process when universe 1 transforms into the universe 2 with amplitude $\mathcal{Y}(V_1, V_2)$. The third term describes a process of splitting of one universe into two universes, or fusion of two universes into one universe. The similar interpretation may be given for the subsequent not written explicitly terms in (3).

As we have mentioned above, our aim is to find mainly the topological content of the action (3). This means that we shall establish at what topologies of V_1 and V_2 the interaction vertex $\mathcal{Y}(V_1,V_2)$ is nonzero, and propose some answer for it, and the same for $\mathcal{Y}(V_1,V_2,V_3)$. We also claim that there are no further terms in (3). Now we turn to the principles by which we shall find the action (3).

We postulate the answer for some quantities - propagator, for example, - and obtain the action (3) from the requirement that it must give the postulated form for those quantities when used in the functional integral. We shall pay attention only at the topological aspects of all the constructions.

We postulate the following form of the propagator in terms of the functional integral:

$$G(V_1, V_2) = \sum_{M, \partial M = V_1 \cup V_2} \omega_M \int \mathcal{D}\bar{\varphi}\, e^{S_M(\bar{\varphi})} \quad (4)$$

The sum in (4) is over all (d+1)-dimensional manifolds M with a boundary consisting of V_1 and V_2 . Integration goes over all field configurations $\bar{\varphi}$ on M such that they coincide on the boundary of M with given configurations: φ_1 on V_1 and φ_2 on V_2 - $\bar{\varphi}|_{\partial M} = \varphi_1$ on V_1 and φ_2 on V_2 . S_M is the action for fields $\bar{\varphi}$. ω_M are real numbers - the weight with which the manifold M is taken. ω_M depends only on topological class of M. Evidently, one may easily generalize (4) for higher Green functions.

This choice is a simple and natural one. A similar choice for the wave function of the ground state of the Universe is advocated in [13] . This choice is in complete agreement with the answer for propagator in the string theory, where the sum over manifolds M includes the manifolds with arbitrary number of handles.

In this supposed answer for the propagator the sum over different (topologically) manifolds M connecting the initial and final universes cannot run with arbitrary weight ω_M for each topologically distinct manifold. These weights must be the products of coupling constants entering in the classical action (3).

In the string field theory the weight for the manifold M (which in this case is the sphere with k handles and two disks cut out) is λ^{2k} , when λ is coupling constant of the string field theory.

Before beginning the discussion of (4) one may note the possibility of the following, more general approach. One may try to find all possible topological types of universes V and corresponding manifolds M in the representation (4) such, that the r.h.s. of (4) could be obtained from some action like (3) with definite interaction terms. One such possibility is the following: the manifolds V are only d-dimensional spheres, and permitted M are the manifolds, which arise when one takes a (d+1)-dimensional sphere with a number of d-dimensional disks removed and then glue the boundary of a number of (d+1)-dimensional cylinders (without losing orientability) to the boundary of these disks. The boundary of such a manifold consists of a number of spheres (for propagator (4) one has to take this number equal to two). One can readily see that propagator in such a form arises from the action (6) below, where V_i are now only the spheres, and only the first and the last terms in (6) are taken.

Expression (4) represents the complete propagator of the theory in a first-quantized form. We suppose that, as in all known theories, the same quantity has to appear as a "two-point" Green function in the corresponding second-quantized theory.

In our case this second-quantized theory is supposed to have action S (3). So, one expects:

$$G(V_1,V_2) = \int \mathcal{D}\Psi \; \Psi(V_1)\Psi(V_2) e^{S(\Psi)} \qquad (5)$$

Our claim is that the desired action S is given by the following expression:

$$S = S_0 + \sum_{k=d,d-1,\dots,a} \delta_k(V_1,V_2)\Psi(V_1)\Psi(V_2) + \delta_d(V_1,V_2,V_3)\Psi(V_1)\Psi(V_2)\Psi(V_3), \qquad (6)$$

where $a=(d+2)/2$ for d even and $a=(d+1)/2$ for d odd.

The meaning of vertexes δ_k is (in accordance with their notations) that an interaction occurs when some points of one or two universes coincide. The index k denotes the dimensionality of the submanifold (which always will be a sphere, as discussed below) of the universe, which consists of these points, i.e. which "shrinks" to one point.

The vertex δ_k describes the "spherical reconstruction" the meaning of which is that the manifold(s) V_1 , $i > 1$ arises from the manifold V_1 by the following procedure. Let us take some sphere S^{k-1} $(k=1,\dots,d)$ imbedded in V_1 in such a way that it has a vicinity homeomorphic to $S^{k-1} \times D^{d-k+1}$ Then we remove the interior of D^{d-k+1}. The manifold appeared has a boundary $S^{k-1} \times S^{d-k}$. The same boundary has a manifold $D^k \times S^{d-k}$, and we glue these two manifolds over their boundaries and obtain one or two (depending on k and the choice of initial imbedding of sphere S^{k-1} , two manifolds may appear only for k=d) new manifold(s). These are the basi interactions in the field theory of universes. The process of obtaining V_i , $i > 1$ from V_1 can be made continuous, and the (d+1)-dimensional manifold - the "trace" of this process - is described below.

The theory of spherical reconstructions was used for the first time in quantum gravity by A. Kocharyan [14] for describing a notion, which may be called "the space of all universes", introduced in [14] .

The vertex δ_K is nonzero when there exists (d+1)-dimensional manifold M_K obeying the following conditions: a) the arguments V_i of this vertex are the boundary of M_K : $\partial M_K = \bigcup_i V_i$, b) M_K is the standard cobordism of rank k. This last condition means that M_K is diffeomorphic to the manifold obtained by the following procedure. Let $\varphi: S^{K-1} \times D^{d-K+1} \to V_1$ be the imbedding. Let $\bar{M}_K$ be the topological space obtained from ($V_1 \times I$) ($I = [0,1]$) by gluing it with the manifold $D^K \times D^{d-K+1}$ through the manifold $Im\varphi_1$ where $\varphi_1 : S^{K-1} \times D^{d-K+1} \to (V_1 \times I)$ is given by $(\varphi, 1)$. By the standard procedure $\bar{M}_K$ may be endued with the structure of differentiable manifold and that is the desired manifold M_K . The boundary of M_K is $\partial M = \bigcup_i V_i$, where i = 1,2 or i =1,2,3. These two cases differ in the following way: when we use the above procedure, we usually obtain the manifold M_K with a boundary consisting of two pieces - initial manifold V_1 and some "final" one V_2 , whereas in the case $K = d$ and when using the special imbedding φ (such, that it divides V_1 into two pieces), the boundary of M_K may consist of three pieces. One more note is necessary in the case when boundary of M_K consists of two

pieces. In the above procedure the manifolds V_1 and V_2 enter asymmetrically: we start from the V_1 and obtain the other as a part of the boundary of M_K . But actually M_K are symmetric with respect to both components of their boundary in the following sense: the same M_K may be considered as obtained from the other component of its boundary by the same procedure at $K' = d - K$ and some imbedding φ ([16] 4.1.2). This is the reason why only the half of the possible values of K are included in (6).

Would we want to describe the "purely topological" field theory of (interacting) universes, i.e. the theory of the empty (even without metric, i.e. gravitational field) universe, this will be the end of the story - we may finish the definition of vertexes by saying that when the arguments V_i obey the above conditions the value of the vertex δ_K is an arbitrary real number, depending on the topological class of V_i .

The existence of different fields (gravitational, scalar, spinor, etc.) in the universe requires further definitions of δ_K .

Let's consider the manifold M_K , defined above, the boundary of which is $\bigcup_i V_i$. Remember that now on the boundary of M_K a number of fields is defined. We suppose that one may define these fields on the whole M_K in such a way that values of these fields on the boundary of M_K coincide with that on V_i (if it is impossible, then δ_K is defined to be zero. Discussion of corresponding restrictions on fields configurations for some cases is given in Ref.[15]). Now one has to "shrink" the M_K in such a way that eventually we come to the situation when the boundaries of M_K coincide. This means that the values of fields also have to coincide. The δ-function of coincidence of the values of fields on V_i is the vertex δ_K. If this shrinking process could be carried out in different ways, and answers would be different, then all these terms have to be included in the action.

More exactly, instead of considering this shrinking process one may use a theory of Morse functions. It is known that on manifold M_K one may define a (Morse) function f which is a differentiable function with exactly one critical point, taking values 0 on V_1 and 1 on V_i, $i > 1$ [16,17]. Let c be the value of f in that point. Let's denote by M_a the set of all points x in M_K with $f(x) = a$. It is known [16,17] that all M_a, $a < c$ are diffeomorphic to V_1, and M_a, $a > c$ are diffeomorphic to $\bigcup_i V_i$, $i > 1$, so the set $\bigcup_{a \leqslant c-\varepsilon} M_a$ is diffeomorphic to cylinder $V_1 \times I$, and set $\bigcup_{a \geqslant c+\varepsilon} M_a$ is diffeomorphic to cylinder $(\bigcup_{i>1} V_i) \times I$, for arbitrary small $\varepsilon > 0$. The set M_c is not a manifold. The points of M_c are in one-to-one correspondence both with points of V_1 and points of $\bigcup_{i>1} V_i$. This correspondence may be established by the arbitrary gradient like vector field [16], in particular the gradient field of f. The δ-function of coincidence of values of fields φ_1 on V_1 and φ_i on V_i, $i > 1$, in the corresponding points, is the vertex $\delta_K(V_i)$.

So, the picture of interaction is the following: some sphere of dimensionality (k-1) in universe V shrinks to one point and then this point extends in "transversal directions" to a sphere of dimensionality (d-k) (k=1,2,...,d). As a result

the universe V transforms into one or two other universe(s).

3. The Main Statement

Our claim is that these simple interactions are sufficient and necessary to obtain, in perturbation theory of (5), all possible manifolds in (4).

Let's discuss this last statement which is the main claim of this work. Of course, this claim is nothing but a translation of the well-known mathematical facts into a physical language plus some hypotheses about theory (6), which (hypotheses) essentially mean that the properties of that theory are similar to other known theories such as string field theory.

In what follows we shall consider the perturbation theory for the action (6). As a free part of the action we take the first term in (6) (although it is not the only quadratic term in (6)), and all the remaining terms will be considered as interaction vertexes.

Firstly, we suppose that the first term in the action S (6) (i.e. its "free part") gives (through (5)) a free propagator G_0, which may be represented as in (4) where now the sum over manifolds M includes only one term - the term where M is a cylinder. This is a completely normal assumption which actually has to be correct after appropriate definitions of theory and functional integral (inclusion of ghosts, etc.). It may also turn out to be already proved in some works on quantum gravity.

Now we must take into account the further terms in the perturbation theory expansion.

Let us consider, for example, the correction to the propagator which arises when we take into account the vertex once:

$$\delta G(V_1, V_2) = \int \mathcal{D}\psi e^{S_0(\psi)} \psi(V_1)\psi(V_2)\delta_K \simeq$$

$$G_0(V_1, V_1')\delta_K(V_1' V_2')G_0(V_2', V_2') \simeq \qquad (7)$$

$$\int \mathcal{D}\bar{\varphi}_1 e^{S_{N_1}(\bar{\varphi}_1)} \delta_K(V_1', V_2') \int \mathcal{D}\bar{\varphi}_2 e^{S_{N_2}(\bar{\varphi}_2)} \simeq \int \mathcal{D}\bar{\varphi} e^{S_{M_K}(\bar{\varphi})}$$

where N_1 and N_2 are cylinders (i.e. are diffeomorphic to $V_1 \times I$ and $V_2 \times I$, respectively):

$\partial N_1 = V_1 \cup V_1'$, $\partial N_2 = V_2 \cup V_2'$, $\bar{\varphi}|_{V_1} = \bar{\varphi}_1|_{V_1} = \varphi_1$, $\bar{\varphi}|_{V_2} = \bar{\varphi}_2|_{V_2} = \varphi_2$

and in the second and third lines of (7) integration goes over all the configurations of matter fields on N_1 and N_2. The last line of (7) is the desired answer - the term in (4) with $M = M_K$.

Of course, Eq.(7) is but a symbolic one, it is nothing but reminder that the similar equation is correct in string field theory, and one may hope that its generalization (7) on higher dimensions can be made correct as well.

Now, it is evident (on the same level of strictness) that in higher terms of perturbation theory the manifolds M arise, which are the result of gluing an arbitrary number of manifolds M_K with arbitrary K in such a way that the final manifold M has a boundary $\partial M = V_1 \cup V_2$. It remains to use the theorem in mathematics which reads that in such a way one may obtain an arbitrary manifold M [16,17].

So, we give intuitive arguments that the action in the form (6) gives the propagator in the form (4) in the perturbation theory expansion. This has been the main goal of our work.

Now let's consider an example of the above theory at d=2. In this case all manifolds V are classified to be the spheres with g handles ($g = 0,1,...$ is called a genus of the manifold V). Now in (6) there enter three interaction terms obtained from the standard cobordisms with indexes one and two. M_1 connects two manifolds V with genera g and g', $|g-g'|=1$, and δ_2 enter in two terms. Firstly, it may cause the same effect as M_1 - i.e. connects the manifolds with genera g and g', $|g-g'|=1$ (but in a different way - in the process described by M_1, the zero-dimensional sphere S^0 shrinks to one point and then this point extends to one-dimensional sphere S^1, and in the second process the one-dimensional sphere S^1 shrinks to a point and then expands to a zero-dimensional sphere S^0). This process takes place when the circle S^1 which shrinks to zero doesn't divide V into two pieces. Otherwise, the second process caused by δ_2 takes place - the birth of one additional universe. In this latter case M_2 connects manifolds V_1, V_2 and V_3 having such genera g_1, g_2, g_3 that the sum of two of them is equal to the third one.

4. Wave Function of the Ground State of the Field Theory of Interacting Universes

One may easily generalize the Hartly and Hawking's [13] wave function of the ground state of the Universe on the field theory of universes. This may be done as follows. In the subspace of the previously discussed Fock space, consisting of the tensor product of N spaces for one universe, we define in analogy with [13] the following functional:

$$\Psi_N(V_1, V_2, \dots, V_N) = \sum_M \int \mathcal{D}\bar{\varphi}\, e^{S_M(\bar{\varphi})} \tag{8}$$

$$\partial M = \bigcup_{i=1}^{N} V_i\,, \quad \bar{\varphi}\big|_{V_i} = \varphi_i$$

Here M runs over all compact connected manifolds with boundary $\partial M = \bigcup_{i=1}^{N} V_i$, and $S_M(\bar{\varphi})$ is euclidean action on M (i.e. the metric $g_{\mu\nu}$ on M, which is the part of fields $\bar{\varphi}$, has euclidean signature). For N=1 Ψ_N coincides with wave function of Hartly and Hawkings [13].

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Р.Л.МКРТЧЯН

К ПОЛНОМУ ДЕЙСТВИЮ ТЕОРИИ ПОЛЯ ВЗАИМОДЕЙСТВУЮЩИХ ВСЕЛЕННЫХ

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