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LIENARD-WIECHERT FIELD AS COVARIANT DYNAMICS OF ELECTRIC LINES OF FORCE

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ԼԻԵՆԱՐ-ՎԻԽԵՐՏԻ ԴԱՇՏԸ ՈՐՊԵՍ ԷԼԵԿՏՐԱԿԱՆ ԴԱՇՏԻ ՈՒԺԱՅԻՆ ԳԾԵՐԻ ԿՈՎԱՐԻԱՆՏ ԴԻՆԱՄԻԿԱ

Կամայականորեն շարժվող լիցքի Լիենար-Վիխերտի դաշտը ներկայացված է որպես շարժվող էլեկտրական ուժային գծերի Լորենց-կովարիանտ համակարգ: Ցույց է տրված, որ այդ գծերը նկարագրող քառաչափ վեկտորը գրանցվում է դիտման կետին ուղղված՝ լիցքի և իզոտրոպ քառաչափ վեկտորների գումարային տեսքով: Այդ քառաչափ վեկտորի շարժումը նկարագրվում է հավասարմամբ, որը համընկնում է արտաքին դաշտերում մագնիսական մոմենտի շարժման հավասարմանը՝ պայմանով, երբ սեփական մագնիսական մոմենտը հավասար է զրոյի: Ըստ գծերի համակարգի, որոնք լրիվ համապատասխանում են արտաքին դաշտերում մագնիսական մոմենտի շարժման հավասարմանը, վերականգնվում է մագնիսական դաշտը: Պարզվում է, որ այդ դաշտին համապատասխանում է տարածական մագնիսական հոսանք, որը համեմատական է դիտման կետին ուղղված իզոտրոպ քառաչափ վեկտորին:

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LIENARD-WIECHERT FIELD AS COVARIANT DYNAMICS OF ELECTRIC LINES OF FORCE

The Lienard-Wiechert field of an arbitrarily moving charge is presented as a system of Lorentz-covariant moving electric lines of force. It is shown that the 4-vector that describes these lines is written as a sum of the 4-vector of the charge and the isotropic 4-vector directed to the observation point. The motion of this 4-vector is described by the equation coinciding with the equation of motion for magnetic moment in external fields provided that the intrinsic magnetic moment is zero. By the system of lines that corresponds to the complete equation of motion of magnetic moment in external fields the electromagnetic field is restored. It turned out that the spatial magnetic current proportional to the isotropic 4-vector directed to the observation point corresponds to this field.

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С.Г.АРУТЮНЯН

ЛИЕНАР-ВИХЕРТОВСКОЕ ПОЛЕ КАК КОВАРИАНТНАЯ ДИНАМИКА СИЛОВЫХ ЛИНИЙ ЭЛЕКТРИЧЕСКОГО ПОЛЯ

Лиенар-вихертовское поле произвольно движущегося заряда представлено как система лоренц-ковариантных движущихся электрических силовых линий. Показано, что 4-вектор, описывающий эти линии, записывается в виде суммы 4-вектора заряда и изотропного 4-вектора, направленного в точку наблюдения. Движение этого 4-вектора описывается уравнением, совпадающим с уравнением движения магнитного момента во внешних полях при условии, что собственный магнитный момент равен нулю. По системе линий, соответствующей полному уравнению движения магнитного момента во внешних полях, восстанавливается электромагнитное поле. Оказывается, что этому полю соответствует пространственный магнитный ток, пропорциональный направленному в точку наблюдения изотропному 4-вектору.

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1. Remind, how the covariant system of lines of force is constructed for orthogonal fields. We define electromagnetic field tensor $F^{\mu\nu}$ and its dual tensor $F^{*\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ where $\varepsilon^{\mu\nu\alpha\beta}$ is a completely antisymmetric unit 4-vector in four dimensions.

The system of Lorentz-covariant electric lines of force is determined by exact differential equations [1,2] :

$$F^{*}_{\mu\nu}\, dx^{\mu} = 0 \,, \tag{1}$$

$dx^{\mu} = (cdt\,,\, d\vec{r})$. Component equations (1) are of the form:

$$\begin{aligned} &[d\vec{r} \times \vec{E}] - cdt\,\vec{H} = 0, \\ &(\vec{H}\, d\vec{r}) = 0 \,, \end{aligned} \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic fields, respectively.

The system (1) has nontrivial solutions if $\det F^{\mu\nu} = 0$, i.e. in case when the electric and magnetic fields are orthogonal.

The integrability condition for system (1)

$$[\vec{E}\times(rot\,\vec{H}-\frac{1}{c}\frac{\partial\vec{E}}{\partial t})]-\vec{H}\,div\,\vec{E}=0 \tag{3}$$

is fulfilled in space without charges and currents.

Under condition $\vec{E}\cdot\vec{H}=0$ the dimension of antisymmetric tensor $\overset{*}{F}_{\mu\nu}$ is two, hence Eqs. (1) determine a two-dimensional surface $x^{\mu}(\tau,\sigma)$ which is covered by electric lines of force.

We have shown in Ref. [3] that by the system of lines of force given on this surface one can restore the tensor part of the electromagnetic field:

$$F^{\mu\nu}=\lambda(\dot{x}^{\mu}x'^{\nu}-x'^{\mu}\dot{x}^{\nu}), \tag{4}$$

where λ is scalar, the dot and prime denote differentiation with respect to τ and σ, respectively.

We can obtain closed equations corresponding to Maxwell equations if we assume that the system of lines covers the whole space. Introducing additional dependence of on parameters c_1 and c_2 we can rewrite the Maxwell equations for orthogonal fields. The first pair of equations in space without charges and currents is written as

$$\frac{\partial}{\partial\xi^{0}}\left(\lambda G\,\frac{\partial x^{\nu}}{\partial\xi^{1}}\right)-\frac{\partial}{\partial\xi^{1}}\left(\lambda G\,\frac{\partial x^{\nu}}{\partial\xi^{0}}\right)=0, \tag{5}$$

where dimensionless variables ($\tau,\sigma;\ c_1,\ c_2$) are combined into ξ^{κ}, $\kappa=0,1,2,3$; $G=\det(\partial x^{\mu}/\partial\xi^{\kappa})$

The second pair of the Maxwell equations is of the form:

$$\frac{\partial}{\partial\xi^{\ell}}\left[\varepsilon^{ij\kappa\ell}\,G^{-1}x^{\nu}_{,i}(\dot{x}x_{,j})(x'x_{,\kappa})\right]=0. \tag{6}$$

It follows from Eq.(5) that $\lambda \sim G^{-1}$. We'll obtain the correct dimension of field if $\lambda = eG^{-1}$. Then

$$F^{\mu\nu} = \frac{e}{G}(\dot{x}^{\mu}x'^{\nu} - x'^{\mu}\dot{x}^{\nu}). \qquad (7)$$

2. In Refs [4-6] we have constructed a system of electric and magnetic lines of force for an arbitrarily moving charge. The field pattern obtained gives a good representation of its structure. Characteristic narrow localizations of radiation field of ultrarelativistic charge in space are revealed. However the system of lines was determined for a fixed moment of time, and the question of field restoration by the lines of force was not discussed in those works. The field of the arbitrarily moving point charge satisfies the orthogonality condition, and therefore this theory is applicable here.

We parametrize the lines of force in the form:

$$x^{\mu} = R\left(\tau, \frac{\vec{z}_0(\sigma)}{R} + (\tau-\sigma)\vec{n}(\sigma, c_1, c_2)\right) = R\left(\sigma, \frac{\vec{z}_0(\sigma)}{R}\right) + R(\tau-\sigma)(1, \vec{n}), \qquad (8)$$

where R is a multiplier with length dimension, $\tau = ct/R$, $\sigma = ct'/R$, t and t' are current and retarded times, $\vec{z}_0(\sigma)$ is the charge motion trajectory, $\vec{n}$ is a unit vector directed to the observation point. By the current and retarded time parameters we find a point in space that satisfies the retarded equation by construction. The physical solutions that satisfy the retarded potentials are described by a condition: $\sigma < \tau$.

At fixed τ, c_1, c_2 parameters the variation of σ produces a one-dimensional line in the 4-dimensional space. It is required that the spatial part of the tangent to this line coincides with the direction of electric field:

$$R^2\vec{E} = \frac{e}{\gamma^2(\tau-\sigma)^2(1-\vec{\beta}\vec{n})^3}\left(\vec{n}-\vec{\beta}+\gamma^2(\tau-\sigma)\left[\vec{n}\times[(\vec{n}-\vec{\beta})\times\vec{\beta}']\right]\right), \qquad (9)$$

where $\vec{\beta}c = \frac{c}{R}\frac{d\vec{z}_o}{d\sigma}$ is the velocity of motion of the charge, $\gamma = (1-\beta^2)^{-1/2}$.

With account of (8) and (9) we obtain an equation for vector $\vec{n}$:

$$\vec{n}' = -\gamma^2\left[\vec{n}\times[(\vec{n}-\vec{\beta})\times\vec{\beta}']\right], \qquad (10)$$

which describes its motion over the surface of the unit sphere at the given function $\vec{\beta}(\sigma)$. The general solution of (10) depends on two integration constants - let these be parameters C_1 and C_2 .

The resulting system of lines, according to (7), will give a correct expression for tensor $F^{\mu\nu}$ if

$$G = R^4(\tau-\sigma)^2\gamma^2(1-\vec{\beta}\vec{n})^3. \qquad (11)$$

This in turn is true if

$$\left[\frac{\partial\vec{n}}{\partial c_1}\times\frac{\partial\vec{n}}{\partial c_2}\right] = -\vec{n}\gamma^2(1-\vec{\beta}\vec{n})^2. \qquad (12)$$

Differentiating both sides of this equality with respect to and using (10) and (12) one can show that the condition (12) does not contradict the equations of motion (10).

The general solution of (10) can be found, for example for uniform motion of the charge along a circle of radius R.

The 4-vector is written in the form:

$$x^{\mu} = R\left(\sigma, \cos\beta\sigma + (\tau-\sigma)(\nu_1\cos\beta\sigma - \nu_2\sin\beta\sigma), \sin\beta\sigma + (\tau-\sigma)(\nu_1\sin\beta\sigma + \nu_2\cos\beta\sigma), (\tau-\sigma)\nu_3\right), \tag{13}$$

where

$$\nu_1 = \frac{\sqrt{1-c_2^2}}{\gamma} \cdot \frac{\sin(\beta\gamma\sigma + c_1)}{1+\beta\sqrt{1-c_2^2}\cos(\beta\gamma\sigma+c_1)}, \qquad \nu_2 = \frac{\beta + \sqrt{1-c_2^2}\cos(\beta\gamma\sigma + c_1)}{1+\beta\sqrt{1-c_2^2}\cos(\beta\gamma\sigma + c_1)},$$

$$\nu_3 = \frac{c_1}{\gamma} \cdot \frac{1}{1+\beta\sqrt{1-c_2^2}\cos(\beta\gamma\sigma + c_1)} .$$

The dependence of x^{μ} on c_1 and c_2 is chosen such that the condition (12) is satisfied. This means that formula (13) entirely defines the tensor of electromagmetic field.

3. Having made the transformation

$$\vec{\nu} = \frac{\vec{n} + ((\gamma-1)(\vec{\beta}\vec{n})/\beta^2 - \gamma)\vec{\beta}}{\gamma(1-\vec{\beta}\vec{n})} \tag{14}$$

we'll obtain instead of (10) an equation:

$$\vec{\nu}\,' = \frac{\gamma-1}{\beta^2}\left[\vec{\nu} \times [\vec{\beta} \times \vec{\beta}\,']\right] . \tag{15}$$

Introduce the 4-vector a^{μ} whose components in the associated reference system connected with trajectory are $(0, \vec{\nu})$. In the laboratory reference system we have

$$a^{\mu} = \left(\gamma\frac{\vec{\beta}\vec{n} - \beta^2}{1-\vec{\beta}\vec{n}}, \frac{\vec{n}}{\gamma(1-\vec{\beta}\vec{n})} - \vec{\beta}\gamma\right) . \tag{16}$$

Vector x^μ here is written in the form:

$$x^\mu = z^\mu + (\tau - \sigma)\gamma(1-\vec{\beta}\vec{n})(u^\mu + a^\mu), \qquad (17)$$

where $u^\mu = (\gamma, \beta\gamma)$ is 4-velocity of the charge. The product $(\tau-\sigma)\gamma(1-\vec{\beta}\vec{n})$ is Lorentz-invariant being connected with the second invariant of field $\vec{E}^2 - \vec{H}^2 = \varepsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$ via the relation:

$$(\tau-\sigma)\gamma(1-\vec{\beta}\vec{n}) = R\left[(\vec{E}^2 - \vec{H}^2)/e^2\right]^{1/4}. \qquad (18)$$

The equation of motion for vector a^μ, resulting from (15), is written as

$$\frac{c}{R}\gamma\frac{da^\mu}{d\sigma} = \frac{e}{mc}u^\mu f^{\nu\lambda}u_\nu a_\lambda \qquad (19)$$

where $f^{\nu\lambda}$ is the external electromagnetic field tensor that presets the particle motion trajectory. The left-hand side of (19) represents differentiation of a^μ with respect to intrinsic time of the particle. Hence (as mentioned in [6]), Eq.(19) coincides with the equation of spin motion in external fields [7] if we put for the value of intrinsic magnetic moment $\mu = 0$ and for anomalous magnetic moment $\mu' = -e\hbar/(2mc)$.

We shall realize the following procedure: we'll force the vector a^μ (which in our case was introduced not as a vector of the particle polarization state) to follow the general Bargmann-Michel-Telegdi equation and then by (17) and (7) we'll restore the corresponding electromagnetic field. i.e. instead of (19) we'll write

$$\frac{c}{R}\gamma\frac{da^\mu}{d\sigma} = 2\mu f^{\mu\nu}a_\nu - 2\mu' u^\mu f^{\nu\lambda}u_\nu a_\lambda. \qquad (20)$$

The resultant equation for $\vec{n}$ is of the form:

$$\vec{n}' = \bar{\mu}'\gamma^2\left[\vec{n}\times[(\vec{n}-\vec{\beta})\times\vec{\beta}']\right] + \frac{\bar{\mu}}{\gamma}\Big([\vec{n}\times\vec{h}_o] - [\vec{n}\times[\vec{n}\times\vec{e}_o]]\Big); \quad (21)$$

where $\bar{\mu} = \frac{2mc}{e\hbar}\mu$, $\bar{\mu}' = \frac{2mc}{e\hbar}\mu'$, $\bar{\mu}-\bar{\mu}'=1$; $\vec{e}_o$, $\vec{h}_o$ are tensor $f^{\mu\nu}$ electric and magnetic fields normalized to quantity mc^2/eR. The external field is considered on trajectory only, therefore, $\vec{e}_o$ and $\vec{h}_o$ are functions of σ only.

One can readily see that the quantity G does not change, hence the field restored according to (7) is of the form:

$$R^2\vec{E} = \frac{e}{\gamma^2(\tau-\sigma)^2(1-\vec{\beta}\vec{n})^3}\Big(\vec{n}-\vec{\beta}-\bar{\mu}'(\tau-\sigma)\gamma^2[\vec{n}\times[(\vec{n}-\vec{\beta})\times\vec{\beta}']] + \frac{\bar{\mu}}{\gamma}\big([\vec{n}\times[\vec{n}\times\vec{e}_o]] - [\vec{n}\times\vec{h}_o]\big)\Big), \quad (22)$$

$$\vec{H} = [\vec{n}\times\vec{E}].$$

Usually the electromagnetic field is defined by the given currents. And now, on the contrary, we must obtain the currents that correspond to the field (22). The expression for electric current j_E comes out of equation $\partial F^{\mu\nu}/\partial x^\mu = 4\pi j_E^\nu/c$ which in our case are written as

$$\frac{1}{4G}\left\{\frac{\partial}{\partial\xi^0}\left(\lambda G\frac{\partial x^\nu}{\partial\xi^1}\right) - \frac{\partial}{\partial\xi^1}\left(\lambda G\frac{\partial x^\nu}{\partial\xi^0}\right)\right\} = \frac{4\pi}{c}j_E^\nu . \quad (23)$$

The expression for j_E^ν is independent of whether $\vec{n}$ is determined by equations (10) or (21), since at $(\tau-\sigma)\to 0$ (for $(\tau-\sigma)\neq 0$ the right-hand side in (23) turns to zero) in the 4-vector $x'^\mu = R(0,\vec{\beta}-\vec{n}+(\tau-\sigma)\vec{n}')$ the term proportional to $\vec{n}'$ falls out, and vector $\vec{n}$ cancels out with the similar component of the 4-vector $\dot{x}^\mu = R(1,\vec{n})$. We obtain the previous electric current corresponding to the motion of charge e

along the given trajectory proportional to $(1,\vec{\beta})$.

The second pair of Maxwell equations is defined by expression $\partial E^{*\mu\nu}/\partial x^{\mu}$ which is of the form:

$$\frac{\partial F^{*\mu\nu}}{\partial x^{\mu}}=\frac{e}{4G}\,\frac{\partial}{\partial\xi^{\ell}}\left[\varepsilon^{ij\kappa\ell}\,\frac{1}{G}\,\frac{\partial x^{\nu}}{\partial\xi^{i}}(\dot{x}x_{,j})(x'x_{,\kappa})\right]=\frac{2e\bar{\mu}R\left((\vec{n}-\vec{\beta})\vec{h}_0+\vec{\beta}[\vec{n}\times\vec{e}_0]\right)(1,\vec{n})}{\gamma(1-\vec{\beta}\vec{n})} \tag{24}$$

One can see that this expression does not turn to zero; hence it can be interpreted as the occurrence of spatial magnetic current:

$$\dot{j}^{\nu}_{\mu}=\frac{\bar{\mu}e^{2}}{8\pi mc}\cdot\frac{f^{*\lambda\mu}n_{\mu}u_{\lambda}}{R^{2}(\tau-6)^{2}\gamma^{2}(1-\vec{\beta}\,\vec{n})^{2}}\cdot n^{\nu} \tag{25}$$

It is essential that the current $\dot{j}^{\nu}_{M}$ is an isotropic 4-vector, i.e. corresponds to magnetic charges moving with the velocity of light. Note, that the electric current with such a property was introduced by Rohrlich [8] to describe the space-restricted front of uniformly accelerated charge field.

We'd like to note the following: instead of equation (19) we have written down the more general equation (20). This in a sense is similar to the introduction of the monopole that symmetrizes the Maxwell equations. Then we used the procedure worked out for orthogonal fields in order to restore the field by a covariant system of lines. Perhaps, some other, more general procedure for the restoration of not necessarily orthogonal fields should have been used for the general equation (20). Presumably, such fields would have a more ordinary physical meaning than fields (22) that correspond to spatial isotropic magnetic current.

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