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YEREVAN PHYSICS INSTITUTE

L.S.DULYAN, A.Yu.KHODJAMIRIAN, A.D.MAGAKIAN

QUARKONIUM TWO-PHOTON DECAYS IN QCD

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Լ.Ս.ԳՈՒԼՅԱՆ, Ա.Յու.ԽՈՋԱՄԻՐՅԱՆ, Ա.Դ.ՄԱՂԱՔՅԱՆ

ՔՎԱՐԿՈՆԻՈՒՄԻ ԵՐԿՖՈՏՈՆԱՅԻՆ ՏՐՈՀՈՒՄԸ ՔՎԱՆՏԱՅԻՆ ՔՐՈՄՈԴԻՆԱՄԻԿԱՅՈՒՄ

Հաշվարկված է $\chi_{c2} \to 2\gamma$ թենզորային չարմոնիումի երկֆոտոնային տրոհման լայնությունը գլյուոնային կոնդենսատի էֆեկտների հաշվառմամբ: Արդյունքը համաձայնության մեջ է փորձի հետ: Ստացվել են $\eta_g \to 2\gamma$ թյուրսկալյար քվարկոնիումի երկֆոտոնային լայնության գնահատականներ:

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L.S. DULYAN, A.Yu. KHODJAMIRIAN, A.D. MAGAKIAN

QUARKONIUM TWO-PHOTON DECAYS IN QCD

The two-photon decay width of tensor charmonium $\chi_{c2} \to 2\gamma$ is calculated with account of gluon condensate effects. The result is in good agreement with experiment. The two-photon width of pseudoscalar b-quarkonium $\eta_b \to 2\gamma$ is estimated.

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Л.С.ДУЛЬЯН, А.Д.МАГАКЯН, А.Ю.ХОДЖАМИРЯН

ДВУХФОТОННЫЕ РАСПАДЫ КВАРКОНИЯ В КХД

Вычислена ширина двухфотонного распада тензорного чармония $\chi_{c2} \to 2\gamma$ с учетом эффектов глюонного конденсата. Результат находится в хорошем согласии с экспериментом. Получены оценки для двухфотонной ширины псевдоскалярного b-кварކония $\eta_b \to 2\gamma$

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1. Introduction

The annihilation of charmonium or b-quarkonium C-even levels into two photons reveals heavy quark dynamics both at small and large distances.

The first estimates of two-photon widths have been obtained by means of quarkonium nonrelativistic models (see, e.g. detailed discussion in [1]). Reliable calculation of these widths may be carried out by means of QCD sum rules [1,2] . In this way $\eta_c \to 2\gamma$ width [3,4] as well as $\chi_{c0} \to 2\gamma$ width [5,6] were estimated taking into account the leading nonperturbative effects. While data on both widths are not stable yet, there exist two independent measurements of tensor charmonium width $\chi_{c2} \to 2\gamma$ [7,8] giving close results although with large errors (see Table 1). It is interesting to test the reliability of QCD sum rules for this decay.

In this paper we shall calculate $\chi_{c2} \to 2\gamma$ width taking into account gluon condensate contribution. The result obtained without any new parameter turns out to agree with experimental values (see Table 1).

Two-photon widths of b-quarkonium still have not been measured. This problem is for future experiments in photon-photon collisions as well as for proposed B-factories. Unfortunately, standard QCD sum rules are not applicable for b-quarkonium. Nevertheless, as it will be demonstrated in the second part of this paper, the $\eta_b \to 2\gamma$ width may be reliably estimated due to the experimental information about S-wave b-quarkonium spectrum (i.e. Υ-resonances).

2. $\chi_{c2} \to 2\gamma$ Decay

Following the method suggested in [1] consider three-current correlator

$$\Delta_{\mu\nu\lambda\rho}(K_1, K_2) = \int dx\, dy \exp\left[i(K_1 x + K_2 y)\right]$$
$$\langle 0 | T\{ j_{\lambda\rho}(0) j_\mu(x) j_\nu(y)\} | 0 \rangle \qquad (1)$$

where $j_{\lambda\rho} = i\bar{c}(\gamma_\lambda \overleftrightarrow{\partial}_\rho + \gamma_\rho \overleftrightarrow{\partial}_\lambda - \frac{2}{3}\eta_{\lambda\rho}\hat{\overleftrightarrow{\partial}})c$ is the tensor c-quark current ($J^{PC} = 2^{++}$) with momentum $q = K_1 + K_2$; $\eta_{\lambda\rho} = \delta_{\lambda\rho} - q_\lambda q_\rho / q^2$; $j_\mu = \bar{c}\gamma_\mu c$ is the electromagnetic current corresponding to the photon emission with momentum K_1 or K_2 ($K_1^2 = K_2^2 = 0$).

$\chi_{c2} \to 2\gamma$ decay is determined by two invariant amplitudes. The kinematical structures of correlator (1) may be chosen in the following form:

$$e_{1\mu} e_{2\nu} \chi_{\lambda\rho} \Delta_{\mu\nu\lambda\rho} = \chi_{\lambda\rho} \Big[\big\{ K_{2\rho} e_{1\lambda}(K_1 e_2) + K_{1\rho} e_{2\lambda}(K_2 e_1) - e_{1\lambda} e_{2\rho}(K_1 K_2) - K_{1\lambda} K_{2\rho}(e_1 e_2) \big\} \Delta_a(q^2) + \big\{ (K_1 e_2)(K_2 e_1) - (K_1 K_2)(e_1 e_2) \big\} K_{1\lambda} K_{2\rho} \Delta_{\beta}(q^2) \Big] \qquad (2)$$

where e_1 , e_2 are photon polarization vectors, $\chi_{\lambda\rho}$ - is the χ_{c2} wave function. The remaining structures in (1) are inessential for decay width, since they disappear if multiplied by wave functions.

In the region $q^2 \ll 4m_c^2$ the correlator (1) may be presented as a sum of QCD diagrams: simple triangle loop (Fig.1a) and leading $O(\alpha_s)$ corrections. The perturbative corrections are given by Fig.1b diagrams. Leading nonperturbative corrections $\sim\alpha_s < G^2 >$ are determined by Fig.1c,d diagrams, i.e. by virtual c-quark interaction with vacuum gluon condensate.

It is convenient to present the contribution of triangle loop into invariant amplitudes $\Delta_{a,\beta}$ in the form of dispersion integral:

$$\overset{o}{\Delta}_{a,\beta}(q^2) = \frac{1}{\pi} \int \frac{\operatorname{Im} \overset{o}{\Delta}_{a,\beta}(s)\, ds}{s - q^2} ,$$

$$\operatorname{Im} \overset{o}{\Delta}_a(s) = -\frac{3}{4\pi} \left\{ (1-v^2)(3-v^2) \ln\frac{1+v}{1-v} - 6v\left(1-\frac{5}{9}v^2\right) \right\} ,$$

$$\operatorname{Im} \overset{o}{\Delta}_{\beta}(s) = \frac{3}{4\pi m_c^2}(1-v^2)^2 \left\{ (3-v^2) \ln\frac{1+v}{1-v} - 6v \right\} \qquad (3)$$

where $v = \sqrt{1-4m_c^2/s}$.

The bare loop contribution considered in [1] (with slightly different choice of invariant amplitudes) is insufficient for reliable calculation of the width. First of all it is necessary to calculate gluon condensate contribution into (1).

The most convenient way to do this calculation is to use the fixed point gauge for the vacuum gluon field: $x_\mu A_\mu^a = 0$ In our case this gauge essentially simplifies the problem. Really, the $j_{\lambda\rho}$ current contains derivative. Therefore, gauge invariance implies that besides Fig.1c diagrams with two gluon insertions into c-quark line we have to consider also Fig.1d diagrams where one of the gluons is emitted in 2^+-vertex. Nevertheless, if $x = 0$ point is chosen in this vertex, these diagrams have zero contribution, since in the fixed point gauge $A_\rho^a(x) = \frac{1}{2} x_\mu G_{\rho\mu}^a(0) + \dots$. The layout of calculations of remaining six diagrams of Fig.1b coincides with 0^{-+}, $0^{++} \to 2\gamma$ cases considered èarlier [3-6] . At the same time the calculations here are technically much more complicated. We have carried out these calculations by means of "REDUCE" program.

The answer for gluon condensate contribution into both amplitudes (2) have been obtained in the form of double integral having the following structure:

$$\Delta_{a,\beta}^G(q^2) \sim \Phi \sum_i \int_0^1 dx \int_0^x dy \, \frac{f_{a,\beta}^i(x,y)(q^2/m_c^2)^{\lambda_i}}{[1-(q^2/m_c^2)(x-y)(1-x)]^{\mu_i}} \qquad (4)$$

where λ_i , μ_i aré integer numbers $\geqslant 0$, $\Phi = \frac{4\pi^2}{9} \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle (4m_c^2)^{-2}$ is dimensionless gluon condensate density,

$f^i_{a,b}$ are some polynomials over x , y . The explicit form of the answer (4) cover more than two pages and we shall not write it down. Using direct transition to numerical calculation inside the "REDUCE" program and doing numerical integration over x , y it is possible to calculate the amplitudes $\Delta^G_{a,b}$ and all their derivatives over q^2 at arbitrary value of q^2.

For completeness we ought to calculate also the perturbative $O(\alpha_s)$ diagram contributions (Fig.1b). However, calculation of these two-loop diagrams is enormously difficult and we postpone it until two-photon width data are improved. In this case "precision" calculations in QCD will be really needed.

As it will be evident from further discussion neglecting $O(\alpha_s)$ correction leads to $\leqslant 10\%$ uncertainty in the final result.

To obtain sum rules we need also physical dispersion representation for correlator (1). The lowest state that contributes into the imaginary part of correlator (1) is $\chi_{c2}(3550)$ resonance.

The resonance contribution into (1) is

$$\Delta^{res}_{\lambda\rho\mu\nu} = \frac{\langle 0|j_{\lambda\rho}|\chi_2\rangle\langle\chi_2|j_\mu j_\nu|0\rangle}{q^2 - m^2_{\chi_2}} \tag{5}$$

where the coupling constant $\langle 0|j_{\lambda\rho}|\chi_2\rangle = g_{\chi_2} m^3_{\chi_2}\chi_{\lambda\rho}$ may be independently extracted from sum rules [9] for two current correlator $\langle j_{\lambda\rho} j_{\lambda'\rho'}\rangle$.

The matrix element $e_{1\mu}e_{2\nu}\langle\chi_2|j_\mu j_\nu|0\rangle$ is given by the same expression (2) where $q^2 = m^2_{\chi_2}$. Instead of $\Delta_{a,b}$ the invariant dimensionless amplitudes of $\chi_{c2}\to 2\gamma$ decay must be written:

$$\Delta_a \to A_a(\chi_2 \to 2\gamma) m_{\chi_2}^{-1} \quad , \quad \Delta_b \to A_b(\chi_2 \to 2\gamma) m_{\chi_2}^{-3}$$

In terms of these amplitudes the width is expressed as follows:

$$\Gamma(\chi_2 \to 2\gamma) = \frac{\pi \alpha^2 Q_c^4}{30} m_{\chi_2} \left\{ 4A_a^2 - \frac{1}{2} A_a A_b + \frac{1}{64} A_b^2 \right\} \tag{6}$$

Equating the physical representation of correlator (1) to its QCD representation

$$\Delta^{res} + \ldots \simeq \Delta^{0} + \Delta^{G} + \ldots$$

we obtain for each amplitude $A_{a,b}$ an approximate relation valid in the region of small distances $-\infty \leqslant q^2 \ll 4m_c^2$. Differentiating both sides of this equation n times over q^2 at some value of q^2 inside this region, we obtain moments of sum rules for physical amplitudes $A_{a,b}$.

The simplest expression for sum rules is at $q^2=0$. At this point all integrals are solved analytically and the final form for $n > 3$ is as follows:

$$A_a(\chi_2 \to 2\gamma) \simeq \frac{12}{\pi^2 g_{\chi_2}} \left(\frac{m_{\chi_2}}{m_c}\right)^{2n} \frac{n!(n-1)!}{(2n+4)!} (2n^2+5n+4) \cdot \left\{1 - \delta_{an} - c_{an}\phi + \ldots\right\} \tag{7}$$

$$A_b(\chi_2 \to 2\gamma) \simeq -\frac{48}{\pi^2 g_{\chi_2}} \left(\frac{m_{\chi_2}}{m_c}\right)^{2n+2} \frac{(n+1)!\,n!}{(2n+6)!} (n+7) \cdot \left\{1 - \delta_{bn} - c_{bn}\phi + \ldots\right\}$$

where coefficients

$$C_{an}=2(n^6+17n^5+120n^4+474n^3+1014n^2$$

$$+1063n+383)(n+1)/(2n^2+5n+4)(2n+5)(n+3) \qquad (8)$$

$$C_{\theta n}=(4n^5+68n^4+428n^3+1317n^2+2127n+$$

$$+1510)(n+1)(n+2)/(2n+9)(2n+7)(n+7)(n+4)$$

are obtained differentiating (5) at $q^2=0$. δ_{an} and $\delta_{\theta n}$ are corrections taking into account the contribution of higher physical 2^{++} -states. The standard way [1] to estimate these corrections is to replace the contribution of higher states by triangle loop dispersion integral (3) over the interval $S_o < S < \infty$. The continuum threshold S_o is not an independent parameter and is naturally determined (as well as the coupling constant g_{χ_2}) from sum rules for two-current correlator $\langle j_{\lambda\rho}\, j_{\lambda\rho}\rangle$. These last sum rules at $q^2=0$ are [1] :

$$g^2_{\chi_2}=\frac{3}{8\pi^2}\left(\frac{m_{\chi_2}}{m_c}\right)^{2n}\frac{(2n+3)(n-1)!\,2^{n+2}}{(2n+5)!!}\cdot \qquad (9)$$

$$\cdot\left\{1-\delta_n-\theta_n\Phi+\cdots\right\}$$

The coefficients θ_n have been calculated in [9] :

$$\theta_n=\frac{n(n+1)[3(n+2)(n+3)(2n+9)-13n-10]}{(2n+3)(2n+7)},$$

δ_n are higher state corrections analogous to δ_{an} , $\delta_{\theta n}$.

To improve the sum rules (7) it is convenient to divide it by the corresponding two-current correlator sum rules, i.e. by (9) (see, e.g. [6]).

In the resulting ratio the dependence on the c-quark current quark mass almost disappears. At the same time there is mutual cancellation of QCD corrections and the set of applicable moments enlarges. Recall that in the case of $\eta_c \to 2\gamma$ two-photon amplitude [3,5] this procedure leads to perturbative α_s -correction $\leqslant 5\%$ in those moments which are used to estimate this amplitude. Note that the leading Coulomb contribution $\sim 2\pi\alpha_s/3v$ (Fig.1b diagram) being universal, is cancelled in the ratio of sum rules (7) and (9).

Actually, the $q^2=0$ point is not very useful for the numerical analysis of sum rules for P-wave currents as it was noticed earlier in [9] . The reason is that the coefficients (8) determining gluon correction at $q^2=0$ increase too fast with n although the asymptotics of these coefficients is usually $\sim n^3$. Similarly to the scalar charmonium case $\chi_{c0} \to 2\gamma$ considered in [6] the optimal point is $q^2 = -4m_c^2$. In this point at $4 \leqslant n \leqslant 8$ both the gluon correction and continuum contribution are small enough. The values of g_{χ_2} and S_0 are fixed from the sum rules (9):

$$|g_{\chi_2}| = 0.118 , \qquad \sqrt{S_0} = 4 \text{ GeV}.$$

Following [2,5] we choose $m_c(q^2=-4m_c^2)=1.25$ GeV and $\phi = (1.7\pm0.3)\cdot 10^{-3}$. The resulting values of amplitudes are

$$\begin{aligned} A_a(\chi_{c2} \to 2\gamma) &= 0.37 \\ A_B(\chi_{c2} \to 2\gamma) &= -0.13 \end{aligned} \qquad (10)$$

According to Eq.(7) these values correspond to the width

$\Gamma(\chi_{c2} \to 2\gamma) = 2.35$ keV ($\pm 10\%$).

In parentheses the estimated uncertainty of the width is given. The obtained results agree with data of Ref. [7,8] (see Table 1) within theoretical uncertainties and experimental errors.

More precise amplitudes may be calculated if total two-loop α_s -corrections as well as next order power terms $\sim \langle G^3 \rangle$ are taken into account. As in the case of QED further complicated calculations are justified if experimental data are substantially improved.

3. $\eta_b \to 2\gamma$ Decay

The lightest b-quarkonium level $\eta_b(0^{-+})$ still has not been observed. $\eta_b \to 2\gamma$ is one of the modes that will allow to identify η_b in future experiments. Is it possible to apply QCD sum rules for this decay?

In b-quarkonium the gluon condensate interactions are negligibly small and perturbative gluon exchanges dominate. At large numbers of moments when lowest $b\bar{b}$-resonance contribution dominates, we are to sum up all higher α_s -corrections. This procedure is possible only in nonrelativistic Coulomb limit [10]. The first order in α_s (Fig.1b diagrams) is enough only at $n \leqslant 10$ where lowest resonance dominance is almost absent.

At the same time we have experimental information about five Υ -resonances located higher than the lowest one - $\Upsilon(9460)$. Using these data, a few problems have been solved with first ten moments of sum rules in hand. For example, in [9] the mass difference $m_{\Upsilon} - m_{\eta_b} = 60$ MeV has been predicted. In [11] the widths $\Upsilon \to a\gamma$, $\Upsilon \to H\gamma$ have been estimated, where $a(H)$ is the light pseudoscalar (scalar) particle of axion (Higgs boson) type. In the same way we shall evaluate the $\eta_b \to 2\gamma$ width.

First, it is natural to assume that the spectrum of 0^{-+} excited states in b-quarkonium is similar to 1^{--} -spectrum. In other words, every Υ' -resonance has its η_b' -partner so that

$$\frac{m_{\eta_b'}}{m_{\eta_b}} \simeq \frac{m_{\Upsilon'}}{m_{\Upsilon}} , \qquad \frac{g_{\eta_b'}}{g_{\eta_b}} \simeq \frac{g_{\Upsilon'}}{g_{\Upsilon}} \quad (11)$$

where $g_{\eta_b}(g_{\Upsilon})$ is the coupling constant $\langle 0 | j_5 | \eta_b \rangle$ $(\langle 0 | j_\mu | \Upsilon \rangle)$, $j_5 = i\bar{b}\gamma_5 b$, $j_\mu = \bar{b}\gamma_\mu b$.

Sum rules for the amplitude $\eta_b \to 2\gamma$ coincide with those considered in [3-5] for $\eta_c \to 2\gamma$ decay. Their derivation is analogous to the case considered above and is based on the same three-current correlator (Fig.1a,b diagrams) where the q vertex now has $J^{PC} = 0^{-+}$:

$$\Delta_{\mu\nu 5}(K_1, K_2) \equiv \varepsilon_{\mu\nu\alpha\beta} K_{1\alpha} K_{2\beta} \Delta_5(q^2) =$$

$$\int dx\, dy \exp[-i(K_1 x + K_2 y)] \langle 0 | T\{ j_5(0) j_\mu(x) j_\nu(y) \} | 0 \rangle \quad (12)$$

The imaginary part of Δ_5 in order α_s :

$$\Delta_5(q^2) = \frac{1}{\pi} \int \frac{\mathrm{Im}\,\Delta_5(s)}{s - q^2} ds$$

$$\mathrm{Im}\,\Delta_5(s) = \frac{3 m_b^*}{2\pi s} \ln \frac{1+v}{1-v} \left[1 + \alpha_s \left(\frac{2\pi}{3v} + \beta + \cdots \right) \right] \quad (13$$

where m_b^* is b-quark mass normalized "on mass shell":

$$p^2 = +m_g^2\,,\quad m_g^* \simeq m_g(p^2=-m_g^2)/(1-2\alpha_s \ln 2/\pi)$$ [1]

The dots in the r.h.s. of (13) denote terms $\sim v, v^2, \ldots$ that have been neglected, since their contribution into the moments at high n are negligible due to the weight in dispersion integrals $\sim ds/s^{n+1}$ (or $v\,dv(1-v^2)^{n-1}$). We have fitted the coefficient β using the results [12] of α_s -correction calculation for the triangle amplitude corresponding to $J/\Psi \to \eta_c \gamma$ decay. In the particular case of zero mass in vector vertex this amplitude coincides with (12). The obtained value $\beta = -0.89$ (see [11] where the fitting procedure is described in detail).

Differentiating the amplitude (13) over q^2 at $q^2=0$ we obtain the moments of sum rules:

$$g_{\eta_g} A(\eta_g\to 2\gamma)\Big[1+\sum_{\eta_g'}\Big(\frac{g_{\eta_g'}}{g_{\eta_g}}\Big)\frac{A(\eta_g'\to 2\gamma)}{A(\eta_g\to 2\gamma)}\Big(\frac{m_{\eta_g}}{m_{\eta_g'}}\Big)^{2n+1}\Big] = \tag{14}$$

$$= \frac{3(n!)^2}{2\pi^2(2n+2)!}\Big(\frac{m_{\eta_g}}{m_g}\Big)^{2n+1}\Big[1-\delta_{5n}+\frac{\alpha_s}{\pi}d_n\Big]$$

where α_s -corrections both from (13) and from b-quark mass normalization are included into d_n . δ_{5n} is the contribution of continuum located higher than η_g' -resonances.

The amplitude

$$A(\eta_g\to 2\gamma)\,\varepsilon_{\mu\nu\alpha\beta}\,k_{1\alpha}k_{2\beta}\,m_{\eta_g}^{-1} = \langle 0|j_\mu j_\nu|\eta_g\rangle$$

determines the width

$$\Gamma(\eta_g\to 2\gamma) = \frac{\pi\alpha^2 Q_g^2}{4}\, m_{\eta_g} A(\eta_g\to 2\gamma)$$

Following [11] we also assume that

$$\frac{A(\eta_b' \to 2\gamma)}{A(\eta_b \to 2\gamma)} = \frac{g_{\eta_b'}}{g_{\eta_b}} \tag{15}$$

This equation is valid in arbitrary nonrelativistic model of b-quarkonium, since both sides of it are proportional to the same ratio of S-wave functions in the origin $R_S'(0)/R_S(0)$ At the same time it is evident that gluon or relativistic effects cannot influence (15) substantially.

Dividing moments (14) by the corresponding moments of $\langle j_5 j_5 \rangle$ two-current correlator sum rules [2,9] :

$$g_{\eta_b}^2 \left[1 + \sum_{\eta_b'} \left(\frac{g_{\eta_b'}}{g_{\eta_b}} \right)^2 \left(\frac{m_{\eta_b}}{m_{\eta_b'}} \right)^{2n} \right] = \frac{3}{8\pi^2} \left(\frac{m_{\eta_b}}{2m_b} \right)^{2n} \frac{2^n (n-1)!}{(2n+1)!!} \left[1 - \tilde{\delta}_{5n} + a_n \frac{\alpha_s}{\pi} \right] \tag{16}$$

we obtain taking into account (11) and (15)

$$\frac{A(\eta_b \to 2\gamma)}{g_{\eta_b}} = 4 \left(\frac{m_{\eta_b}}{2m_b} \right) \frac{n}{n+1} R^n \cdot \left[1 - (\delta_{5n} - \tilde{\delta}_{5n}) + (d_n - a_n) \frac{\alpha_s}{\pi} \right] \tag{17}$$

where the factor

$$R^n = \left[1 + \sum_{\Upsilon'} \left(\frac{g_{\Upsilon'}}{g_{\Upsilon}} \right)^2 \left(\frac{m_\Upsilon}{m_{\Upsilon'}} \right)^{2n} \right] \Big/ \left[1 + \sum_{\Upsilon'} \left(\frac{g_{\Upsilon'}}{g_{\Upsilon}} \right)^2 \left(\frac{m_\Upsilon}{m_{\Upsilon'}} \right)^{2n+1} \right]$$

is close to unity and is calculated by means of experimental data [13] .

All parameters entering the r.h.s. of (17) are known from the analysis of sum rules for the coupling constant g_Υ^2 [5,14]. In particular, at $\alpha_s(m_b) = 0.15$, $m_b = 4.21$ GeV, $\sqrt{s_0} = 11.3$ GeV these last sum rules reproduce very stable value of

$$\Gamma(\Upsilon \to \mu^+\mu^-) = \frac{4\pi\alpha^2 Q_b^2}{3} g_\Upsilon^2 m_\Upsilon = 1.15 - 1.2 \text{ keV}$$

in the whole interval $2 \leqslant n \leqslant 9$.

The g_{η_b} constant is fixed by the ratio of sum rules for two two-current correlators. The resonance factor drops out from this ratio:

$$\frac{g_{\eta_b}^2}{g_\Upsilon^2} = \frac{n+3/2}{n+1}\left[1-(\tilde{\delta}_{5n}-\tilde{\delta}_{5n}^{(\Upsilon)})+(a_n-a_n^{(\Upsilon)})\frac{\alpha_s}{\pi}\right] \qquad (18)$$

The numerical analysis of (17) and (18) gives in the whole interval $3 \leqslant n \leqslant 9$:

$$\frac{A(\eta_b \to 2\gamma)}{g_{\eta_b}} = 3.81(\pm 0.01), \qquad \frac{g_{\eta_b}^2}{g_\Upsilon^2} = 1.11(\pm 0.01)$$

For these moments both the higher state contribution and α_s - corrections are less than 10% (due to cancellation of order of 20 ÷ 30 % contributions from numerators and denominators).

Excluding g_{η_b} by means of (18) we finally obtain

$$\Gamma(\eta_b \to 2\gamma) = (1.01 \pm 0.1) 3Q_b^2 \Gamma(\Upsilon \to \mu^+\mu^-) \simeq 0.4 \text{ keV} \qquad (19)$$

As it was expected, the ratio $\Gamma(\eta_b \to 2\gamma)/\Gamma(\Upsilon \to \mu^+\mu^-)$ is very close to its nonrelativistic limit $3Q_b^2$ although it was obtained in an independent way. We underline that QCD sum rules reproduce both widths entering (19) separately.

4. Conclusion

The obtained results allow to discuss the status of two photon widths as well as all related quarkonium annihilation widths in nonrelativistic approach.

The most general predictions concern the ratios of widths determined by the ratios of wave functions in the origin. Thus in arbitrary nonrelativistic model

$$\frac{\Gamma(\chi_{c0} \to 2\gamma)}{\Gamma(\chi_{c2} \to 2\gamma)} = \frac{15}{4} \tag{20}$$

(see, e.g. [1]). More or less definite are α_s-corrections to these ratios. In particular, the correction factor to the r.h.s. of (20) is equal [15] to $(1 + 5.5 \frac{\alpha_s}{\pi}) \simeq 1.35$ at $\alpha_s(m_c) = 0.2$. Relativistic corrections are model-dependent (see, e.g. [16] , where corrections to the ratio $\Gamma(J/\psi \to H\gamma)/\Gamma(J/\psi \to \mu^+\mu^-)$ turn out to be essential). The question open is whether there is interference between relativistic and α_s -corrections.

Returning to the ratio (20) we see that it substantially differs from QCD prediction 3.0 ± 0.4 keV / 2.35 ± 0.2 keV. Possibly, (20) already contradicts experiment if we take seriously the only measurement [20] of $\chi_0 \to 2\gamma$ width (see Table 1). Note that α_s -correction increases this deviation!

In the case of b-quarkonium the situation with QCD sum rules is closer to the nonrelativistic limit as demonstrated above. α_s -correction factor is small here: $3Q_b^2$ is to be multiplied by $1 + 1.96 \frac{\alpha_s}{\pi} \simeq 1.09$ at $\alpha_s(m_b) = 0.15$.

Even larger is the discrepancy between QCD sum rules and nonrelativistic potential in the case of two photon widths taken separately. For example, phenomenological potentials that nicely reproduce both ψ and Υ levels give $\Gamma(\chi_2 \to 2\gamma)$ in the interval $0.5 \div 1.0$ keV (see, e.g. [17]).

The question of what is the α_s-correction to the individual annihilation width remains unresolved in nonrelativistic approach for both $c\bar{c}$ and $b\bar{b}$-states.

At the same time we have been convinced that charmonium two-photon widths may be safely calculated from QCD sum rules with controllable accuracy in α_s and in purely relativistic invariant way. For b-quarkonium this method is applicable if experimental information on spectrum of levels is available.

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Table 1. Charmonium two-photon widths

Decay mode	Prediction of QCD sum rules (keV)	Experiment (keV)
$\eta_c \to 2\gamma$	4.6 ± 0.4 [3]	$4.5^{+5.5}_{-3.6}$ [19]
		$4.3^{+3.4}_{-3.7} \pm 2.4$ [7]
		6^{+6}_{-5} [13]
$\chi_{c0} \to 2\gamma$	3.7 ± 0.5 [5]	< 20 [13]
	3.0 ± 0.4 [6]	$4 \pm 2 \pm 2$ [7]
$\chi_{c2} \to 2\gamma$	2.35 ± 0.2	$2.9^{+1.3}_{-1.0} \pm 1.7$ [8]
		2.8 ± 2.0 [7]
		2.86 ± 1.6 [13]

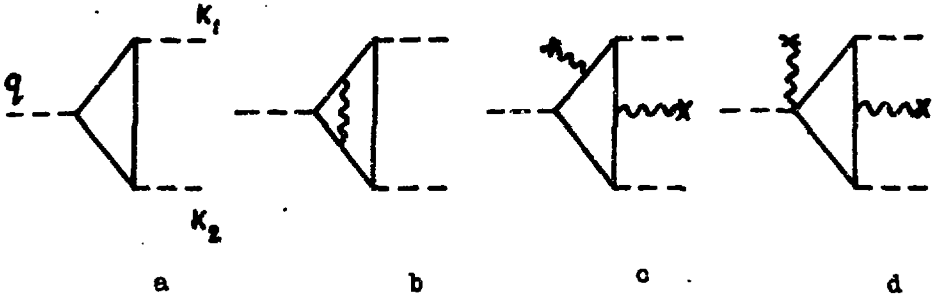

Fig. 1

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Figure Captions

Fig.1. a) Diagram corresponding to three-current correlator in zero order in α_s ; b) one of the diagrams of $O(\alpha_s)$ perturbative corrections; c,d) examples of diagrams corresponding to the interaction with gluon condensate.

Л.С.ДУЛЬЯН, А.Д.МАГАКЯН, А.Ю.ХОДЖАМИРЯН

ДВУХФОТОННЫЕ РАСПАДЫ КВАРКОНИЯ В КХД

(на английском языке, перевод З.Н.Асланян)

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The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR

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