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Al.R. KAVALOV, R.L. MKRTCHYAN

THE LATTICE CONSTRUCTION FOR ABELIAN CHERN-SIMONS GAUGE THEORY

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ԱԼ.Ռ.ԿԱՎԱԼՈՎ, Ռ.Լ.ՄԿՐՏՉՅԱՆ

ՑԱՆՑԱՅԻՆ ԿԱՌՈՒՑՎԱԾՔ՝ ՉԵՐՆ-ՍԱՅՄՈՆԻ ԱԲԵԼՅԱՆ ՏԵՍՈՒԹՅԱՆ ՀԱՄԱՐ

Կառուցված է աբելյան տրամաչափա-ինվարիանտ ցանցի տեսություն, որը անընդհատության սահմանում U(1) խմբի դեպքում՝ ձգտում է Չերն-Սայմոնի աբելյան տրամաչափային տեսությանը:

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Ал.Р.КАВАЛОВ, Р.Л.МКРТЧЯН

РЕШЕТОЧНАЯ КОНСТРУКЦИЯ ДЛЯ АБЕЛЕВОЙ ТЕОРИИ ЧЕРНА-САЙМОНА

Построена калибровочная абелева решеточная теория, переходящая в непрерывном пределе и в случае калибровочной группы $U(1)$ в топологическую калибровочную теорию Черна-Саймона.

Ереванский физический институт
Ереван 1990

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Al.R. KAVALOV, R.L. MKRTCHYAN

THE LATTICE CONSTRUCTION FOR ABELIAN CHERN-SIMONS GAUGE THEORY

The Abelian gauge-invariant lattice theory is constructed, which in the continuum limit and in the case of U(1) gauge group tends to the Chern-Simons topological gauge theory.

Yerevan Physics Institute
Yerevan 1990

1.The aim of the present paper is to describe a simple lattice version of the three-dimensional topological Abelian gauge theory with the action given entirely by the Chern-Simons term. This last theory was first considered by A.Schwarz [1] who has shown that the partition function is determined in terms of the Ray-Singer torsion of the corresponding "space-time" manifold. It has also been studied more recently by A.Polyakov [2] in connection with the Fermi-Bose transmutation and has been brought to the prominence by E.Witten who has shown, that non-Abelian Chern-Simons (CS) theory provides a natural three-dimensional framework for a knot theory [3], a new description of three-dimensional gravity [4], and, most intriguing, is closely related to the two-dimensional conformal theories [3]. These points have been further studied in a number of works.

There are many motivations for the lattice approach to the Chern-Simons theory. The first is the observation that the big part of the relevant information about the topology of the manifold is maintained when replacing the manifold by the corresponding simplicial or cell complex. Thus, one may hope to get the simpler description of the results of the continuous topological theory. The second, as all other lattice theories, our one provides a natural regularization of the continuous theory. The completely new point is that the lattice approach permits one to define a topological CS gauge theory for discrete groups, e.g. Z_n. Finally, the lattice analog of topologically massive (Abelian) gauge theories of Ref. [5] is now possible.

The first construction of the lattice topological theory was given by J.F.Wheater in the work [6] where he considered a simple two-dimensional topological Ising (i.e. with variables taking values ±1) theory on some triangulated manifold. The important point of the approach of Ref. [6] was that the weight exp(-action) was constructed to be independent of the triangulation - recall that in two dimensions summing over all possible triangulations of the manifolds is equivalent to integrating over all possible metrics in the continuum [7-9]. Our construction differs from that of Ref. [6].

2. Consider the simplicial complex corresponding to some closed orientable triangulated manifold M^3 (see any textbook in homology theory e.g. [10]). The standard and useful notation for the n-simplex is by writing down its vertexes in some order. The order of vertexes corresponds to the orientation of the simplex, the two simplexes with the order of vertexes connected by the even (odd) permutation having the same (opposite) orientation. The boundary operator ∂ : (n-chains) → ((n-1)-chains) acts on the simplexes in the following way

$$\partial\,[\alpha_0\alpha_1\alpha_2\ldots\alpha_n] = [\alpha_1\alpha_2\ldots\alpha_n] - [\alpha_0\alpha_2\ldots\alpha_n] + \ldots \qquad (1)$$

$$(-1)^n[\alpha_0\alpha_1\ldots\alpha_{n-1}]$$

and has the property $\partial^2 = 0$. Evidently, the orientation of the simplex induces the orientations of the components of its boundary. The orientations of the two neighbouring n-simplexes are said to be equal (opposite) if they induce the opposite (equal) orientation on their common (n-1)-simplexes. For the orientable manifolds M^n it is possible to choose the same orientation for all n-simplexes. In the present work we will always make such a choice.

The gauge field A is a 1-cochain, i.e. a linear map of 1-simplexes to the ring of coefficients. We take it to be antisymmetric:

$$A([\alpha_0\alpha_1]) = -A([\alpha_1\alpha_0]). \tag{2}$$

The coboundary operator δ is defined to be conjugated to ∂, i.e. for arbitrary n-cochain F

$$\delta F([\alpha_0 \ldots \alpha_{n+1}]) = F(\partial [\alpha_0 \ldots \alpha_{n+1}]). \tag{3}$$

Evidently, $\delta^2 = 0$ as follows from $\partial^2 = 0$.
A gauge transformation of gauge field A is defined to be

$$A \to A + \delta\Phi \tag{4}$$

for some 0-cochain Φ. Note that 0-cochains are defined on 0-simplexes i.e. vertexes and eq. (4) means

$$A([\alpha_0\alpha_1]) \to A([\alpha_0\alpha_1]) + \Phi([\alpha_1]) - \Phi([\alpha_0]) \tag{5}$$

As another example consider the 2-cochain δA :

$$\delta A([\alpha_0\alpha_1\alpha_2]) = A([\alpha_0\alpha_1]) + A([\alpha_1\alpha_2]) + A([\alpha_2\alpha_0]) \tag{6}$$

which is the (Abelian) lattice analog for the field strength.

Let's define now the so-called Kolmogorov-Alexander product, also called the cup-product ($\cup$-product) in the space of cochains. Given the p-cochain P and s-cochain S one can form a p+s cochain $P \cup S$ by the formula

$$P \cup S([\alpha_0 \ldots \alpha_{p+s}]) = P([\alpha_0 \ldots \alpha_p]) S([\alpha_p \ldots \alpha_{p+s}]). \tag{7}$$

The important properties of this product are the associativity and the graded Leibnitz rule with respect to δ:

$$P \cup (Q \cup S) = (P \cup Q) \cup S \tag{8}$$

$$\delta(P \cup S) = \delta P \cup S + (-1)^p P \cup \delta S \tag{9}$$

3. We turn now to the construction of the action of our theory. The action will be a sum of the values of some 3-cochain L over the 3-simplexes of the manifold, which we denote $\sum L$. Exactly, we define a value of 3-cochain L on one 3-simplex $[\alpha_0 \ldots \alpha_3]$ by taking a value of L on the simplex $[\alpha_i \alpha_j \alpha_k \alpha_l]$ where i,j,k,l is the permutation of 0,1,2,3 and summing over this permutations with the sign +,- for even and odd permutations, respectively. To obtain the action one has to take the sum of just defined values of L on all tetrahedrons of triangulation.

The following property holds for M^3 without boundary:

$$\sum \delta P = 0 \tag{10}$$

for arbitrary 2-cochain P.

Now the action for the gauge field (1-cochain) A is

$$S = k \sum A \cup \delta A \tag{11}$$

It is invariant with respect to the gauge transformation (4) due to the properties (10) and $\delta^2 = 0$. It's easy to see that the sum over permutations encoded in $\sum$ produces the following value for the action on the tetrahedron with vertexes $\alpha_0, \ldots, \alpha_3$

$$4k \left(A([\alpha_0\alpha_1])A([\alpha_2\alpha_3]) + A([\alpha_2\alpha_0])A([\alpha_1\alpha_3]) + \right. \\ \left. + A([\alpha_0\alpha_3])A([\alpha_1\alpha_2]) \right) \tag{12}$$

The geometrical picture is simple - action is the sum of products of pairs of non-intersecting links. 4. The formulae (11),(4) constitute the main result of our paper. They are the simplicial cohomology analog of the corresponding expressions introduced by E.Witten in context of the string field theory [11]. The final part of the construction is the proof of the statement that in the continuum limit when the size of the simplexes tends to zero the formulae (11), (4) tend to corresponding ones in topological U(1) Chern-Simons theory.

Take the 3-simplex $[\alpha_0 \ldots \alpha_3]$ and choose an arbitrary point x in its interior. Denote by e_i^{μ} the vectors from x to the α_i.

In the continuum limit the gauge field may be expanded near the point x :

$$A([\alpha_i \alpha_j]) = (e_j^{\nu} - e_i^{\nu})\,(A_{\nu} + (1/2)(e_i^{\mu} + e_j^{\mu})\,\partial_{\mu}A_{\nu} + \ldots) \qquad (13)$$

Substituting (13) into (11) we find, up to irrelevant in continuum limit terms of forth order over link length, that the action (12) gives

$$12k\; A_{\alpha}\partial_{\beta}A_{\gamma}\; \varepsilon^{\alpha\beta\gamma}\, v \qquad (14)$$

where $v = (1/6)(e_0^{\alpha}e_1^{\beta}e_2^{\gamma} - e_0^{\alpha}e_1^{\beta}e_3^{\gamma} + e_0^{\alpha}e_2^{\beta}e_3^{\gamma} - e_1^{\alpha}e_2^{\beta}e_3^{\gamma})\varepsilon_{\alpha\beta\gamma}$ is the oriented volume of 3-simplex.

5. There exist different ways of development of the present results. On may generalize them to a lattice construction of BF systems [12-14]. The correlators of Wilson loops may be calculated and the connection with linking number may be shown, and also the connection with two-dimensional lattice systems may be established in the case of M^3 with boundary; corresponding results will be published elsewhere. The important problem is the non-Abelian generalization of action (11). It seems, at first sight, that it may be obtained along the lines of general approach of Ref.[11], but actually the cup-product (generalized to include matrix product) does not posses the property of being (anti)commutative under the symbol of "integration" $\sum$ – the property, widely used in [11].

References

1. A.S.Schwarz The partition function of degenerate quadratic functional and Ray-Singer invariants. - Lett.Math.Phys. 2 (1978) 247, Comm.Math.Phys. 67 (1979) 1.
2. A.M.Polyakov Fermi-Bose transmutations induced by gauge fields. - Mod.Phys.Lett. A3 (1988) 325.
3. E.Witten Quantum field theory and the Jones polynomials. - IAS preprint HEP-88/33.
4. E.Witten 2+1 dimensional gravity as an exactly soluble system. - Nucl.Phys. B311 (1988/89) 46-78.
5. S.Deser, R.Jackiw and S.Templeton Phys.Rev.Lett. 48 (1983) 975; Ann.Phys.NY 140 (1984) 372.
6. J.F.Wheater A topological lattice model. - University of Oxford preprint OUTP-89-08P, February 1989.
7. V.A.Kazakov, I.K.Kostov and A.A.Migdal Phys.Lett. 157B (1985) 295.
8. V.A.Kazakov and A.A.Migdal Nucl.Phys. B311 (1988/89) 171.
9. V.G.Knizhnik, A.M.Polyakov and A.B.Zamolodchikov Mod.Phys.Lett. A3 (1989) 819.
10. A.Dold Lectures on algebraic geometry. - Springer-Verlag 1972.
11. E.Witten Non-commutative geometry and string field theory. Nucl.Phys. B268 (1986) 253.
12. G.T.Horowitz Exactly soluble diffeomorphysm invariant theories Santa-Barbara preprint ITP-NSF-88-178.
13. M.Blau and G.Thompson Topological gauge theories of antisymmetric tensor fields. - PAR LPTHE 89-18, S.I.S.S.A. 44/89/FM Paris.
14. M.Blau and G.Thompson A new class of topological field theories and the Ray-Singer torsion. - Phys.Lett. B228 (1989) 64.

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Ал.Р.КАВАЛОВ, Р.Л.МКРТЧЯН

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(на английском языке, перевод авторов)
Редактор Л.П.Мукаян
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The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR

ИНДЕКС 3649

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ