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YEREVAN PHYSICS INSTITUTE

Ed.Sh. EGORIAN, R.P. MANVELIAN

CANONICAL FORMULATION OF 2D INDUCED GRAVITY

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ՄԱԿԱԾՎԱԾ ԵՐԿՉԱՓ ԳՐԱՎԻՏԱՑԻԱՅԻ ԿԱՆՈՆԱԿԱՆ ՁԵՎԱԿԵՐՊՈՒՄԸ

Ներքոհիշյալ աշխատանքում, կապերով համակարգերի տեսության տեսակետից, ուսումնասիրվում է SL(2,R) համաչափության ակունքը՝ մակածված երկչափ գրավիտացիայում:

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Эд.Ш. ЕГОРЯН, Р.П. МАНВЕЛЯН

КАНОНИЧЕСКАЯ ФОРМУЛИРОВКА 2D ИНДУЦИРОВАННОЙ ГРАВИТАЦИИ

В работе с точки зрения теории систем со связями изучается происхождение SL(2,R) симметрии в двухмерной индуцированной гравитации.

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CANONICAL FORMULATION OF 2D INDUCED GRAVITY

The origin of SL(2,R) symmetry in the 2D induced gravity from the viewpoint of the theory of systems with constraints is investigated in this paper.

Yerevan Physics Institute

Yerevan 1990

1. Introduction

A canonical formulation of a two-dimensional induced gravity [1,3] on a classical level is investigated.

The origin of SL(2,R) symmetry in this model is considered from the viewpoint of the theory of systems with constraints.

The symmetry of theory is the two-dimensional diffeomorphism. The gauge fixation is produced by fixation of Lagrangian factors of initial constraints, which realize the diffeomorphism group. At first glance no residue system is left.

In section 3, a canonical formalism for the gauge fixed Lagrangian with different time interpretation (the x^+ coordinate is taken as time) is constructed. Other constraints arise in this formalism and for the resolution of second-class ones, the SL(2,R) symmetry emerges.

2. Canonical Formulation and Gauge Fixation

Let us consider the action for the induced 2D gravity [1,3]

$$S = -K \int \sqrt{-g}\, R \Box^{-1} R d\tau d\sigma \qquad (1)$$

and remove non-locality by introducing the auxiliary field $\varphi(\tau,\sigma)$:

$$S = -\frac{1}{2}\int\sqrt{-g}\;[g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi+\alpha R\varphi]\,d\tau d\sigma\ . \qquad (2)$$

Note, that in the action (2) the second derivatives of metric enter $\sqrt{-g}R$ only through full derivatives, hence (2) can be rewritten in components as:

$$S = \int L\ d\tau d\sigma$$

$$L=\frac{1}{2\sqrt{-g}}\,[2g_{01}\dot{\varphi}\varphi'-g_{11}\dot{\varphi}^2-g_{00}\varphi'^2+\alpha(\dot{g}_{11}\dot{\varphi}+g_{00}'\varphi'-2g_{01}'\dot{\varphi})+ \qquad (3)$$

$$+\frac{\alpha g_{01}}{g_{11}}\,[g_{11}'\dot{\varphi}-\dot{g}_{11}\varphi']\ .$$

The expressions for the momenta follow directly from (3):

$$P_\varphi=\frac{1}{\sqrt{-g}}\,(g_{01}\varphi'-g_{11}\dot{\varphi})+\frac{2}{2\sqrt{-g}}\,\dot{g}_{11}-\frac{2g_{01}'}{\sqrt{-g}}+\frac{\alpha g_{01}}{g_{11}\sqrt{-g}}\,g_{11}'$$

$$P^{11}=\frac{\alpha\dot{\varphi}}{2\sqrt{-g}}-\frac{\alpha g_{01}\varphi'}{2g_{11}\sqrt{-g}},\quad P^{0\alpha}=0\ , \qquad (4)$$

From the first two expressions, $\dot{\varphi}$, $\dot{g}$, can be expresses through the coordinate and momentum. The last two equations do not allow us to do so for $\dot{g}_{0\alpha}$, consequently, there are primary constraints.

Let us calculate the initial Hamiltonian

$$H_0=\int(P_\varphi\dot{\varphi}+P^{\alpha\beta}g_{\alpha\beta}'-L)d\sigma=\int\{-\frac{\sqrt{-g}}{g_{11}}\,\Phi_1+\frac{g_{01}}{g_{11}}\,\Phi_2\}d\sigma\ , \qquad (5)$$

where

$$\Phi_1=\frac{1}{2}\,[\varphi'^2-\frac{4}{\alpha^2}\,(P^{11}g_{11})^2-\frac{4}{\alpha}(P^{11}g_{11})P_\varphi-\frac{\alpha g_{11}'\varphi'}{g_{11}}+2\alpha\varphi'']$$

$$\Phi_2=P_\varphi\varphi'-2P^{11\prime}g_{11}-P^{11}g_{11}' \qquad (6)$$

When one follows the Dirac procedure [4] and checks the conditions of conservation of the primary constraints $P_{0\alpha}$. Then, $\Phi_1(\tau,\sigma)$ and $\Phi_2(\tau,\sigma)$ (see Eq.(6)) turn into secondary constraints, which realize the diffeomorphism group

$$\begin{aligned}
\{\Phi_1(\sigma),\ \Phi_1(\sigma')\} &= \delta'(\sigma-\sigma')[\Phi_2(\sigma)+\Phi_2(\sigma')], \\
\{\Phi_1(\sigma),\ \Phi_2(\sigma')\} &= \delta'(\sigma-\sigma')[\Phi_1(\sigma)+\Phi_1(\sigma')], \qquad (7) \\
\{\Phi_2(\sigma),\ \Phi_2(\sigma')\} &= \delta'(\sigma-\sigma')[\Phi_2(\sigma)+\Phi_2(\sigma')].
\end{aligned}$$

Moreover, since the expressions for Φ_1 and Φ_2 do not include the components $(g_{0\alpha})$ and their momenta, it is obvious that these two metric components can be gauged out (by means of the generators $P^{0\alpha}$) from the theory. In result there arises a theory with an action in the first-order formalism

$$S = \int d\tau d\sigma \{P_\varphi \dot{\varphi} + \mathcal{P}\dot{q} - \lambda_1\Phi_1 - \lambda_2\Phi_2\}, \qquad (8)$$

where $q=g_{11}$, $\mathcal{P}=p^{11}$. The gauge fields λ^i, $i=1,2$, are transformed over the conjugate representation of algebra (7):

$$\delta\lambda^i = \dot{\varepsilon}^i + \varepsilon^k f^i_{ke}\lambda^e,$$

where f^i_{ke} are defined from (7).

The following gauge constraints impose on λ^i:

$$\lambda_1 = \frac{1}{q}, \qquad \lambda_2 = 1 - \frac{1}{q}. \qquad (9)$$

After integration over the momenta, the action (8) in this gauge takes the form:

$$S = \int d^2x\ [\partial_+\varphi\partial_-\varphi + q(\partial_-\varphi)^2 - \alpha\partial_-\varphi\partial_- q], \qquad (10)$$

where

$$x^{\pm} = \frac{\tau\pm\sigma}{\sqrt{2}}, \qquad \partial_{\pm} = \frac{\partial_\tau \pm \partial_\sigma}{\sqrt{2}}$$

and a shift, $q\to q+1$, is performed.

It is clear from (10) in the gauge (9), which corresponds to the light-cone gauge, there are already no constraints. Really, defining the momenta from (10) again,

$$P_{\varphi} = \frac{\delta\mathcal{L}}{\delta\dot{\varphi}} = \dot{\varphi} + q(\dot{\varphi}-\varphi') - \frac{\alpha}{2}(\dot{q}-q'),$$

$$P_{q} = \frac{\delta\mathcal{L}}{\delta\dot{q}} = -\frac{\alpha}{2}(\dot{\varphi}-\varphi') \qquad (11)$$

It is obvious that all the velocities are defined through coordinate and momentum functions, consequently, there are no constraints in the theory and the origin of the symmetry SL(2,R) is not clear, which is found in Refs.[1-3].

3. Hidden SL(2,R) Symmetry

Now let us change the time interpretation, namely let us try to develop for (10) a canonical formalism taking the variable x^{+} as the time:

$$P_{\varphi} = \frac{\delta\mathcal{L}}{\delta\partial_{+}\varphi} = \partial_{-}\varphi$$

$$P_{q} = \frac{\delta\mathcal{L}}{\delta\partial_{+}q} = 0 \qquad (12)$$

$$H_0 = \int[P_{\varphi}\partial_{+}\varphi + P_{q}\partial_{+}\varphi - \mathcal{L}]dx = \int[\alpha\partial_{-}q\partial_{-}\varphi - q(\partial_{-}\varphi)^2]dx^{-}$$

It is seen from (12) that the first two equations are primary constraints Then perform the standard procedure, construct $H^{(1)}$:

$$H^{(1)} = H_0 + \int\lambda_0 P_q dx^{-} + \int\lambda_1(P_{\varphi}-\partial_{-}\varphi)dx^{-} \qquad (13)$$

From conservation of the constraint $P_q=0$ in time

$$\{P_q, H^{(1)}\} \approx 0 \qquad (14)$$

we obtain the secondary constraint

$$\Phi^{(2)} = (\partial_-\varphi)^2 + \alpha\partial_-^2 \varphi \approx 0 \quad (15)$$

and from the condition

$$\{P_\varphi - \partial_- \varphi, \quad H^{(1)}\} \approx 0$$

λ_1 is defined

$$\lambda_1 = \frac{1}{2}\alpha\partial_- q - q\partial_-\varphi + f(x^+), \quad (16)$$

where $f(x^+)$ is an arbitrary function of the time x^+. Introducing (17) into (14) we obtain:

$$H^{(1)} = \int\{\frac{\alpha}{2}\partial_- q[P_\varphi+\partial_-\varphi]-q P_\varphi\partial_-\varphi+\lambda_0 P_q+f(x^+)P_\varphi\}dx^-$$

Then, from the condition $\{\Phi_1^{(2)}, H^{(1)}\}$ we obtain the tertiary constraint

$$\partial_-^3 q = 0 \quad (17)$$

and from the condition of its conservation, $\{\partial_-^3 q, \quad H^{(1)}\}\approx 0$, we obtain the constraint on λ_0:

$$\partial_-^3 \lambda_0 = 0 \quad (18)$$

i.e.,

$$\lambda_0=\lambda_0^+(x^+)+2x^-\lambda_0^0(x^+)+(x^-)^2 \lambda_0^-(x^+) \quad (19)$$

Now let us resolve the constraint (17)

$$q(x^+,x^-)=I^+(x^+)+2x^- I^0(x^+)+(x^-)^2 I^-(x^+) \quad (20)$$

and substitute into (13), then we obtain:

$$S = \int dx^+dx^- P_\varphi\partial_+\varphi + \int dx^+[P^-\partial_+ I^+ + P^0\partial_+ I^0 + P^+\partial_+ I^-$$
$$-\lambda_0^+P^- - \lambda_0^0 P^0 - \lambda_0^- P^+] - \int f(x^+)P_\varphi(x^+,x^-)dx^+dx^- \quad (21)$$
$$-\int dx^+ [I^+(x^+)\Phi^-(x^+) + 2I^0(x^+)\Phi^0(x^+) + I^-(x^+)\Phi^+(x^+)],$$

where

$$P^- = \int P_q dx^- , \quad P^0 = \int 2x^- P_q dx^- , \quad P^+ = \int (x^-)^2 P_q dx^-$$

$$\Phi^+ = \int [(x^-)^2 P_\varphi \partial_- \varphi - \alpha x^- P_\varphi + \alpha\varphi] dx^-$$

$$\Phi^0 = \frac{1}{2} \int (2x^- P_\varphi \partial_- \varphi - \alpha P_\varphi) dx^- \qquad (22)$$

$$\Phi^- = \int P_\varphi \partial_- \varphi dx^- .$$

The momenta of I^a are zero from the condition of the variation of the action (21) over λ^a. Then I^a are Lagrangian multiples, and there arises the following effective theory:

$$S = \int dx^+ dx^- (P_\varphi \partial_+ \varphi - f(x^+) P_\varphi) - \int dx^+ [I^+ \Phi^- + 2I^0 \Phi^0 + I^- \Phi^+] , \qquad (23)$$

where Φ^a, a=+ −0 are constraints forming SL(2,R) algebra

$$\{\Phi^a(x^+), \; \Phi^b(x^+)\} = f^{abc} \, \Phi^c(x^+)$$

on the constraint $\int P_\varphi dx^- \simeq 0$. f^{abc} are the structure constants of SL(2,R) algebra. I^a are transformed as $\delta I^a = \partial_+ \varepsilon^a + f^{abc} \varepsilon^b I^c$, i.e. over the conjugate representation of SL(2,R).

In conclusion note that SL(2,R) symmetry arises on the classical level only when x^+ is taken as the time. In this sense it is not a usual symmetry and has a dynamical origin.

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Эд. Ш. ЕГОРЯН, Р. П. МАНВЕЛЯН

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The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR

ИНДЕКС

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ