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YEREVAN PHYSICS INSTITUTE

A.R.AVAKIAN, A.S.HARUTYUNIAN, A.G.HOVANESSIAN
S.M.SHAHINIAN, C.YANG

MOTION OF CHANNELING PARTICLES IN A BENT CRYSTAL

Yerevan 1990

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Հ.Ռ.ԱՎԱԳՅԱՆ, Ա.Ա.ՀԱՐՈՒԹՅՈՒՆՅԱՆ, Ա.Գ.ՀՈՎՀԱՆՆԵՍՅԱՆ,
Ս.Մ.ՇԱՀԻՆՅԱՆ, ՅԱՆ ՇԻ

ՄԱՍՆԻԿՆԵՐԻ ԴԻՆԱՄԻԿԱՆ ՃԿՎԱԾ ԲՅՈՒՐԵՂՈՒՄ ՄԻՋԱՆՑՄԱՆ ԺԱՄԱՆԱԿ

Բյուրեղային հարթությունների անընդհատ պոտենցիալի մոտավորությամբ կատարված է ճկված բյուրեղում միջանցվող մեծ էներգիայով լիցքավորված մասնիկների շարժման հետազոտություն, միջավայրի ատոմների վրա ոչ կոհերենտ ցրումների հաշվառմամբ: Հետազոտված է բյուրեղից դուրս թռչող մասնիկների անկյունային բաշխումը՝ կախված շեղման առավելագույն անկյունից և մասնիկների էներգիայից: Ոչ կոհերենտ ցրման հաշվառման պարզագույն մոդելի շրջանակներում ստացված է միջանցվող մասնիկների բաժինը՝ կախված բյուրեղի հաստությունից, կորության շառավղից և մասնիկների էներգիայից: Հետազոտված է բյուրեղի կորության ազդեցությունը բյուրեղում մասնիկների կոհերենտ ցրման վրա: Քննարկված է բյուրեղի օպտիմալ հաստության հարցը՝ տրված անկյան տակ առավելագույն թվով մասնիկներ շեղելու իմաստով: Ստացված արդյունքները համեմատվում են վերջերս կատարելագործված մեթոդիկայով կատարված [1] փորձի արդյունքների հետ: Քննարկված են նաև ճկված բյուրեղում մասնիկների միջանցման երևույթի գործնական կիրառության հնարավորությունները:

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MOTION OF CHANNELING PARTICLES IN A BENT CRYSTAL

The motion of high-energy charged particles in a bent rystal is investigated in the approximation of the model of ontinuous potential of crystallographic planes and with ccount of incoherent scattering on the atoms of media. Angular istribution of charged particle beams is investigated at the xit of the bent region of the crystal in dependence with the aximum deflection angle and energy of particles. The dependence of the fraction of channeling particles on crystal thickness, crystal curvature and particle energy is found in a simple model approximation. The influence of crystal curvature on incoherent scattering of particles in the crystal is analyzed. The concept of an optimal thickness for the maximum number of particles deflected at a given angle is considered. The results are compared with recent experimental data [1], where an improved method of obtaining a constant radius of curvature has been used. Practical applications of the channeling effect in bent crystals are discussed.

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А.Р.АВАКЯН, А.С.АРУТЮНЯН, А.Г.ОГАНИСЯН, С.М.ШАГИНЯН, ЯН ШЬ

ДИНАМИКА ЧАСТИЦ ПРИ КАНАЛИРОВАНИИ В ИЗОГНУТОМ КРИСТАЛЛЕ

В приближении непрерывного потенциала кристаллографических плоскостей проведено исследование динамики заряженных частиц высоких энергий в изогнутом кристалле с учетом некогерентного рассеяния на атомах среды. Исследовано угловое распределение заряженных частиц на выходе из изогнутого кристалла в зависимости от максимального угла отклонения и энергии частиц. В простой модели учета некогерентного рассеяния получена зависимость доли каналированных частиц от толщины кристалла, кривизны кристалла и энергии частиц. Проведен анализ влияния кривизны кристалла на некогерентное рассеяние частиц в кристалле. Рассмотрено понятие оптимальной толщины кристалла дляотклонения максимально большего количества частиц на заданный угол. Полученные результаты сравниваются с данными недавнего эксперимента [1], где использовалась улучшенная методика для получения кристалла с постоянным радиусом кривизны. В работе обсуждаются также возможности практического применения эффекта каналирования в изогнутых кристаллах.

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Recently experimental investigations of charged particle beam channeling in bent crystals were carried out [1-4]. This problem was theoretically considered [5-6] in a continuous potential approximation of crystallographic planes.

In the continuous potential approximation the particle motion is considered in the field of an effective potential $U_c(x)$ which is obtained by adding a centrifugal term to the continuous potential.

In this case the minimum of the potential energy for positively charged particles is displaced from the centre between the planes (Fig.1).

For investigation of the motion of charged particles channeling in a real crystal it is important to take into account the influence of incoherent multiple scattering of particles at nuclei and electrons of media.

Dynamical description of motion can be carried out by solving the differential stochastic equations of particles motion [7]

$$dv_x = -\frac{dU_c(x)}{m dx}\,dt + \sigma_x(x)\,dW_1(t)$$

$$dv_y = \sigma_y(x)\,dW_2(t)\ , \qquad (1)$$

$$dx = v_x dt, \qquad dy = v_y dt,$$

where dW_1 and dW_2 are Winner's random quantities, and $\sigma_{x,y}$ depend on the mean square changes of transverse components of particle velocity

$$\sigma^2_{x,y} = \frac{\overline{v^2_{x,y}}}{\Delta t} = \frac{1}{\Delta t}\int_{\Delta t}(dv_{x,y})^2, \qquad ($$

where the averaging interval Δt is as large as the time [illegible] for a particle to travel a path considerably greater than atomic distances, but is considerably smaller than distances at which the potential energy of particles changes essentially. The quantities $\langle\sigma_x\rangle_c$ and $\langle\sigma_y\rangle_c$ averaged over the particle oscillating period, determine the mean square angle of particle multiple scattering in crystal.

In case of a slight curvature of crystal a parabolic potential may be used

$$U_c(x,R) = U_0\left(\frac{2}{d_x}\right)^2(x-x_0)^2, \qquad (3)$$

where U_0 is the potential well depth, d_x is the interplanar distance, $x_0 = E(d_x/2)^2/2U_0R$ is the minimum displacement due to the centrifugal force, R is the radius of curvature.

Particles with a transverse energy less than the critical ε_c are trapped in the channel [6] and their trajectories follow the crystal bending.

In the parabolic potential approximation the fraction of channeled positive particles moving at an angle φ to the crystallographic planes is given by

$$N(\varphi)d\varphi = \begin{cases} \sqrt{z_c\theta_L^2 - \varphi^2}\; d\varphi/\theta_L^2 & \varphi^2 < z_c\theta_L^2 \\ 0 & \varphi^2 > z_c\theta_L^2 \end{cases} \qquad (4)$$

where $z_c = \varepsilon_c/U_0$ is dimensionless critical energy and θ_L is the Lindhard angle. If the particle angular distribution in a large penetration depth l is considered as Gaussian

$$\Psi(\varphi,l) = \frac{1}{\sqrt{2\pi}\theta_R}\exp(-\varphi^2/2\theta_R^2)$$

where θ_R^2 is the mean square of angular distribution in crystal, we have

$$N_1(\theta_R) = \int_0^{\sqrt{z_c}\theta_L} N(\varphi)\Psi(\varphi,l)d\varphi = \sqrt{\frac{\pi}{8}}\,\frac{{}_{c}\;{}_{L}}{\theta_R} \qquad (5)$$

for the fraction of channeled particles in this simple model at $Z_c<\theta_R^2/\theta_L^2$.

The mean square of particles angular distribution increases gradually with the depth of penetration in crystal. At the depth l the magnitude of θ_R^2 may be determined approximately as

$$\theta_R^2(l)=\theta_0^2+\overline{\theta^2}(l)$$

where θ_0 is the incident distribution parameter, $\overline{\theta^2}(l)$ is the mean square of multiple scattering distribution angle approximately described by [8]

$$\overline{\theta^2}(l)=\frac{E_s^2}{E^2}\,\frac{l}{l_R}\,\eta \tag{6}$$

where E_s=14.1 MeV, l_R is the radiation length.

For the ratio $\eta=\langle\theta_x\rangle_c/\langle\theta_a\rangle_c$ of the mean square angles in crystals and amorphous media for positive particles we have

$$\eta(\varepsilon)=1+\frac{1}{ZL'+L''}\sum_{m=1}^{\infty}(-1)^m\cos\left(\frac{2\pi m x_0}{d_x}\right)J_0(\pi m\sqrt{Z_c})(ZL'\nu_m+L''\mu_m)\quad , \tag{7}$$

Here ν_m and μ_m are coefficients of Fourier series of number densities of nuclei and electrons, L′ and L″ are the corresponding radiation logarithms [8].

The magnitude of η depends on the particles transverse energy ε (Fig.2). For positive channeled particles which do not approach the planes, $\eta\simeq L'/(ZL'+L'')$. For approximate estimations one can use $\bar{\eta}$ (Fig.3) which is the value of η averaged over the particle distribution function in a channel.

In an unbent crystal the increase in the particles energy leads to a decrease in dechanneling caused by incoherent scattering. In a bent crystal the energy increase under a fixed maximum deflection angle leads to a decrease of incoheren scattering, on the one hand, and to an increase of the fraction of particles dechanneled at the entrance of the bent region on the other hand. The potential well minimum is shifted from the center of planes and the effective depth of well decreases. This leads to an increase of scattering.

The fraction of particles dechanneled due to bending is $1-N_1(\theta_0)$ (see Eq.(5)), which is in a good agreement (Fig.4) with the experiment [1] and with the theoretical data of a more complicated model [6].

The length of dechanneling may be defined as that on which the fraction of channeled particles decreases e times as compared to the same magnitude at incidence. The results of calculations are in a good agreement with the experiment [1] (Fig.5).

There is a maximum in the dependence of the penetration depth on the fraction of particles deflected at a given angle θ_c(Fig.6). It is not difficult to define the optimal thickness at a given energy:

$$L_{opt} = \frac{5}{8} d_x \frac{E\theta_c}{U_0} \left\{ 1 + \left(1 + \frac{64}{25} \theta_0^2 \frac{2\, l_R U_0^2}{d_x E_s^2 \theta_c \eta} \right)^{1/2} \right\} \tag{8}$$

In the simplified model considered the magnitude for particles escaping at angles from θ to $\theta+d\theta$ is defined as:

$$P(\theta)d\theta = \frac{\sqrt{\pi}\ \alpha}{\sqrt{32}\ \theta_c \left[1+\alpha\frac{\theta}{\theta_c} \right]^{3/2}} (1-\beta\theta_c)^2 d\theta \tag{9}$$

where

$$\alpha = \frac{E_s^2 \eta L}{2EU_0 l_R \theta_0^2} \ ; \qquad \beta = \frac{d_x}{2L\theta_L^2}$$

L is the length of the crystal. Particles escaping at θ travel the length $l=L\theta/\theta_c$. The angular distribution of particles (9) is shown in Fig.7.

Particle channeling in bent crystals will find a wide application in physical experiment in the nearest future. Calibration of experimental installation and high-energy particle detectors may be possible examples of such application. The transverse size of detectors for such energies is too large, and huge mechanical devices are needed to scan the detectors by particle beams of a given energy. In this paper the property of bent crystals to deflect particles at large angles is suggested for calibration of large detectors.

Consider an experimental set up where one end of the crystal is fixed and the other end vibrates with frequency Ω. The coordinate of the free end of the crystal will oscillate in time as

$$x(t) = x_m \sin\Omega t \tag{10}$$

The crystal curvature will not change during one particle's passing through the crystal. At the same time, different particles traverse the crystal at different values of x. If considered that particles enter the crystal at a random moment, the probability of having given x, hence that of having the given deflection angle $\theta_c=x/L$, is

$$Q(x)dx = \frac{1}{\pi}\,\frac{d(x/x_m)}{\sqrt{1-(x/x_m)^2}} \tag{11}$$

The angular distribution of particles at the exit (Fig.8) is obtained by averaging the Eq.(9) with account of crystal's oscillation.

A silicon crystal with a bent part length of 8cm, the curvature of which will be periodically changed by a vibrator with an oscillation amplitude of the exit end of 2-3cm and with 50m far from the calibrating device, provides sufficiently uniform dispersing of particles with momentum 500GeV/c along the axial direction of a 3m high detector. The particle spectrometer magnet necessary for all experiments may be used for horizontal shift of beams. The solution of Eq.(1) by means of numerical simulation with account of the return capture of dechanneled particles may be a more accurate consideration of this problem.

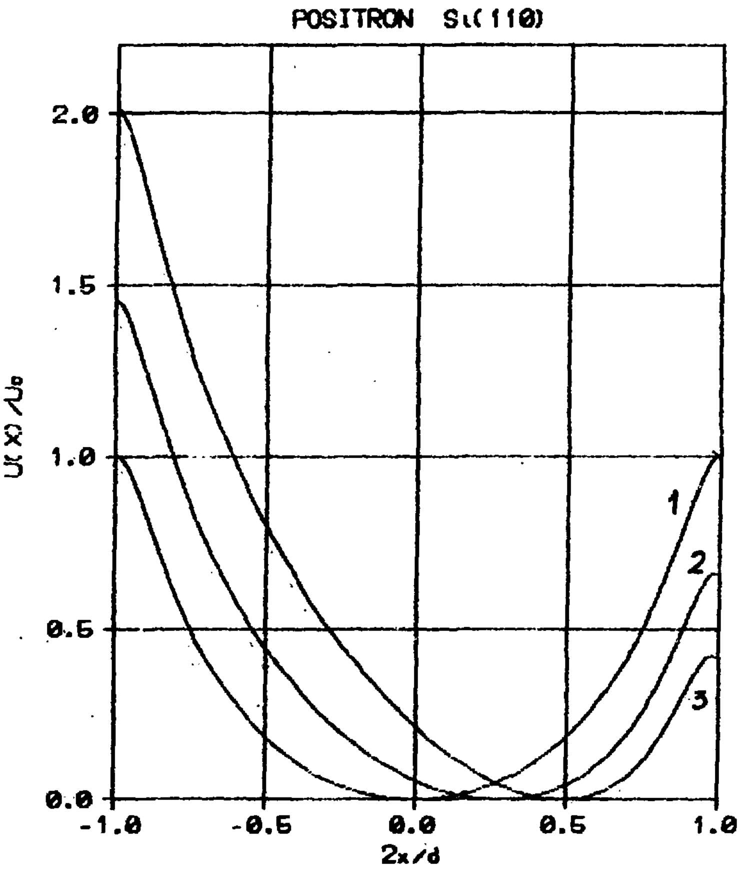

Fig.1

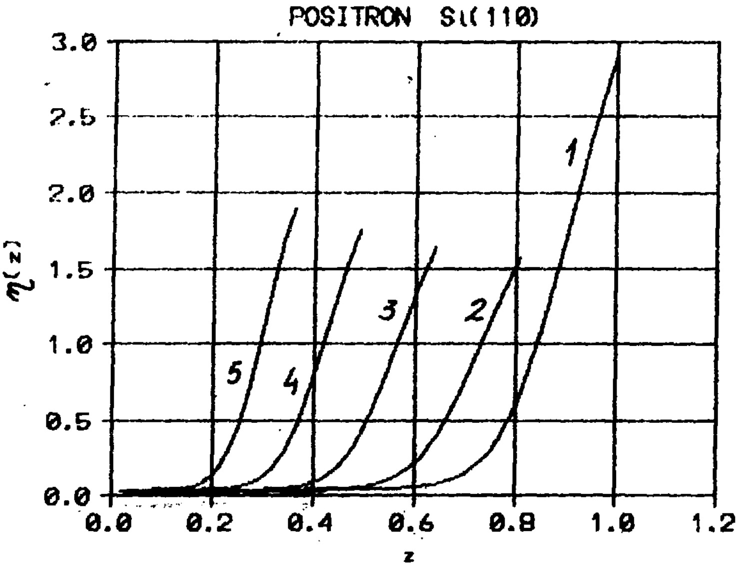
POSITRON Si(110)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\eta(z)$
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
z

Fig.2

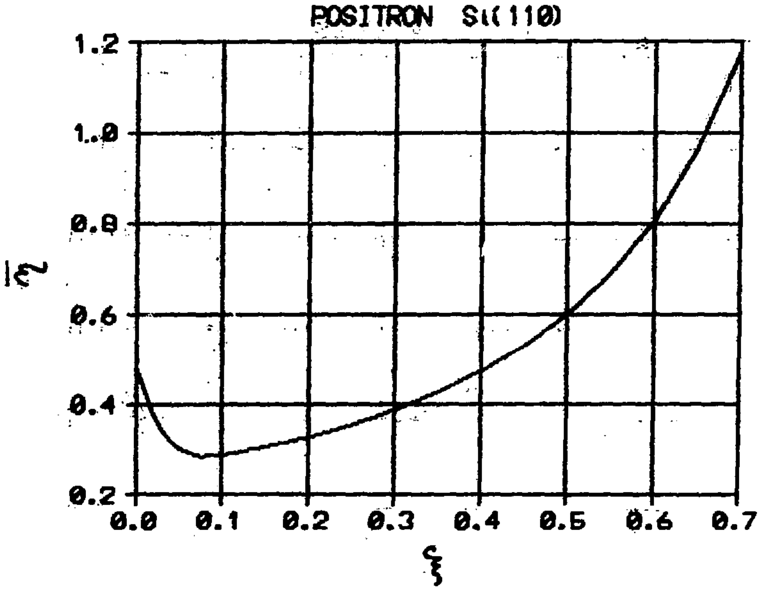
POSITRON Si(110)
1.2
1.0
0.8
0.6
0.4
0.2
$\bar{\eta}$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
ξ

Fig.3

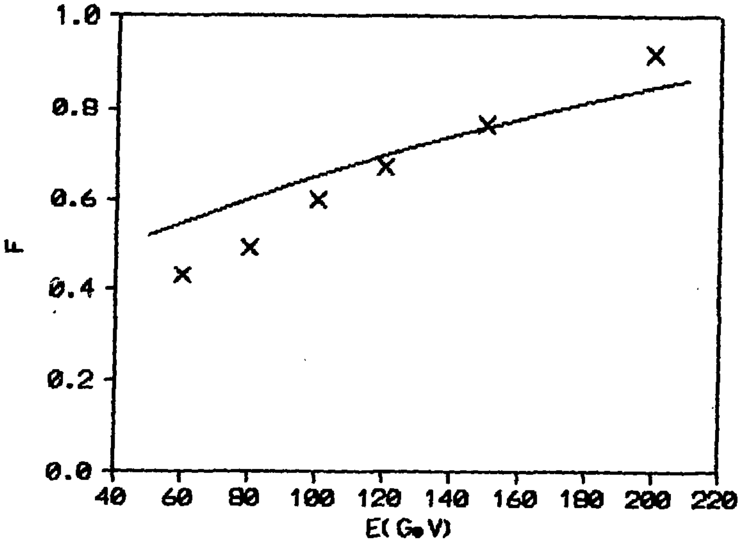
1.0
0.8
0.6
0.4
0.2
0.0
F
40
60
80
100
120
140
160
180
200
220
E(GeV)

Fig.4

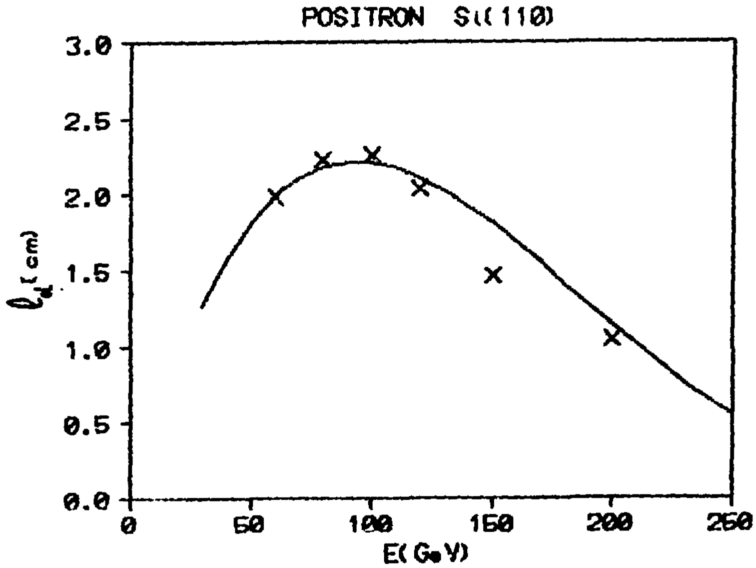
POSITRON Si(110)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
l_d(cm)
0
50
100
150
200
250
E(GeV)

Fig.5

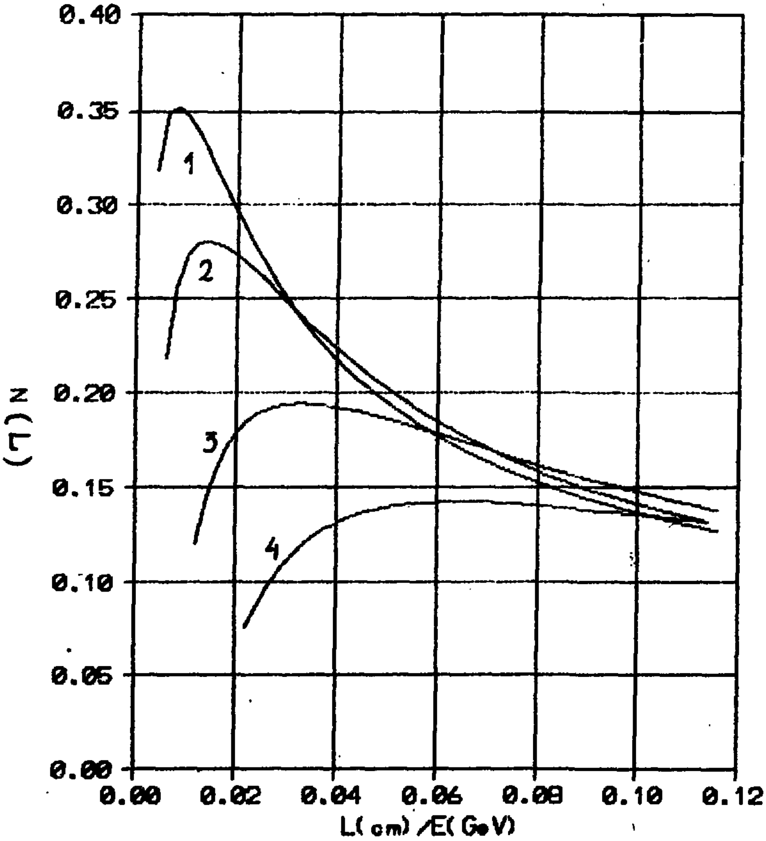
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
N (L)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
L(cm)/E(GeV)
1
2
3
4

Fig.6

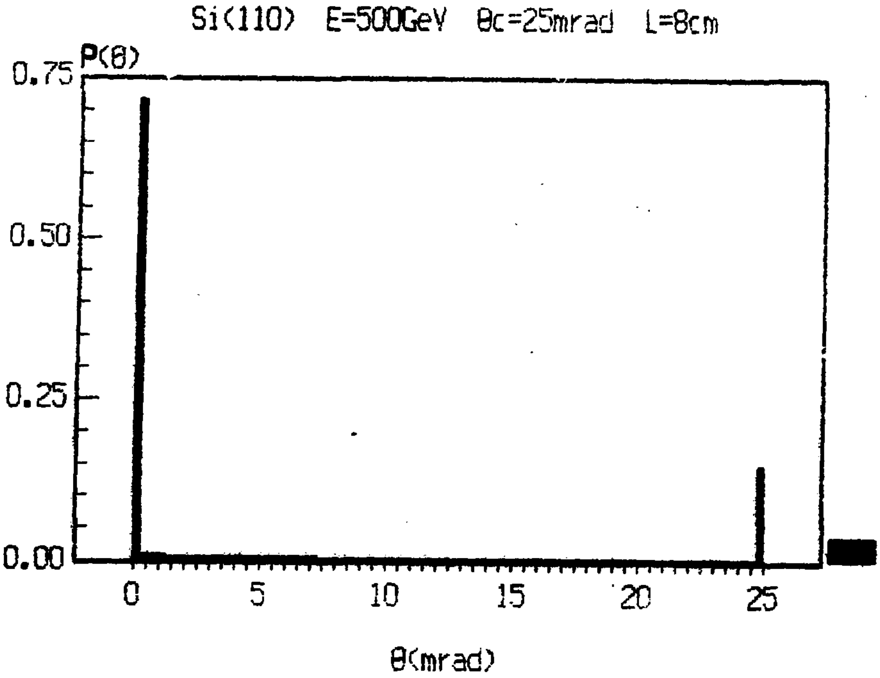
Si(110) E=500GeV θc=25mrad L=8cm
P(θ)
0.75
0.50
0.25
0.00
0
5
10
15
20
25
θ(mrad)

Fig.7

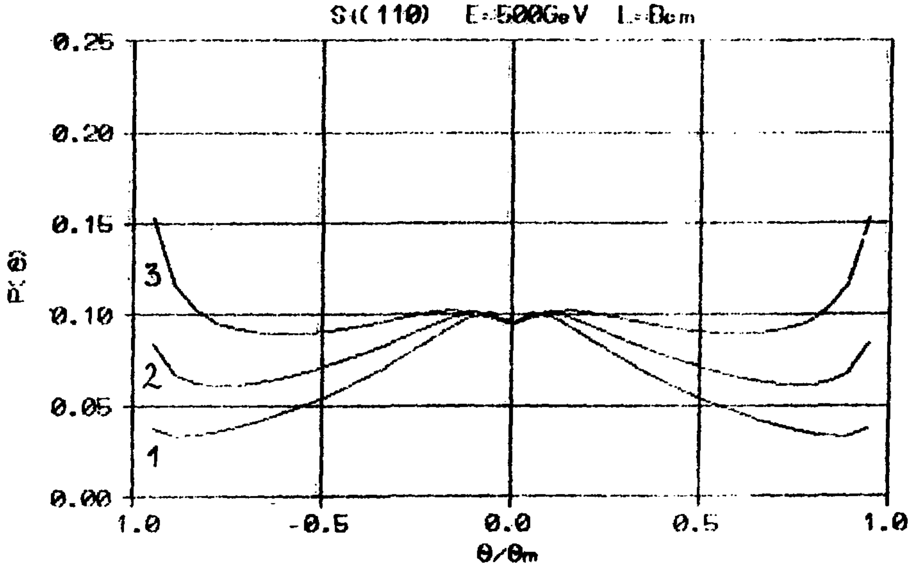
Si(110) E=500GeV L=8cm
P(θ)
0.25
0.20
0.15
0.10
0.05
0.00
3
2
1
1.0
-0.5
0.0
0.5
1.0
θ/θm

Fig.8

FIGURE CAPTIONS

Fig.1 Potential energy $U(x)/U_0$ of positive charged particlesin the averaged field of (110) plane of a silicon crystal. Numbers 1,2,3, refer to $\xi=2x_0/d_x=0$, 0.2, 0.4.

Fig.2 Dependence of $\bar{\eta}$ on the transverse energy $z=\varepsilon/U_0$ at different values of the parameter ξ. The numbers 1,2,3,4,5 refer to $\xi=0$, 0.1, 0.2, 0.3, 0.4.

Fig.3 Dependence of η averaged over the transverse energies on the parameter ξ.

Fig.4 The fraction F of unchanneled particles at the entrance of a bent crystal. The solid curve corresponds to the calculation by means of Eq.(5), the crosses correspond to the experiment [1], for the crystal length L=2.6cm and the deflection angle $\theta_c=32.5$ mrad.

Fig.5 The dechanneling length l_d calculated according to Eq.(5) (the solid curve) and the experimental data[1] (the crosses).

Fig.6 The fraction of particles escaping from a bent crystal $N_1(L)$ as a function of L/E (L is the length of crystal in cm, E is the particle energy in GeV). Numbers 1,2,3,4 refer to deflection angles $\theta_c=5,10,25,50$ mrad.

Fig.7 Angular distribution $P(\theta)$ of particles with energy 500GeV escaping from the bent crystal of 8cm length at a deflection angle $\theta_c=25$ mrad.

Fig.8 Angular distribution $Q(\theta)$ of channeled particles with energy 500GeV at the exit of a 8cm-long oscillating crystal. Numbers 1, 2, 3 refer to the maximum shift angle $\theta_m=x_m/L=50$, 37.5, 25 mrad.

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А.Р.АВАКЯН, А.С.АРУТЮНЯН, А.Г.ОГАНИСЯН, С.М.ШАГИНЯН, ЯН ШИ
ДИНАМИКА ЧАСТИЦ ПРИ КАНАЛИРОВАНИИ В ИЗОГНУТОМ КРИСТАЛЛЕ
(на английском языке, перевод Г.А. Папяна)
Редактор Л.П.Мукаян
Технический редактор А.С.Абрамян

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The address for requests:
Information Department
Yerevan Physics Institute
Alikhanian Brothers 2,
Yrevan, 375036
Armenia, USSR

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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ