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CONSTRUCTION OF THE FIELD THEORY OF
2+1 DIMENSIONAL UNIVERSES FILLED BY
TOPOLOGICAL YANG-MILSS FIELDS

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CONSTRUCTION OF THE FIELD THEORY OF 2+1 DIMENSIONAL
UNIVERSES FILLED BY TOPOLOGICAL YANG-MILLS FIELDS

The action for the field theory of 2+1 dimensional universes filled by topological Yang-Mills fields is constructed.

Yerevan Physics Institute
Yerevan 1990

А. А. КАВАЛОВ, А. П. КАВАЛОВ, Р. Л. МКРТЧЯН

ПОСТРОЕНИЕ ТЕОРИИ ПОЛЯ ВЗАИМОДЕЙСТВУЮЩИХ ВСЕЛЕННЫХ, ЗАПОЛНЕННЫХ
ТОПОЛОГИЧЕСКИМИ КАЛИБРОВОЧНЫМИ ПОЛЯМИ, В РАЗМЕРНОСТИ $2+1$

Построено полное действие теории поля взаимодействующих вселенных в размерности $2+1$, заполненных топологическими калибровочными полями Черна-Саймона.

Ереванский физический институт

Ереван 1990

The logical development of quantum field theory leads to the concept of the field theory of interacting universes, i.e. the theory describing the processes of creation and annihilation of universes [1-10]. By the universe one usually means some closed (i.e. compact and boundaryless) d -dimensional manifold filled by some quantum fields. The integer d is the number of space-like dimensions. Calculating some amplitude in the field theory of universes means summing over all relevant $(d+1)$ -dimensional manifolds, representing the history of universe(s). The simplest case of direct product manifold $\Sigma^d \times R^1$ describes the universe Σ^d which develops without changing its topology or splitting/joining on/with some other universes. Taking all other possible manifolds into account one naturally obtains the "first-quantized" (i.e. described by "Feynman diagrams") formulation of the field theory of universes. For example, the propagator is given by the functional integral

$$G(v_1, v_2) = \sum_{M, \partial M = v_1 \cup v_2} \omega_M \int D\bar{\varphi} e^{i S_M(\bar{\varphi})} \quad (1)$$

The sum in (1) is over all $(d+1)$ -dimensional compact manifolds

M with boundary consisting from V_1 and V_2 . Integration is over all field configurations $\bar{\varphi}$ on M, such that they coincide on the boundary of M with given configurations: φ_1 on V_1 and φ_2 on V_2 , $\bar{\varphi}|_{V_\alpha} = \varphi_\alpha$, $\alpha=1,2$. S_M is the action for fields $\bar{\varphi}$. ω_M are some numbers - the weights, with which the manifold M is taken. They depend only on topological class of M and are given by the combination of coupling constants of the second-quantised theory. The whole expression (1) describes the coordinate representation for "the amplitude for the universe V_1 with the field configuration φ_1 on it to evolve to the universe V_2 with the field configuration φ_2 " and can be understood as an operator from $H(V_1)$ to $H(V_2)$, where $H(V_\alpha)$ ($\alpha=1,2$) is the Hilbert space of quantum states of fields on manifold V_α .

An expression (1) gives a propagator in coordinate representation. One may take the scalar product of (1) with some basis set of wave functions $\Psi_{i_1}(V_1)$, $\Psi_{i_2}(V_2)$ (they depend on φ_1 and φ_2 , respectively) to obtain the propagator G_{ij} in i-representation.

One may easily generalize (1) for higher Green function. Now, when given the values of all terms in expansion (1) and the similar ones for higher Green functions one can try to construct the "second quantized" field theory of universes, in which the Green functions (with the weights ω_M fixed) will appear as a result of perturbation expansion. The generic n-universe interaction term in the action of such field theory will be given by the expression of the type

$$\sum_{V_1 \dots V_n} \sum_{i_1 \dots i_n} \Psi_{i_1}(V_1) \dots \Psi_{i_n}(V_n) \cdot Y(V_1, i_1; \dots; V_n, i_n)$$

and will contain the sum over the manifolds V and the sum/integral over the states of the corresponding Hilbert spaces. The general form of the action for the arbitrary dimension d which will be used below is established in [11], where it is shown that it is sufficient to consider definite set of quadratic and cubic interaction terms. An important simplification occurs if the dimensions of the Hilbert spaces are finite, in which case the second sum/integral will be just the finite sum. This is the case for topological field theories (TFT-s).

The TFT-s were first considered by A.Schwarz [12] then more recently by A.Polyakov [13] and by E.Witten [14], who has shown their numerous connections with various fields of physics and mathematics. The important point which deserves further study is that the TFT-s may be considered as theories of unbroken phase of general relativity [15]. No a priori choice of metric is needed to calculate the action. This immediately rises the question of symmetry breaking. One possible mechanism is through wormholes (see e.g. the discussion in [16]), i.e. by considering the field theory of the universes.

The aim of the present paper is to construct the explicit action for the field theory of 2-dimensional universes filled by the topological Chern-Simons Yang-Mills field. The quantization of Chern-Simons theory leads to a finite dimensional Hilbert space, isomorphic to the space of conformal blocks of the WZW model [14]. The wave functions are the vectors in this space with coordinates which we denote by $\Psi_{g,i}$ (here g is the genus of the unpunctured Riemann surface Σ , and i takes n_g values, where n_g is the number of linearly independent conformal blocks of WZW model on Σ). We take $\Psi_{g,i}$

to satisfy the reality condition which will be written explicitly below. The action S of the field theory is the function of this vectors $\Psi_{g,i}$. Its general form established in [1i] and explained below, is

$$\begin{aligned}
 S = & \sum_{g_1, i_1, i_2} \Psi_{g_1, i_1} \Psi_{g_2, i_2} H^{g_1, i_1, i_2} + \Psi_{g_1, i_1} \Psi_{g_1+1, i_2} Y^{g_1, i_1, i_2} + \\
 & \sum_{g_1, g_2} \Psi_{g_1, i_1} \Psi_{g_2, i_2} \Psi_{g_1+g_2, i_3} Y^{g_1, i_1, g_2, i_2, i_3}
 \end{aligned}
 \tag{2}$$

This expression means that there are terms describing the propagation without changing the topology of the universe and also two types of interaction vertexes - the one which changes the genus $g \rightarrow g \pm 1$ (second term in (2)), and the other given by the last term of (2), describing the process of creation/annihilation of the universes, when two universes with genera g_1 and g_2 , respectively, join together to form the new one with genus $g_1 + g_2$. The important fact is that in the case of the Chern-Simons theory all the terms in (2) are actually known exactly - they may be calculated using the methods of [14,17].

It is worthwhile to note explicitly that we shall write down the vertexes and the whole theory in terms of the Hilbert space arising from the quantization of the reduced phase space variables (with the gauge volume factored out). In stringy terms it means that we construct the analog of Kaku-Kikkawa [18] rather than Witten's BRST [19] field theory.

One may note also that the three-dimensional gravity may be described as a topological (Chern-Simons) gauge theory [20]

too. The corresponding gauge group is, however, non-compact and the Hilbert spaces are infinite-dimensional. The vertexes for the corresponding field theory of universes are calculated in [21].

We turn now to the more detailed explanation of the form of the action (2). Let's denote by G_M the following path integral

$$G_M(V_1, V_2, \dots) = \int D\bar{\varphi} e^{S_M(\bar{\varphi})} \quad (3)$$

$$\partial M = \bigcup_i U_i$$

The integration is over all field configurations $\bar{\varphi}$ on M , such that they coincide on the boundary of M with given configurations φ . $\bar{\varphi} = \varphi$. The Green functions similar to

(3) are the ones of D_n with coefficients w_{ij} depending linearly on the tensor product of a pair (a_i, a_j) of elements of boundary ∂M .

$$w_{ij} = \alpha_{ij} + \beta_{ij} \varphi_i \varphi_j$$

(in oriented manifolds, in such a way, however, that the α_{ij} and β_{ij} are antisymmetric). The first term α_{ij} corresponds to dual operators (as mentioned in [21]), the second term β_{ij} makes equal all the operators α_{ij} from different U_i .

It is unnecessary to mention explicitly that α_{ij} and β_{ij} are found by $\int_M \varphi_i \varphi_j$ for arbitrary compact manifold M with boundary ∂M . The main fact is that there is one α_{ij} for each pair of adjacent U_i and U_j . Similarly, there are β_{ij} for each pair of adjacent U_i and U_j . The corresponding α_{ij} and β_{ij} are determined by the boundary conditions $\varphi_i = \varphi_j$ on $U_i \cap U_j$. The corresponding α_{ij} and β_{ij} are determined by the boundary conditions $\varphi_i = \varphi_j$ on $U_i \cap U_j$. The corresponding α_{ij} and β_{ij} are determined by the boundary conditions $\varphi_i = \varphi_j$ on $U_i \cap U_j$.

dual: $H_i^M = (H_j^N)^\vee$, and one may take the scalar product of G_M and G_N over these spaces: $(G_M \cdot G_N)_{ij}$. In such a way one obtains the answer for the manifold $M \# N$, obtained by gluing M and N over V_i^M and V_j^N (with opposite orientations): $G_{M \# N} = (G_M \cdot G_N)_{ij}$. If one glues these boundaries making previously some diffeomorphism K on V_j^N , then $G_{M \# N} = (G_M \cdot \hat{K} G_N)_{ij}$ where \hat{K} is the matrix, representing the diffeomorphism K in a given basis.

So, one can try to find the answer for arbitrary manifold by finding the set of "standard" manifolds such that any other can be obtained by gluing some number of these standard manifolds (using the diffeomorphisms K if necessary). Exactly such a construction is prepared to us by mathematicians - see, e.g. [22,23]. The necessary manifolds are called standard cobordisms. "Physically", each of them is the trace of the process when some circle S^1 imbedded into some two-dimensional manifold Σ_g , shrinks to a point. Depending on the triviality or non-triviality of homology classes defined by this circle one obtains eventually two or one manifold(s), respectively (this explains why only the terms up to third order in Ψ are needed in the action S). In the first case the sum of the genera of the manifolds obtained is equal to the genus of the original manifold, $g_1 + g_2 = g$, and in the second case the genus gets reduced by one, $g + g - 1$. All three-dimensional manifolds obtained in this way (with different initial manifolds Σ_g and different imbeddings of S^1) constitute the set of standard cobordisms. We shall see later that the set of standard manifolds required in expansions like (1) is the subset of these standard cobordisms. More formally one may define standard cobordisms in the following way. The general definitions, for an arbitrary

dimension d of the universe, may be found in [22,23], here we specialize for $d=2$.

Let V_1 be an arbitrary two-dimensional universe, and let $\varphi: S^1 \times D^1 \rightarrow V_1$ be the imbedding. Let \bar{M}_φ be the topological space, obtained from $V_1 \times I$ ($I = [0,1]$) by gluing it with the manifold $D^2 \times D^1$ through the manifold $\text{Im}\varphi_1$ where $\varphi_1: S^1 \times D^1 \rightarrow (V_1 \times I)$ is given by $(\varphi, 1)$. By the standard procedure \bar{M}_φ may be endowed with the structure of differentiable manifold and that is the desired manifold M_φ . The boundary of M_φ is $\partial M = \bigcup_i V_i$, where $i=1,2$ or $i=1,2,3$. These two cases differs in the following way: when we use the above procedure, we usually obtain the manifold M_φ with boundary, consisting of two pieces - initial manifold V_1 and some "final" one V_2 , but for some imbedding φ (such, that it divides V_1 on two pieces), the boundary of M_φ consists of three pieces.

Now we note that if one is allowed to make an arbitrary diffeomorphism of one of the boundary components before gluing, then one may not include into the consideration all topologically different imbeddings $\varphi: S^1 \rightarrow \Sigma$. Instead one may consider a much smaller set of standard imbeddings. This follows from the fact that by a number of Dehn twists (i.e. large diffeomorphisms) one can transform an arbitrary imbedded curve γ in a following ways:

1) if γ represents non-trivial homological class, then it can be transformed to an arbitrary homologically non-trivial curve γ_1 . We thus choose a standard one, and the corresponding standard cobordism (with curve γ drawn) is shown on Fig.1. This manifold is simply the handlebody with boundary Σ_g (the filled surface Σ_g) with some other handlebody with boundary Σ_{g-1}

cutted out. Note that curve γ is contractable one.

2) if γ is homologically trivial, then it divides (remember, that γ is an imbedded curve) Σ into two manifolds, which (after gluing the disks on their boundary) have genus g_1 and g_2 . $g_1 + g_2 = g$. In that case we can transform γ into an arbitrary other curve with same property. The corresponding standard curve γ and standard cobordism are shown on Fig.2. Again this is a handlebody with boundary Σ_g with the definite handlebodies with boundaries Σ_{g_1} and Σ_{g_2} ($g_1 + g_2 = g$) cutted out. The curve γ is again contractable one - this follows from the definition of the standard cobordisms.

Now we have to determine the terms in the action S , corresponding to the (a) free propagation of the universe Σ , (b) the terms taking into account the possibility of making the diffeomorphisms K , and (c), (d) - the terms, corresponding to the interaction vertexes given by the standard cobordisms of Figs.1,2. To this end we have to choose some basis on the space of the wave functions of the universe.

According to Ref.[24] the WZW conformal blocks on the unpunctured Riemann surface of genus g are constructed through the contraction of the punctures of some number of the punctured spheres - the chiral vertex operators. Since we consider the universes without Wilson loops (no charged matter fields), the remaining punctures have to be taken with unit operators, hence, disappear. It is well-known that the way of constructing the surface with given genus g is inessential - different ways give the same space of conformal blocks but in different bases. We choose some concrete basis which as shown in [17] may be represented as the functional integral of Chern-Simons theory over the three-dimensional manifold - the

handlebody with the configuration of Wilson lines in it shown in Fig.3. Each line bears the indexes denoting some integrable irreducible representations of corresponding affine algebra and the Wilson line is taken in that representation. The indexes corresponding to the vertexes choose one of N_{abc} possible invariant couplings of the lines of that vertex. As was shown in [17] for very similar configuration of Wilson lines, these vectors constitute the basis in the Hilbert space for the surface Σ . The proof may be trivially modified for our configuration of Wilson loops to prove that these vectors, at all possible choices of representations and couplings, constitute an orthonormal basis of the Hilbert space. The space of wave functions of the universe Σ is the linear combination with complex coefficients $\Psi_{g,i}$ (here index i denotes the whole configuration of representations and couplings of Fig.3). We impose the reality condition on $\Psi_{g,i}$ in the form $(\Psi_{g,i})^* = \Psi_{g,i'}$ where i' is the same configuration of Fig.3 with all the representations and couplings changed on the dual ones.

Now we shall find the terms of type (a) i.e. terms describing the free propagation. By definition, these are given by G_M where M is direct product $M = \Sigma \times I$, $I = [0,1]$. For such M $(G_M)_i^j = \delta_i^j$ [17], as follows from the fact that Hamiltonian is equal to zero in this generally covariant topological theory. The corresponding term in the action S is equal to

$$\sum_{g,i} (\Psi_{g,i})^* \Psi_{g,i} \quad (4)$$

The terms of the second type (b) describe the possibility of making large diffeomorphism before gluing. The action of modular transformations (large diffeomorphisms) on the space of conformal blocks, is well-known. Let's denote by S_g this group

for genus g . The terms in the action, which generate the modular transformations, are written as

$$\sum_{g, s \in S_g} \lambda_{g,s} (\Psi_{g,j})^* \epsilon_j^i \Psi_{g,i} + c.c. \quad (5)$$

Here $\lambda_{g,s}$ are an arbitrary complex coupling constants, the sum is over the genera g and also over some set of elements of S_g . This set must be choosed to include a generating set of the modular group (and for definiteness we imply such a choice) but may also be larger. The difference is in the number of coupling parameters of the field theory.

The remaining terms in the action correspond to the vertexes defined by standard cobordisms (types (c), (d)). To obtain them in a basis described above, one has to take the scalar product of these vertexes (Figs.1,2) with the basic vectors (Fig.3) i.e. all the boundaries on Figs.1,2 have to be glued with the manifold of the type shown on Fig.3. For manifold of Fig.1 we first glue the basis vector to the "smaller" (internal on Fig.1) component of the boundary, i.e. Σ_{g-1} , and obtain the handlebody with some configuration of Wilson loops (Fig.4). Note, that this is exactly one of the basic vectors of Fig.3 with particular configuration of Wilson loops, namely, $e=1=0$ (the trivial representation, so the corresponding Wilson lines disappear), $c=d$, and coupling s' trivial. Then, taking the scalar product of this vector with general vector (Fig.3) from the basis of other component of vertex of Fig.1, we find the answer from the orthonormality of the basis vectors. In other words, the vertex in i -representation is delta-function of coincidence of configurations of Wilson loops of Figs.4 and 3. We denote the configuration of Wilson lines on Fig.4 by l and that of Fig.3 (the one having non-zero scalar product with l)

through $e(i)$. Evidently, $e(i)$ is the same configuration i (which is one of the basis vectors for the surface of genus $g-1$), but interpreted as a configuration from the basis set of the surface with genus g . In the same way one obtains the answer for the vertex of Fig.2. It is the delta-function of coincidence of configurations of Wilson loops of Figs.3 and 5. If we denote the configurations for two small components of the boundary of manifold on Fig.2 (i.e. Σ_{g_1} and Σ_{g_2}) by i and j , respectively, then the corresponding configuration of Fig.3, which gives non-zero answer in scalar product will be denoted as $i+j$. These results give the following terms in the action S :

$$\sum_{g,j} \lambda_g (\Psi_{g,e(i)})^* \Psi_{g-1,j} + \quad (6)$$

$$\sum_{g_1, g_2, i, j} \lambda_{g_1, g_2} (\Psi_{g_1+g_2, i+j})^* \Psi_{g_1, i} \Psi_{g_2, j} + \text{c.c.}$$

Note that due to the reality condition these terms may be written in a different ways. For example, the first term we can write as $\Psi_{g,e(i)} \Psi_{g-1,j}^*$ (remember that prime here means the dual configuration). In general case all these terms have to be included in the action separately.

The complete action is given by the sum of (4), (5) and (6):

$$S = \sum_{g,i} (\Psi_{g,i})^* \Psi_{g,i} + \sum_{g, s \in S_g} \lambda_s (\Psi_{g,s})^* s_j^i \Psi_{g,i} +$$

$$\sum_{g,j} \lambda_g (\Psi_{g,e(j)})^* \Psi_{g-1,j} + \quad (7)$$

$$\sum_{g_1, g_2, i, j} \lambda_{g_1, g_2} (\Psi_{g_1+g_2, i+j})^* \Psi_{g_1, i} \Psi_{g_2, j} + \text{c.c.}$$

In conclusion, we have discussed the two approaches to the theory of interacting universes: the first quantized "Feynman diagram" approach of eqs.(2), (3) and the field theory approach of eq.(7). The last approach, as has been shown above, reconstructs the same Feynman diagrams. It is well-known, however, that the field theory approach has an advantage of permitting the study of nonperturbative phenomena usually based on the finding of classical solutions of the equations of motion of the field theory. This problem together with the problem of renormalization of quantum theory has to be the main direction for farther investigation of (7).

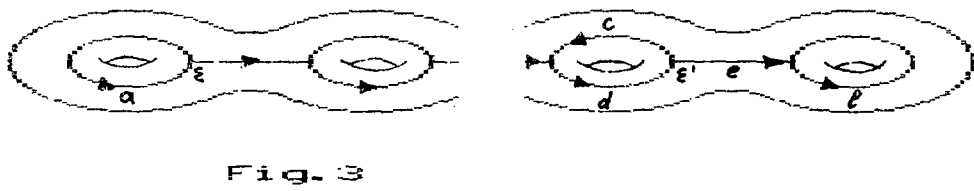
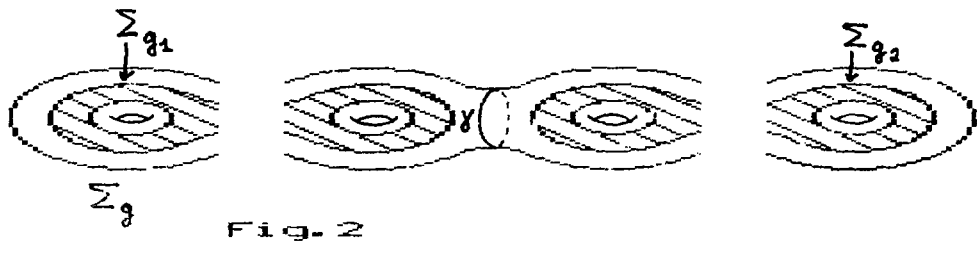
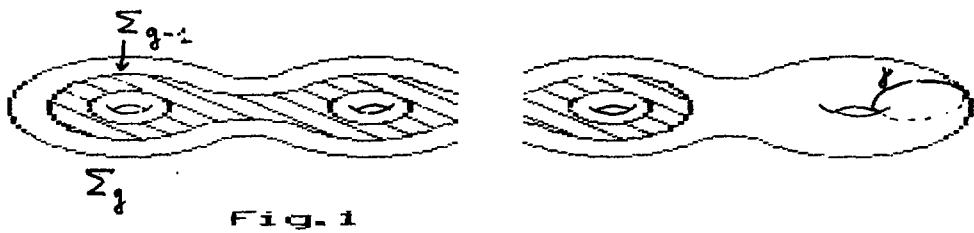




Fig. 7



Fig. 8

References

1. Hawking S.W. Phys.Lett. B195 (1987) 337.
2. Hawking S.W. Phys.Rev.D37 (1988) 904.
3. Hawking S.W., Page D., Pope C. Nucl.Phys. B170 (1980) 283.
4. Lavrelashvili G.V., Rubakov V.A., Tinyakov P.G. JETP Lett. 46 (1987) 167; Mod.Phys.Lett. A3 (1988) 1231.
5. Giddings S., Strominger A. Harvard preprints HUTP-87/A067, HUTP-88/A006; Nucl.Phys. B306 (1988) 890.
6. Coleman S. Nucl.Phys. B307 (1988) 867.
7. Linde A.D. Phys. Lett. B200 (1988) 272.
8. Banks T. Nucl.Phys.B309,(1988) 493.
9. Coleman S. Nucl.Phys. B310 (1988) 643.
10. Klebanov I., Susskind L., Banks T. Preprint SLAC-PUB-4705 (1988).
11. Mkrtchyan R.L. Towards the complete action of the field theory of interacting universes, preprint YERPHY-1148(25)-89 (1989).
12. Schwarz A.S. Lett.Math.Phys. 2 (1978) 247; Comm.Math.Phys. 67 (1979) 1.
13. Polyakov A.M. Mod.Phys.Lett. A3 (1988) 325.
14. Witten E. Comm.Math.Phys. 121 (1989) 351.
15. Witten E. Comm.Math.Phys. 117 (1988) 353; preprint IASSNS-HEP-88/85 The search for higher symmetry in string theory.
16. Brooks R. Nucl.Phys. B325 (1989) 481.
17. Witten E. Nucl.Phys. B322 (1989) 629.
18. Kaku M., Kikkawa K. Phys. Rev. D10,(1974) 1110,1823.
19. Witten E. Nucl.Phys. B268 (1986) 253.

20. Witten E. Nucl.Phys. B311 (1988/89) 46-78.
21. Witten E. Nucl.Phys. B323 (1989) 113.
22. Browder E., Surgery on simply-connected manifolds. Springer-verlag 1972.
23. Рохлин В.А., Фукс Д.Б. Начальный курс топологии. М. "Наука" 1977.
24. Moore G. and Seiberg N. Comm.Math.Phys. 123 (1989) 177.

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**ПОСТРОЕНИЕ ТЕОРИИ ПОЛЯ ВЗАИМОДЕЙСТВУЮЩИХ ВСЕЛЕННЫХ, ЗАПОЛНЕННЫХ
ТОПОЛОГИЧЕСКИМИ КАЛИБРОВОЧНЫМИ ПОЛЯМИ, В РАЗМЕРНОСТИ 2+1**

(на английском языке, перевод авторов)

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