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RENORMALIZATION GROUP INVESTIGATION OF HEAVY QUARKS AND HIGGS BOSONS MASSES

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ԾԱՆՐ ՔՎԱՐԿՆԵՐԻ ԵՎ ՀԻԳՍԻ ԲՈԶՈՆՆԵՐԻ ԶԱՆԳՎԱԾՆԵՐԻ ՎԵՐԱՆՈՐՄԱԽՄԲԱՅԻՆ ՀԵՏԱԶՈՏՈՒՄԸ

Քննարկվում են վերանորմավորումների խմբի հավասարումները տրամաչափային, յուկավայի, սկալյար կապերի համար՝ ֆերմիոնների երեք և չորս սերունդների և Հիգսի դաշտի երկու դուբլետների դեպքում: Ելնելով նրանից, որ տեսության մոդելը և խոտորումների տեսությունը պետք է ճիշտ մնան մինչև մեծ միավորման էներգիան՝ ստացված են սահմանափակումներ մասնիկների զանգվածների համար:

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RENORMALIZATION GROUP INVESTIGATION OF HEAVY QUARKS AND HIGGS BOSONS MASSES

The renormalization group equations for the gauge, Yukawa and scalar coupling constants for three and four generations of fermions, one and two Higgs field doublets are considered. Bounds on the particle masses are obtained from the validity condition of the standard model and perturbation theory up to grand unification energies.

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РЕНОРМГРУППОВЫЕ ИССЛЕДОВАНИЯ МАСС ТЯЖЕЛЫХ КВАРКОВ И ХИГГСОВСКИХ БОЗОНОВ

Рассматриваются уравнения ренормгруппы для калибровочных юкавских, скалярных констант связи для трех и четырех поколений фермионов, одного и двух дублетов поля Хиггса. Из условия справедливости стандартной модели и теории возмущений до энергий великого объединения получены ограничения на масс частиц.

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1. Introduction. Two approaches to the problem

The main problem of the standard model today consists in minimization of the large number of its free parameters. Especially urgent is to obtain information on the masses of the t-quark (and other possible heavy fermions) and of the Higgs boson(s).

As for the t-quark, already the experimental bound is $m_t > 78$ GeV [1] [*].

The study of electroweak radiative corrections to the $\sin^2\theta_W$ gives the range of expected m_t ... m_t ... [2].

As for the Higgs boson mass m_H, ... and theory lead to less definite ... tions. At the same time, in the framework ... for the calculation of standard model ... correlate in some way or other.

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[*] According to the new data from C... Mhedjan... rian, private communication.

Most evidently this correlation can be seen in the renormalization group equations where the running Yukawa coupling constant of the t-quarks with the Higgs field $h(q^2)$ ($m_t = h(\eta^2)\eta$, η is the Higgs field vacuum expectation value) is connected with the running self-interaction constant $\lambda(q^2)$ of this field ($m_H^2 = 4\lambda(q^2)\eta^2$, η = 175 GeV).

Therefore, it is no mere chance that the renormalization group approach to the problem of determination of restrictions on m_t and m_H and correlation between them has been attracting attention for a long time.

Here one can clearly single out two approaches that differ in the assumptions concerning the regimes of ultraviolet behavior of the standard model running constants.

The first approach which goes beyond the standard model and which can be called a scheme of nonperturbative unification is based on the idea that the low-energy coupling constants effectively are close to the possible infrared stable fixed points of the renormalization group equations [3] . In this case in the scale of this "nonperturbative" unification $\Lambda \gg \eta$ the strong-coupling regime is inevitably realised, and the low-energy physics is independent of theory details on the scales Λ .

In such a scheme, due to the large number of fermions and Higgs bosons[2)] one can obtain [3,4] a satisfactory description

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It is appropriate to mention that this approach has close connection with L.D. Landau's idea [5] about the possibility to explain the smallness of the fine-structure constant by the large number of fermions if taking the Planck mass as [illegible] ultraviolet cutoff Λ in QED.

of some parameters of the standard model and applying this idea to $h(q^2)$ and $\lambda(q^2)$ obtain restrictions on the heavy quark masses and the Higgs boson mass.

This approach seems to be nontrivial and interesting in view of the "new physics" manifestations; however the necessity to have a large number of the generations of quarks and leptons (N_G = 8-9) and Higgs bosons ($N_H \approx 3$) [6] as well as the restriction to the lowest loops when studying the evolution of the coupling constants from the strong-coupling region ($q \approx \Lambda$) to $q \approx 10^2$ GeV reduce the value of the model.

The other approach to the restriction on heavy quark and Higgs boson masses is by the definition perturbative, so here one has no difficulties with the highest (> 2) loops. Besides, here one has not to introduce artificially a large number of generations of fermions (and Higgs bosons). We mean the "trivial" grand unified models in which the asymptotic freedom is preserved to the end and which make practical sense precisely for the reason that all interaction constants in them including h and λ remain small up to the M_X -unification scale. Just on such a way, demanding though the smallness of h and λ in the region $q \approx \eta$ only, the restriction on m_t , m_H ($\lesssim$ 1 TeV) was determined for the first time [7] .

One should bear in mind that the Higgs self-action $\lambda\varphi^4$ introduced in the renormalization group equations makes them, generally speaking, unstable. From this point of view, the requirement that λ would be small up to M_X giving a restriction on m_H implies that the renormalization group equations are stable in the whole energy region from η to M_X (the "great desert" hypothesis).

In Ref. 8 the above-mentioned idea was successively realized with the resulting interesting restrictions on the standard model parameters, in particular on m_t and m_H.

In the present paper we also use the approaches of that pioneer work and apply them to some unified models.

2. The Study of the Renormalization Group Equations for the Minimal Model

We are considering the renormalization group equations for the gauge, Yukawa and Higgs coupling constants in the standard model based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

As distinct from Refs [8,3,4] where only the case with one scalar field doublet connected with $SU(2)_L \times U(1)_Y$ breaking was considered, we are studying here also the case with two scalar doublets.

Our further consideration is based on the assumption that between the scale η = 175 GeV and some scale $\Lambda \gg \eta$ the standard model of electroweak and strong interactions remains a correct theory. The scale Λ may coincide with the grand unification scale or with the Planck mass. This scale may lie considerably lower on the energy scale if the new physics is connected, say, with the emergence of new gauge degrees of freedom of the type of $SU(2)_L \times SU(2)_R \times U(1)$ or additional U(1). Here we assume that all coupling constants in the energy range from η to Λ are relatively small, i.e. we assume the validity of the perturbation theory in this energy range. Precisely this condition enables us to obtain some bounds on particle masses. These bounds are interesting due in particular to the fact that they allow to obtain information on the structure of interactions at $E \gg \eta$. Thus, if some relations derived for particle masses at $\Lambda = 10^{14}$ GeV are not confirmed experimentally, this will speak in favour of the existence of new interactions in the region between η and 10^{14} GeV.

We shall consider in what follows that the number of fermion generations is three or four. In this case the smallness of the gauge coupling constants α_1 , α_2 , α_3 in the energy range from η to grand unification scale (10^{14} - 10^{15} GeV) and higher in accordance with the renormalization group equations is guaranteed. And the perturbation theory validity condition for the Yukawa and Higgs coupling constants enables us to obtain restrictions on fermion and Higgs boson masses.

At first, we'll consider the case when the electroweak group is broken by the vacuum expectation value of only one doublet of Higgs fields. Then, as is well known, after spontaneous symmetry breaking there arises one neutral scalar field.

The part of the Lagrangian that describes the interaction of the scalar particle doublet φ with the fermions and with itself is of the form:

$$\mathcal{L} = -\lambda(\varphi^+\varphi - \eta^2)^2 + h_{ij}\bar{q}_{Li}\varphi u_{Rj} + g_{ij}\bar{q}_{Li}\tilde{\varphi} d_{Rj} + f_{ij}\bar{L}_{Li}\tilde{\varphi} e_{Rj} , \tag{1}$$

where i , j denote the number of generations of quarks and leptons:

$$q_{L1} = \binom{u}{d}_L , \quad L_{L1} = \binom{\nu}{e}_L , \quad u_{R1} = u_R , \; d_{R1} = d_R ,$$

$$e_{R1} = e_R , \quad \varphi = \binom{\varphi^0}{\varphi^-} , \quad \tilde{\varphi} = \varphi^* \varepsilon , \quad \varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} . \qquad (2)$$

The behavior of the coupling constants λ , h_{ij} , g_{ij} , f_{ij} , α_1 , α_2 , α_3 is given by the renormalization group equations. It is reasonable to study for three generations of fermions the energy behavior of the Yukawa constant h_t only, because of the large (compared to other quarks and leptons) mass of the t-quark. For the same reason, the contribution of other Yukawa coupling constants may be disregarded in the equations for λ either. In this case the one-loop renormalization group equations are as follows:

$$32\pi^2 \frac{dg_i}{dt} = \beta_i g_i^3 \qquad (i = 1,2,3)$$

$$32\pi^2 \frac{dh_t^2}{dt} = h_t^2 \left(9h_t^2 - 16g_3^2 - \frac{9}{2} g_2^2 - 1.7 g_1^2 \right)$$

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 + 12\lambda h_t - 6h_t^2 - \lambda \left(\frac{3}{5} g_1^2 + 9g_2^2 \right) + \qquad (3)$$

$$+ \frac{9}{24} \left(g_2^4 + \frac{2}{5} g_1^2 g_2^2 + \frac{3}{25} g_1^4 \right) ,$$

$$\beta_1 = -\left(\frac{4}{3} N_f + 0.1 \right), \quad \beta_2 = \frac{22}{3} - \frac{4}{3} N_f - \frac{1}{6} , \quad \beta_3 = 11 - \frac{4}{3} N_f ,$$

where N_f is the number of quark generations.

In our notation the quantity λ/π may be regarded a parameter of expansion in λ , and $h_t^2/4\pi$ - in h_t . This means that for the perturbation theory validity in the energy range from η to Λ these parameters must be smaller than unity.

Besides, the vacuum stability condition is important; in the given case it is simply $\lambda > 0$.

Equations (1) were studied in Refs [3,4,8], where on the basis of the validity condition of the perturbation theory in the region from η to the grand unification scale the bounds on the region of variation of parameters h_t and λ and hence on $m_t = h_t \eta$ and $m_H^2 = 4\lambda\eta^2$ were obtained.

Here we are investigating the renormalization group equations for the coupling constants in the two-loop approximation. On account of awkwardness, we present here only a two-loop equation for h_t [9-12]:

$$\frac{32\pi^2}{h_t^2}\frac{dh_t^2}{dt} = 9h_t^2 - 16g_3^2 - \frac{9}{2}g_2^2 - 1.7g_1^2 + \frac{1}{16\pi^2}\Big[-24h_t^4 + 2h_t^2\Big(\frac{223}{86}g_1^2 + \frac{135}{16}g_2^2 + 16g_3^2\Big) + 5h_t^2\Big(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{20}g_1^2\Big) + \frac{1205}{600}g_1^4 - \frac{9}{20}g_1^2g_2^2 + \frac{19}{15}g_1^2g_3^2 - \frac{23}{4}g_2^4 + 9g_2^2g_3^2 - 108g_3^4 + \frac{3}{8}\lambda^2 - 3\lambda h_t^2\Big]. \quad (4)$$

Now we turn to the analysis of the two-loop equations for the coupling constants.

If we believe that the grand unification scenario is realized without intermediate symmetries, then we must take $\Lambda = 10^{14}$ GeV (see, e.g. [12]), which corresponds to the grand unification scale. The same validity conditions of the perturbation theory provide us with bounds on m_t and m_H.

Fig. 1 shows the region of admissible values of these masses. The results differ little from the consequences of one-loop approximation.

As can be seen from Fig. 1, the value of the t-quark mass is limited: $m_t < 233$ GeV. The upper curve in Fig. 1 refers to the validity condition of perturbation theory $\lambda < \pi$. The lower curve corresponds to the vacuum stability condition. The upper boundary on m_t corresponds to the smallness condition of the Yukawa coupling constant $h_t^2 < 4\pi$.

More strong restrictions on m_t and m_H can be obtained if we assume that the standard model and the perturbation theory are valid up to the Planck mass. The corresponding region of values of m_t , m_H is given in Fig. 2 (curve 2).

Now we consider the case when the standard model abandons being valid up to the grand unification scale. In this case we obtain weaker restrictions on m_t and m_H . The regions of their admissible values for $\Lambda = 10^5$ GeV are presented in Fig. 2 (curve 1).

Fig. 3 shows the upper limit on m_t as a function of scale Λ after which the new physics begins to display.

A question may arise here: how much strong is the dependence of the obtained results on the very quantities of maximally admissible values of coupling constants? This dependence is rather weak. Thus, for the upper limit of the t-quark mass for the case $h_t^2/4\pi < 2, 1, 1/2$ we obtain respectively 235, 233, 229 GeV for $\Lambda = 10^{14}$ GeV.

3. Minimal Model with Four Generations

Now we pass to the study of renormalization group equations for four generations of fermions. We should take into account that in this case the contribution to the equations comes from the Yukawa coupling constants of the Higgs field not only with quarks but with leptons too, because the τ' lepton (a fourth charged lepton) may have a large mass. Then the one-loop renormalization group equations for the coupling constants h_t, $h_{t'}$, $h_{b'}$, $h_{\tau'}$ can be written down as follows:

$$\frac{32\pi^2}{h_t^2}\frac{dh_t^2}{dt} = 9h_t^2 + 6h_{t'}^2 + 6h_{b'}^2 + 2h_{\tau'}^2 - 16g_3^2 - \frac{9}{2}g_2^2 - 1.7g_1^2$$

$$\frac{32\pi^2}{h_{t'}^2}\frac{dh_{t'}^2}{dt} = 6h_t^2 + 9h_{t'}^2 + 3h_{b'}^2 + 2h_{\tau'}^2 - 16g_3^2 - \frac{9}{2}g_2^2 - 1.7g_1^2 \tag{5}$$

$$\frac{32\pi^2}{h_{b'}^2}\frac{dh_{b'}^2}{dt} = 6h_t^2 + 3h_{t'}^2 + 9h_{b'}^2 + 2h_{\tau'}^2 - 16g_3^2 - \frac{9}{2}g_2^2 - \frac{1}{2}g_1^2$$

$$\frac{32\pi^2}{h_{\tau'}^2}\frac{dh_{\tau'}^2}{dt} = 6h_t^2 + 6h_{\tau'}^2 + 6h_{b'}^2 + 5h_{\tau'}^2 - \frac{9}{2}g_2^2 - \frac{9}{2}g_1^2$$

In (5) we have made an essential assumption that the mixing angles between the third and the fourth generations may be ignored. If we believe that all Yukawa coupling constants up to 10^{14} GeV are small (h_t^2, $h_{t'}^2$, $h_{b'}^2$, $h_{\tau'}^2 < 4\pi$), then we'll obtain the following restrictions on the masses of b' and τ' : $m_{b'} < 220$ GeV, $m_{\tau'} < 180$ GeV.

As for the t'-quark, one should distinguish two possibilities: a) $m_t \approx m_{t'}$, and b) $m_t < m_{t'}$. In the first case we obtain m_t , $m_{t'} < 175$ GeV. In the second case for $m_t = 100$ GeV we have $m_{t'} < 215$ GeV, and for $m_t = 125$ GeV $m_{t'} < 205$ GeV. One can see that with increasing m_t the upper limit on $m_{t'}$ is reducing; in other words, at large values of t-quark masses ($m_t > 125$ GeV) the t- and t'-quarks may be rather close in mass.

4. Two-Higgs-Boson Case

Now we pass to the possibility when $SU(2)_L \times U(1)_Y$ is broken with the use of two scalar field doublets. In this case the situation is considerably complicated. The interaction potential of scalar fields $\varphi = \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix}$ and $\xi = \begin{pmatrix} \xi^0 \\ \xi^- \end{pmatrix}$ is of the form:

$$V(\varphi,\xi) = \lambda_1(\varphi^+\varphi - a^2) + \lambda_2(\xi^+\xi - b^2) +$$

$$+ 2\lambda_5(\varphi\xi)(\varphi^+\xi^+) + 2\lambda_3(\varphi^+\varphi - a^2)(\xi^+\xi - b^2) + \qquad (6)$$

$$+ 2\lambda_4(\varphi^+\xi - ab)(\xi^+\varphi - ab) +$$

$$+ \lambda_6[(\varphi^+\xi - ab)^2 + (\xi^+\varphi - ab)^2]$$

where we assumed the presence of symmetry $\varphi \to -\varphi$ and $\xi \to -\xi$. The minimum of potential (6) is attained at $\langle \varphi^0 \rangle = a$ and

$\langle \xi^\circ \rangle = \beta$ (for simplicity we assume that α and β are real numbers), $\alpha^2 + \beta^2 = \eta^2$.

After spontaneous symmetry breaking there remain three neutral and one charged scalar fields. Their masses are given by the following formulae:

$$M^2_{H^-} = 2\lambda_5 \eta^2 , \qquad M^2_{H^\circ_1} = 2(\lambda_4 - \lambda_6)\eta^2 ,$$

$$M^2_{H^\circ_2, H^\circ_3} = 2(\lambda_1 \alpha^2 + \lambda_2 \beta^2) + (\lambda_4 + \lambda_6)(\alpha^2 + \beta^2) \tag{7}$$

$$\mp \left\{ [2(\lambda_1 \alpha^2 - \lambda_2 \beta^2) - (\lambda_4 + \lambda_6)(\alpha^2 - \beta^2)]^2 + 4\alpha^2 \beta^2 (\lambda_4 + \lambda_6 + 2\lambda_3)^2 \right\}^{1/2} .$$

The vacuum stability condition imposes restriction on the potential so as at large values of $\langle \varphi^\circ \rangle$ and $\langle \xi^\circ \rangle$ the potential $V(\varphi, \xi)$ would be positive. This implies that constants λ_i must satisfy one of the following conditions:

$$\lambda_1 > 0 , \qquad \lambda_2 > 0 , \qquad \lambda_3 + \lambda_4 + \lambda_6 > 0 \tag{8a}$$

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 + \lambda_4 + \lambda_6 < 0, \quad \lambda_1 \lambda_2 > (\lambda_3 + \lambda_4 + \lambda_6)^2 . \tag{8b}$$

Besides these conditions, λ_i , certainly, must satisfy the conditions following from the mass positivity requirements $M^2_{H^-}$, $M^2_{H^\circ_i}$.

Now we proceed with the analysis of the renormalization group equations. First, we notice that in the case of two Higgs doublets we have two possibilities to avoid in a natural way the occurrence of strangeness-changing neutral currents in theory: I - the φ doublet interacts with upper quarks only, and ξ - with lower quarks only; II - only the φ doublet interacts with quarks, and ξ - does not. The one-loop renormalization group equations for the coupling constants and h_β are of the form:

$$\frac{32\pi^2}{h_t^2}\frac{dh_t^2}{dt} = 9h_t^2 + h_\beta^2 - 16g_3^2 - \frac{9}{2}g_2^2 - 1.7g_1^2 ,$$
$$\frac{32\pi^2}{h_\beta^2}\frac{dh_\beta^2}{dt} = h_t^2 + 9h_\beta^2 - 16g_3^2 - \frac{9}{2}g_2^2 - \frac{1}{2}g_1^2 , \qquad (9)$$

where $h_t(\eta) = m_{t/a}$, $h_\beta(\eta) = m_{\beta/\beta}$ in the case I, and $h_t(\eta) = m_{t/a}$, $h_\beta(\eta) = m_{\beta/a}$ in the case II. Equations (9) enable us to obtain a restriction on the t-quark mass versus the quantity β/a . Just as before, we'll believe that the coupling constants h_t , h_β in the range from η to A are small: $h_t^2 , h_\beta^2 < 4\pi$.

The restrictions on m_t that result from (9) in the case I are given in Fig. 4 for $\Lambda = 10^5$, 10^{14}, $1.2 \cdot 10^{19}$ GeV versus a/β . The general restriction on m_t is $m_t < 225$ GeV for $\Lambda = 10^{14}$ GeV. It differs little from the upper limit that was obtained by us in the one-Higgs field case. One can see that in this case the ratio of the Higgs fields vacuum expectation values a/β is also limited: $a/\beta < 53$.

The case II can be analyzed similarly. The result is presented in Fig. 5. In this case the quantity a/β is not limited, and the restriction on m_t remains practically unchanged.

The coincidence of the upper bound of m_t in cases I and II and in the one-Higgs case is due to the fact that in the renormalization group equation for h_t , h_β the Higgs fields make no contribution in the first loop, and in the second loop their contribution is negligible.

Now we pass to the analysis of the restrictions emergent on Higgs field masses. To do that, it is necessary to consider along with (9) also the renormalization group equations for the coupling constants λ_i :

$$32\pi^2 \frac{d\lambda_1}{dt} = 4[6\lambda_1^2 + (\lambda_3+\lambda_4)^2 + (\lambda_3+\lambda_5)^2 + \lambda_6^2] + $$
$$+ 12\lambda_1 h_t^2 - 6h_t^4 - 3\lambda_1(3g^2+g'^2) + \frac{3}{8}(2g^4 + (g^2+g'^2)^2)$$

$$32\pi^2 \frac{d\lambda_2}{dt} = 4[6\lambda_2^2 + (\lambda_3+\lambda_4)^2 + (\lambda_3+\lambda_5)^2 + \lambda_6^2] + 12\lambda_2 h_b^2 -$$
$$-6h_b^4 - 3\lambda_2(3g^2+g'^2) + \frac{3}{8}(2g^4 + (g^2+g'^2)^2)$$

$$32\pi^2 \frac{d\lambda_3}{dt} = 4[2\lambda_3^2 + \lambda_4^2 + \lambda_5^2 + (\lambda_1+\lambda_2)(3\lambda_3+\lambda_4) + \lambda_6^2] + \tag{10}$$
$$+ 6(h_t^2 + h_b^2) - 3\lambda_3(3g^2+g'^2) + \frac{3}{8}(2g^4 + (g^2+g'^2)^2)$$

$$32\pi^2 \frac{d\lambda_4}{dt} = 4[2\lambda_4^2 + 2\lambda_4(2\lambda_3 - \lambda_5) + \lambda_4(\lambda_1+\lambda_2) + 4\lambda_6^2] +$$
$$+ 6\lambda_4(h_t^2 + h_b^2) - 3\lambda_4(3g^2 + g'^2)$$

$$32\pi^2 \frac{d\lambda_5}{dt} = 4[2\lambda_5 + \lambda_1 + \lambda_2 + 2\lambda_4 + 4\lambda_3]\lambda_5 +$$
$$+ 6\lambda_5(h_t^2 + h_b^2) - 3\lambda_5(3g^2+g'^2)$$

$$32\pi^2 \frac{d\lambda_6}{dt} = [16\lambda_3 + 6\lambda_4 + 4(\lambda_1+\lambda_2) +$$
$$+ 6(h_t^2 + h_b^2) - 3(3g^2+g'^2)]\lambda_6$$

Analysis of these equations shows that the coupling constants behave differently with energy variation. The constants λ_1 , λ_2 at relatively small (0.2 ÷ 0.3) (depending on the value of m_t) initial (at M_W) values of λ_1 , λ_2 grow with energy. Whereas the constants λ_4 , λ_5 , λ_6 fall off with energy at large initial values (1 ÷ 3) and negative λ_3 . The constant λ_3 at large in magnitude (-1 ÷ (-3)) initial values grows with energy. It is worthy of note that the constants λ_i ($i = 3,\ldots, 6$) enter the right-hand side of the equations for λ_1 , λ_2 in the form of the expression:

$$A = 4\left[(\lambda_3 + \lambda_4)^2 + (\lambda_3 + \lambda_5)^2 + \lambda_6^2 \right]. \qquad (11)$$

This means that the initial values of λ_6 , $\lambda_3 + \lambda_4$, $\lambda_3 + \lambda_5$ cannot be too large; otherwise λ_1 , λ_2 will grow intensely with energy and will rapidly go out of the validity limits of the perturbation theory. Thus, if we believe that the validity condition of the perturbation theory must hold up to 10^{14} GeV, then these $|\lambda_6|$, $|\lambda_3 + \lambda_4|$, $|\lambda_3 + \lambda_5|$ must be smaller than 0.5.

It follows from the above-said that if we impose on the magnitudes of λ the restriction $|\lambda_i| < \pi$, then for masses $M^2_{H^-}$, $M^2_{H_1^0}$, $M^2_{H_3^0}$ we shall have:

$$M^2_{H^-} , \ M^2_{H_1^0} , \ M^2_{H_3^0} < 2\pi\eta^2. \qquad (12)$$

If a weaker condition $|\lambda_i| < 2\pi$ is imposed on λ_i , then the upper limit of M^2 in (12) will also increase by a factor of two. We can say that the renormalization group equations practically give no restrictions on masses of these Higgs fields:

the restriction (12) is the validity condition of the perturbation theory at energies of the order of M_W .

Now we turn to the study of restrictions on the mass of H_2^0 -field. The expression for the mass of this field can be somewhat simplified if we consider three limiting cases: (a) $a \gg b$, (b) $a \ll b$, (c) $a = b$. We'll obtain:

$$M^2_{H_2^0} = 4\lambda_1 a^2 + 2(\lambda_4 + \lambda_6) b^2 - -2\,\frac{\lambda_4 + \lambda_6 + 2\lambda_3}{\lambda_4 + \lambda_6 - 2\lambda_1}\, b^2 \qquad \text{if} \quad a \gg b \tag{13a}$$

$$M^2_{H_2^0} = 4\lambda_2 b^2 + 2(\lambda_4 + \lambda_6) a^2 - -2\,\frac{\lambda_4 + \lambda_6 + 2\lambda_3}{\lambda_4 + \lambda_6 - 2\lambda_2}\, a^2 \qquad \text{if} \quad a \ll b \tag{13b}$$

$$M^2_{H_2^0} = [\lambda_1 + \lambda_2 + 2(\lambda_3 + \lambda_4)]\,\eta^2 \qquad \text{if} \quad a = b \tag{13c}$$

$$\lambda_4 + 2\lambda_3 < 0$$

One can see from (13) that $M^2_{H_2^0}$ cannot take large values because the quantities λ_1 , λ_2 , $\lambda_3 + \lambda_4$ are small.

It should be emphasized that the analysis of equations (10) can be carried out also for the case when the initial conditions on the coupling constants λ_i are imposed not at the point M_W as we had done hitherto, but at the point Λ . Then at sufficiently large values of constants λ_3 , λ_4 , λ_5 (> 1) at some point E ($M_W < E < \Lambda$) an infrared pole arises: the constants increase unlimitedly when approaching that point. Presumably this implies that in this case only one Higgs field "survives" at low energies of the order of M_W. It is not excluded that the presence of this scale E corresponds to some additional intermediate symmetry.

Concrete numerical solution of renormalization group equations for constants λ_i (i = 1,..., 6) from M_W to 10^{14} GeV gives the restrictions from above on the mass H_2^0 that are similar to the one-Higgs field case we have considered. These restrictions are given in Fig. 6.

We can conclude that the renormalization group equations both for one and two Higgs fields give practically the same upper bounds on the mass of one of the neutral Higgs fields. Thus we can claim that if the standard model and the perturbation theory work up to the grand unification energy of 10^{14} GeV, then the mass of one of the neutral Higgs fields does not exceed 150 - 220 GeV depending on the t-quark mass and irrespective of the value of mass of the other Higgs bosons. If m_t < 180 GeV, then this mass too must be less than 180 GeV.

5. Supersymmetric Generalization of the Two Higgs Doublet Model

Our consideration can be generalized also for the supersymmetric case when the model contains two Higgs doublets. Equations for the coupling constants in this case are as follows:

$$32\pi^2 \frac{dg_1}{dt} = 13{,}2 g_1^3$$

$$32\pi^2 \frac{dg_2}{dt} = 2 g_2^3$$

$$32\pi^2 \frac{dg_3}{dt} = -6 g_3^2 \qquad (14)$$

$$32\pi^2 \frac{dh_t^2}{dt} = h_t^2 [12 h_t^2 + 2 h_b^2 - \frac{32}{3} g_3^2 - 6 g_2^2 - \frac{26}{15} g_1^2]$$

$$32\pi^2 \frac{dh_b^2}{dt} = h_b^2 [2 h_t^2 + 12 h_b^2 - \frac{32}{3} g_3^2 - 6 g_2^2 - \frac{14}{15} g_1^2]$$

We'll consider as before that constants $h_t = m_{t/a}$, $h_b = m_{b/b}$; ($a^2 + b^2 = \eta^2$) satisfy the conditions $h_t^2/4\pi < 1$, $h_b^2/4\pi < 1$ in the whole region from M_W to Λ . (The supersymmetry breaking effects are assumed to be of the order of M_W .) This, as before, gives a restriction on the mass m_t . The results for $\Lambda = 10^5,\ 10^{15},\ 1.2 \cdot 10^{19}$ GeV are presented in Fig. 7. One can see that in the supersymmetric case we obtain stronger restrictions on m_t and a/b .

Thus, our analysis of the renormalization group equations allows one to obtain definite restrictions for particle masses if the standard model and the perturbation theory hold up to grand unification energies M_X . The case with the symmetries intermediate between M_W and M_X will be considered elsewhere.

In conclusion, the authors are thankful to A.G. Sedrakyan for the useful discussion.

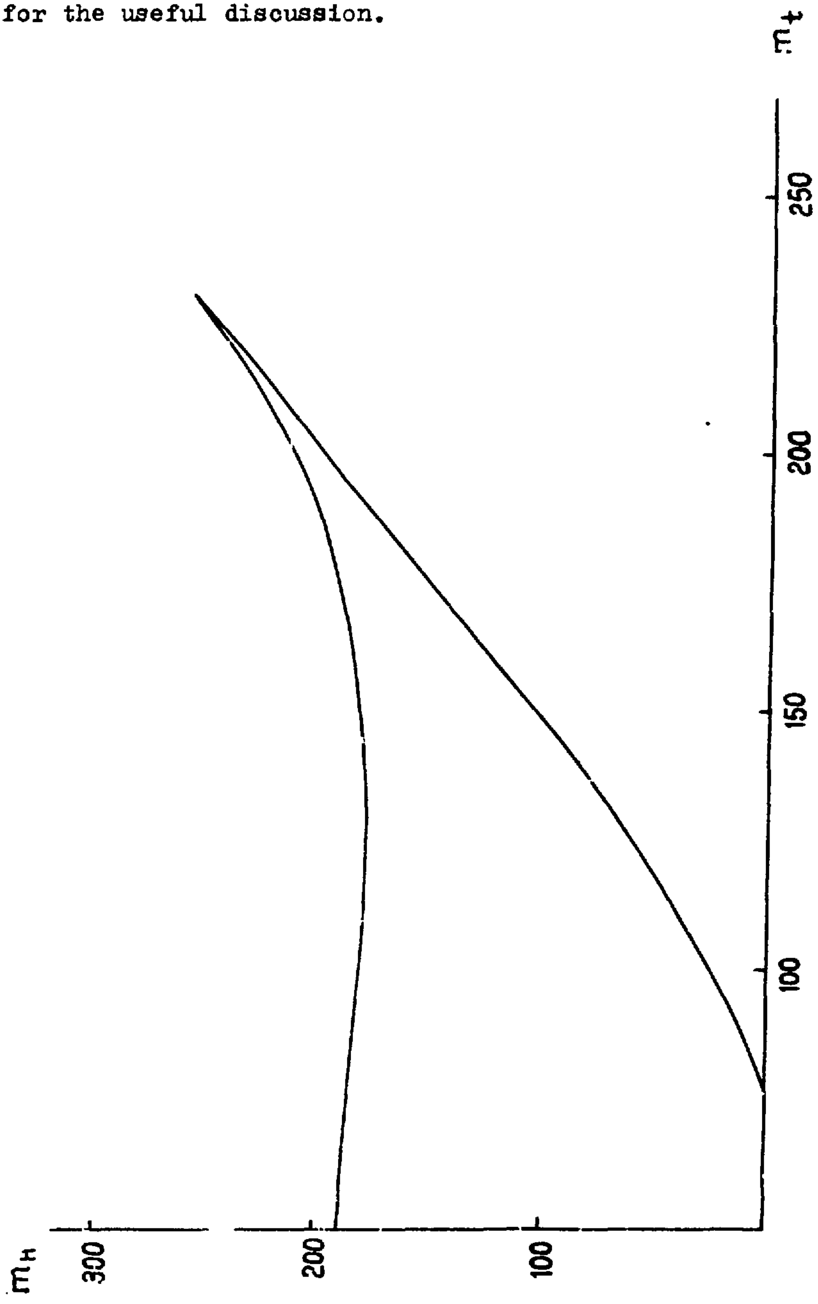

Fig.1

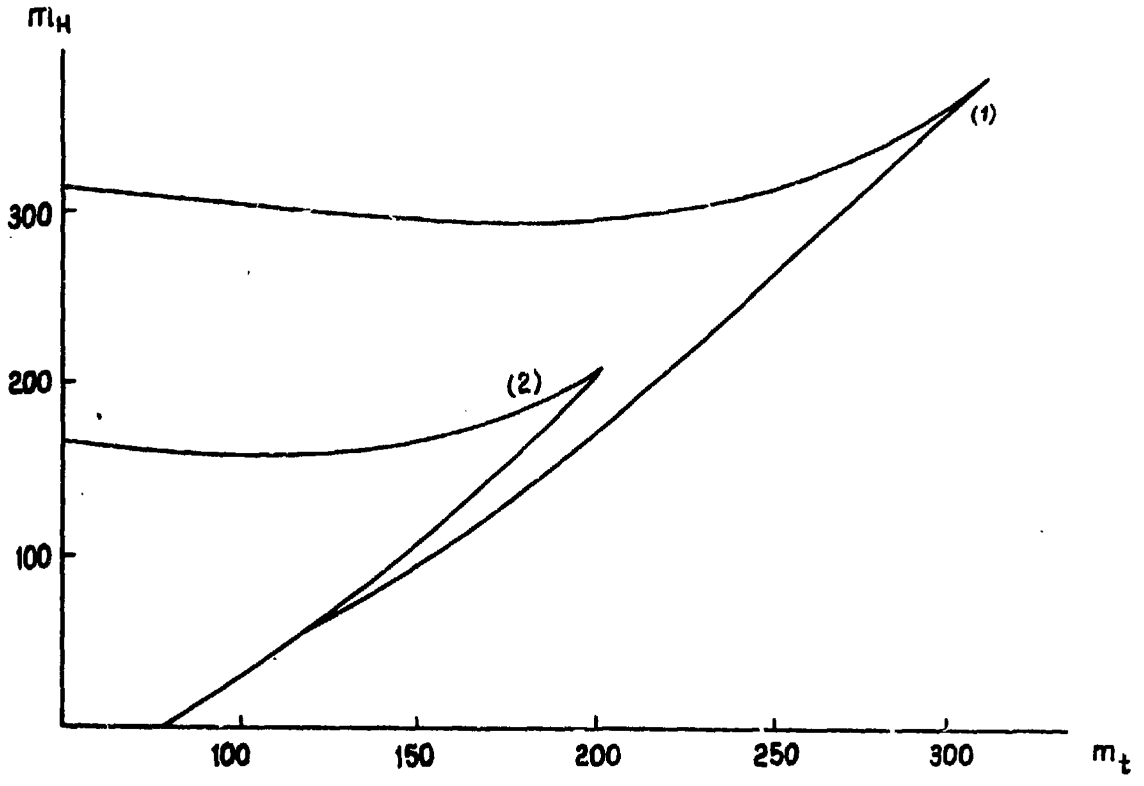

Fig.2

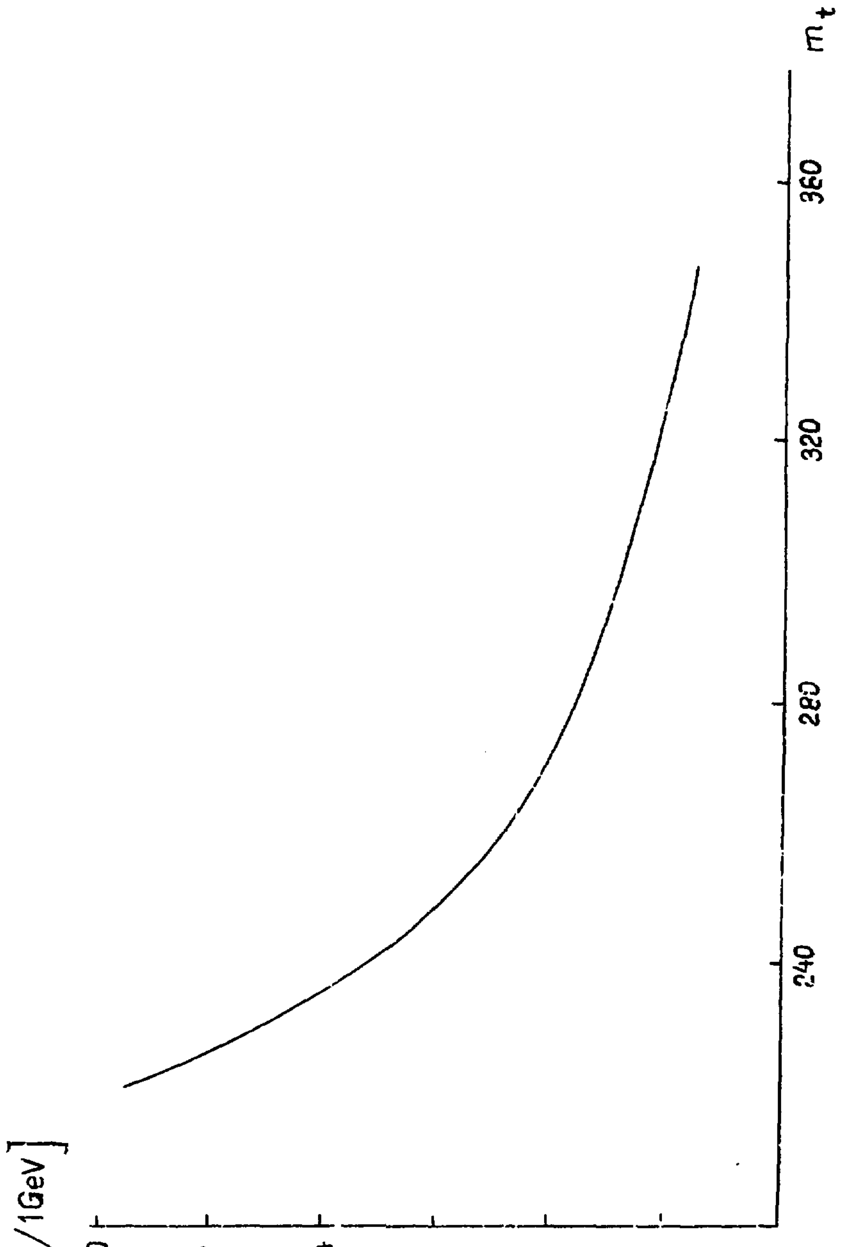

Fig.3

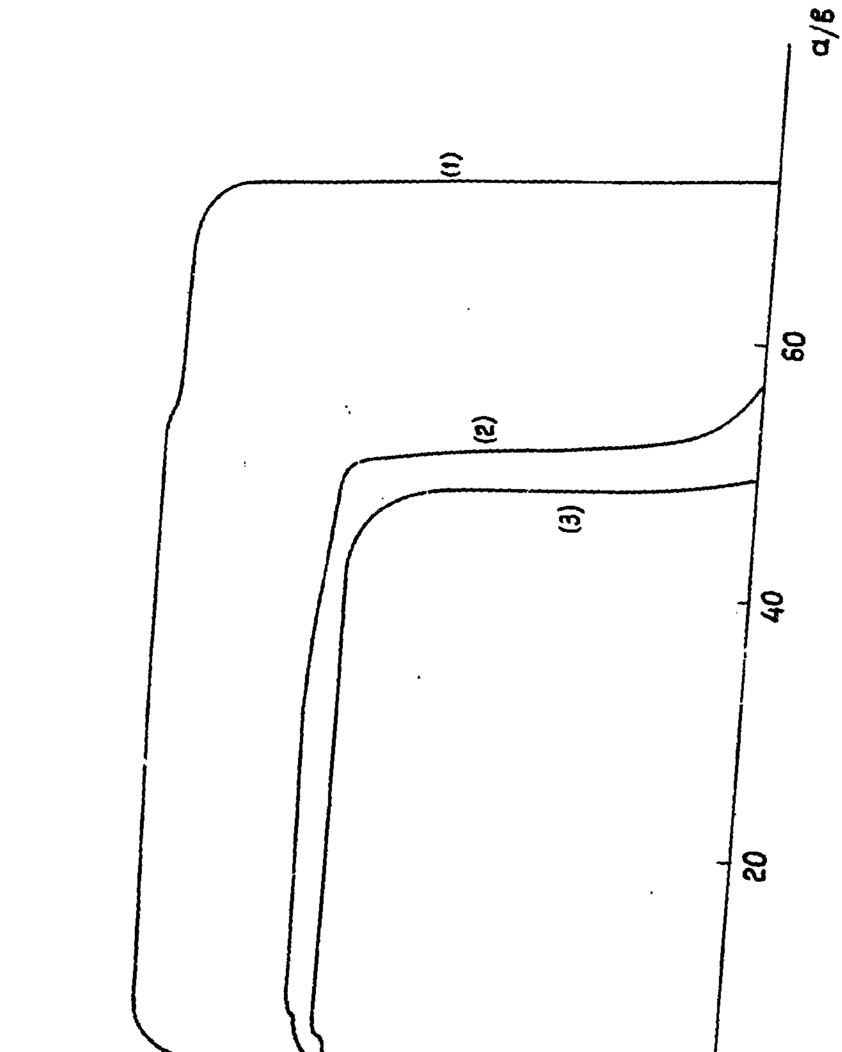

Fig.4

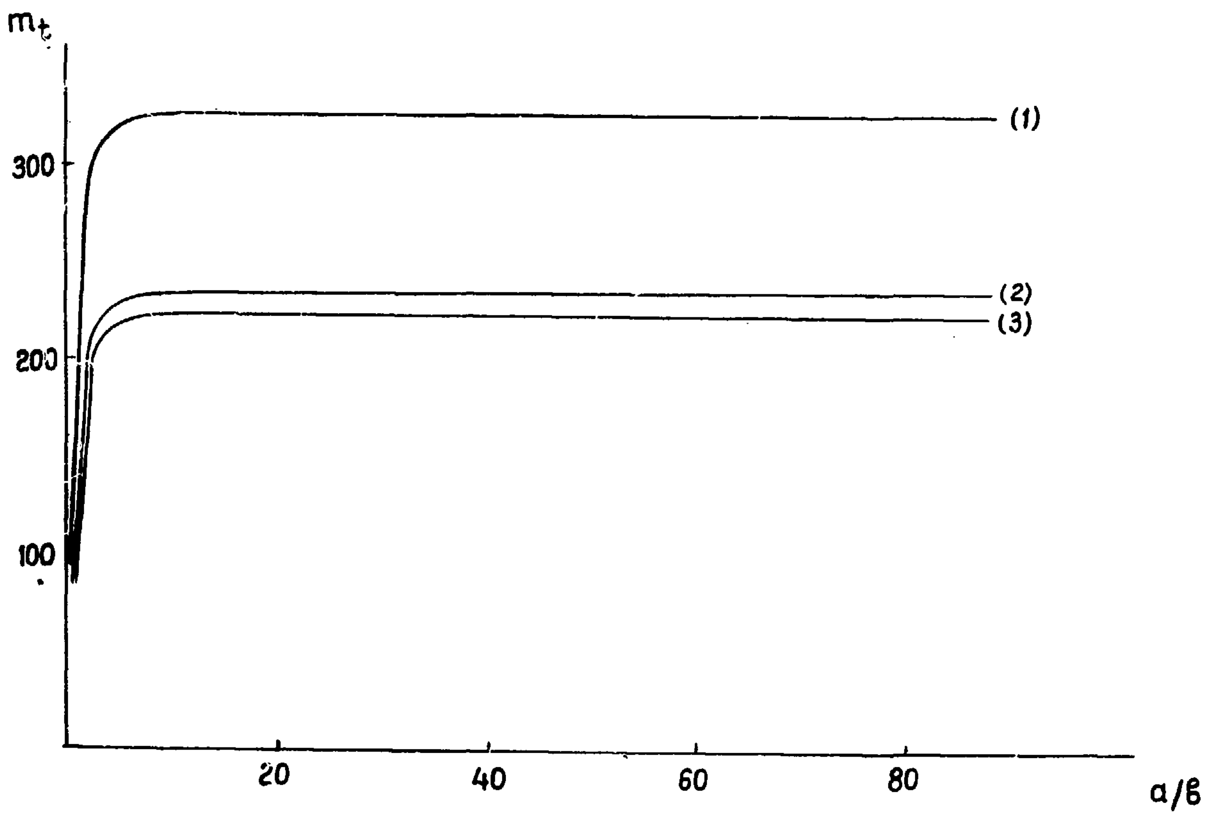

Fig.5

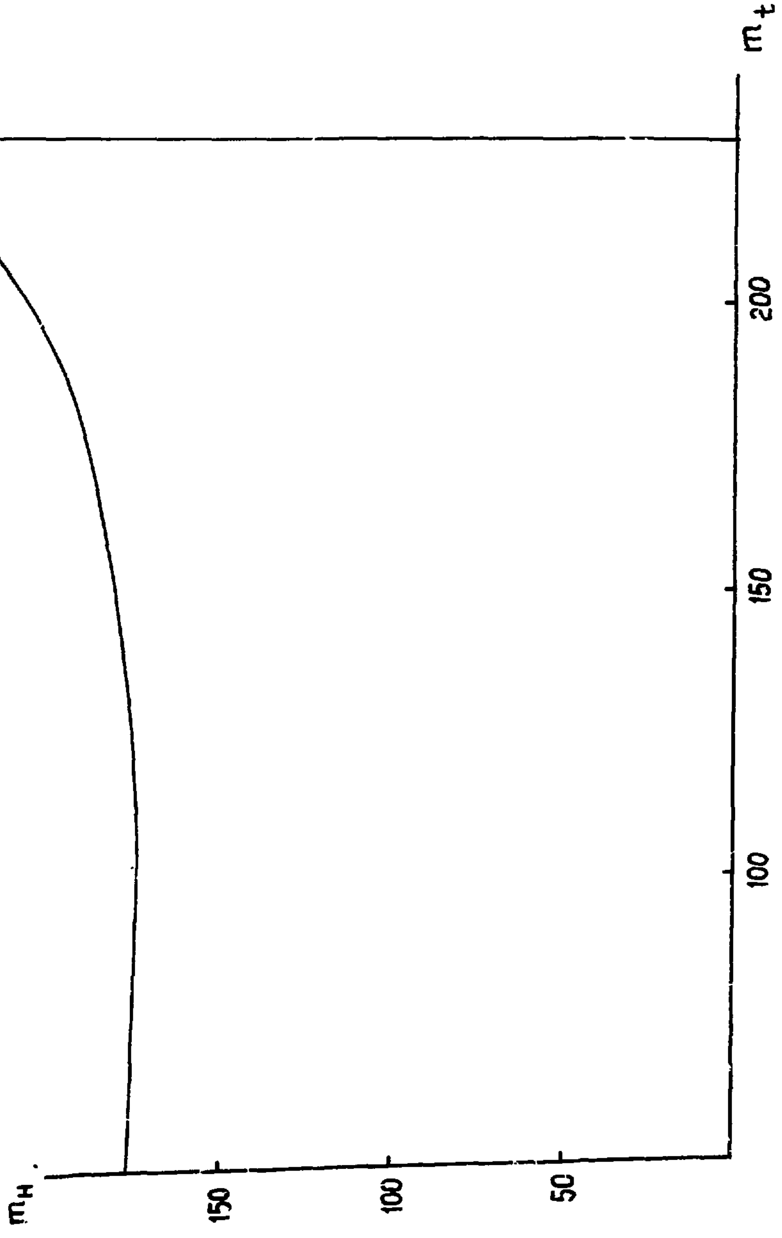

Fig.6

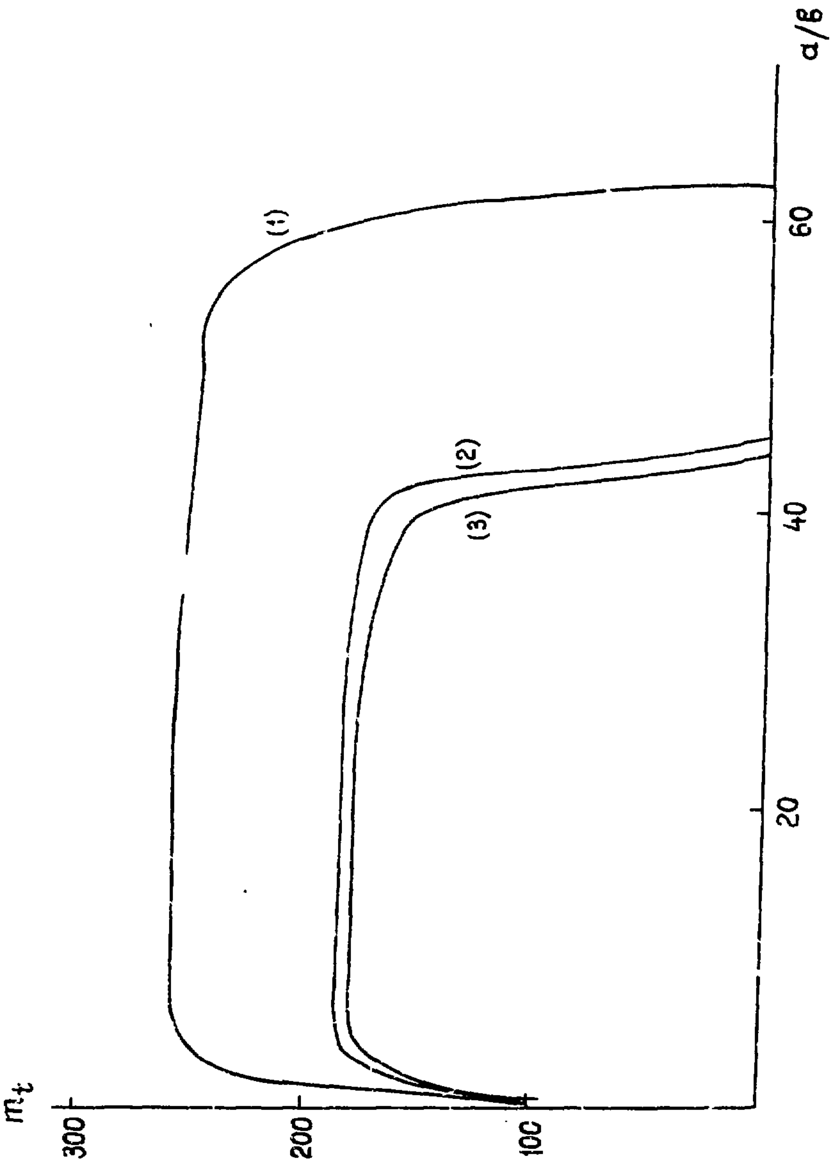

Fig.7

Figure Captions

Fig. 1. The region of admissible values of m_t, m_H for $\Lambda = 10^{14}$ GeV.

Fig. 2. The regions of admissible values of m_t, m_H for $\Lambda = 10^5$ GeV (curve 1), and $\Lambda = 1.2 \cdot 10^{19}$ GeV (curve 2).

Fig. 3. Restriction on m_t versus the values of $\lg \Lambda / 1$ GeV.

Fig. 4. The region of admissible values of the t-quark mass for case I versus a/b for $\Lambda = 10^5$ GeV (curve 1), for $\Lambda = 10^{14}$ GeV (curve 2), $\Lambda = 1.2 \cdot 10^{19}$ GeV (curve 3).

Fig. 5. The region of admissible values of the t-quark mass for case II versus a/b for $\Lambda = 10^5$ GeV (curve 1), $\Lambda = 10^{14}$ GeV (curve 2), $\Lambda = 1.2 \cdot 10^{19}$ GeV (curve 3).

Fig. 6. The region of admissible values of m_t, m_H in the two Higgs doublets case for $\Lambda = 10^{14}$ GeV.

Fig. 7. The region of admissible values of the t-quark mass in the supersymmetric case for $\Lambda = 10^5$ GeV (curve 1), $\Lambda = 10^{15}$ GeV (curve 2), $\Lambda = 1.2 \cdot 10^{19}$ GeV (curve 3).

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РЕНОРМГРУППОВЫЕ ИССЛЕДОВАНИЯ МАСС ТЯЖЕЛЫХ КВАРКОВ И ХИГГСОВСКИХ БОЗОНОВ

(на английском языке, перевод Асланян З.Н.)

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