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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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PECULIARITIES OF RESONATOR EXCITATION BY A TRAIN OF CHARGED PARTICLE BUNCHES

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ՌԵԶՈՆԱՏՈՐԻ ԳՐԳՌՄԱՆ ՅՈՒՐԱՀԱՏԿՈՒԹՅՈՒՆՆԵՐԸ ԼԻՑՔԱՎՈՐՎԱԾ ՄԱՍՆԻԿՆԵՐԻ ԹԱՆՁՐՈՒԿՆԵՐԻ ՀԱՋՈՐԴԱԿԱՆՈՒԹՅԱՄԲ

Քննարկված է գլանաձև ռեզոնատորի գրգռումը վերջնական լավորակությամբ՝ լիցքավորված թանձրուկների հաջորդականությամբ։ Ցույց է տրված, որ ռեզոնատորի լավագույնացումը ըստ էլեկտրական դաշտի երկայնակի բաղադրիչի մաքսիմումի, թույլ է տալիս ներսում ստեղծել 200 ՄէՎ/մ կարգի դաշտեր։ Գնահատված է արդյունավետ ճառագայթող թանձրուկների թիվը։

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Resonator excitation by high-charged electron bunches is the object of investigations of many authors [1,2]. This is not in the last place connected with the possibilities that arise in wake-field or two-beam accelerator schemes [3]. It should be noted that the process of a high-charged electron bunch interaction with resonators must be considered only in a self-consistent formulation of the problem [4], but in some cases one may obtain results that allow to reveal the characteristics of excitation and optimize resonator sizes over the maximum strength of the electric field longitudinal component, in the first stage disregarding the resonator field effect on the exciting bunches.

Let us consider a cylindrical resonator with radius R and height a, which along its oz axis is traversed by a train of cylindrical bunches with length L and radius b. Current density in resonator has the form

$$\vec{j} = \int_0^L \vec{j}\,(\xi)\,d\xi = \frac{\sigma_0\sigma(r)\vec{v}}{\pi b^2}\sum_{n=0}^{N-1}\int_0^L f(\xi)\delta(z+nd+\xi-vt) \qquad (1)$$

where σ_0 is linear charge density in a bunch, d is distance between the centres of two neighbouring bunches, ξ is the bunch element coordinate along its axis, $f(\xi)$ is the charge distribution function, n is ordinal number of a bunch, N is the number of bunches, $\sigma(r)=1$ at $r\le b$, $\sigma(r)=0$ at $r>b$. For the electric field z-component Fourier-component $E^{(\xi)}_{z,\omega}$ excited by the ξ-th element of the n-th bunch we have:

$$\Delta E_{z\omega}(\xi) + \frac{\omega^2}{c^2}\varepsilon E_{z\omega}(\xi) = \frac{4\pi i}{\omega\varepsilon}\left(\frac{\omega^2}{c^2} - \frac{\partial^2}{\partial z^2}\right) j_{z\omega}(\xi) \qquad (2)$$

where

$$j_{z\omega}(\xi) = \frac{\sigma_0 q(z)}{2\pi^2 b^2}\,(\Phi(\xi))\, e^{-i\omega/v(z+\xi+nd)} .$$

Solution of eq.(2) we have in the form of $E_{z,\omega}(\xi)$ and $j_{z,\omega}(\xi)$ expansion over the resonator eigenfunctions, and for the Fourier component of radiation fields we have:

$$E_{z,\omega}(\xi) = -\frac{4\sigma_0 v\Phi(\xi)}{\pi\varepsilon baR^2}\sum_{m,l}\frac{\varepsilon_l J_1(\lambda_{0m}b)J_0(\lambda_{0m}r)\cos\varkappa_l z}{\lambda_{0m}J_1^2(\lambda_{0m}R)}\times$$

$$\times\int\frac{\left(\frac{\omega^2}{c^2}\varepsilon - \varkappa_l^2\right)}{\left(\frac{\omega^2}{c^2}\varepsilon - \lambda_{0m}^2 - \varkappa_l^2\right)}\,\frac{\left(1^{-i(\frac{\omega}{v} - \varkappa_l)a} - 1\right)}{\omega^2 - \varkappa_l^2 v^2}\, e^{i\omega(t - \frac{\xi+nd}{v})}\cdot d\omega \qquad (3)$$

where $\lambda_{0m}=\mu_{0m}/R$, μ_{0m} is the m-th root of the zero-order Bessel function $\varkappa_l=\pi l/a$, $l=0,1,2,3...$ is the field model index that determines the number of variations along oz, m is the field mode index that determines the number of variations along resonator's radius, $\varepsilon_l=2$, $l\neq0$; $\varepsilon_l=1$, $l=0$. Due to the problem symmetry, $E_{z\omega}(\xi)$ is independent of the azimuth angle φ.

Integration over ω in (3) will be performed in the complex plane $\omega=\omega'+i\omega''$. The integrand in (3) has a singularity in the form of simple poles at frequencies that satisfy the dispersion equation

$$\omega = \omega'_{ml} = \pm\, c/\sqrt{\varepsilon}\,\sqrt{\frac{\mu_{0m}^2}{R^2} + \varkappa_l^2} \qquad (4)$$

of resonators placed on the real ω' axis.

As real resonators have a finite Q-factor, the ω_{ml} poles of

the integrand in (3) have the form

$$\omega_{ml} = \pm\omega'_{ml}\left(1 \pm \frac{i}{2Q}\right) \qquad (5)$$

and are shifted into the upper half-plane ω.

1. A Loss-Free Resonator

Closing the path of integration in the upper half-plane by a semicircle of an infinitely large radius with a following summation over all N bunches and integration over ξ, for an ideal resonator we eventually obtain

$$E_z(r,z,t) = -\frac{8qV}{\varepsilon baR^2}\sum_{m,l}\frac{\lambda_{0m}\varepsilon_l}{\omega_l}\,\frac{J_1(\lambda_{0m}b)J_0(\lambda_{0m}r)\cos\varkappa_l z}{J_1^2(\mu_{0m})[\lambda_{0m}^2+\varkappa_l^2(1-\beta^2\varepsilon)]}\times$$

$$\times\frac{\sin\frac{\omega_{ml}L}{2V}}{\frac{\omega_{ml}L}{2V}}\,\frac{\sin\left(N\frac{\omega_{ml}d}{2V}\right)}{\sin\frac{\omega_{ml}d}{2V}}\left[e^{i\varkappa_l a}\sin\omega_{ml}\left(t - \frac{2a+(N-1)d+L}{2V}\right) -\right.$$

$$\left.- \sin\omega_{ml}\left(t - \frac{(N-1)d+L}{2V}\right)\right] \qquad (6)$$

at

$$t > \frac{(N-1)d+L+a}{V},$$

$$E_z(r,z,t) = -\frac{8qV}{\varepsilon baR^2}\sum_{m,l}\frac{\varepsilon_l\, J_1(\lambda_{0m}b)J_0(\lambda_{0m}r)\cos\varkappa_l t}{\lambda_{0m}\, J_1^2(\mu_{0m})[\lambda_{0m}^2+\varkappa_l^2(1-\beta^2\varepsilon)]}\times$$

$$\times\left[\frac{\varkappa_l(\beta^2\varepsilon-1)}{V}\,\frac{\sin\frac{\varkappa_l L}{2}}{\frac{\varkappa_l L}{2}}\,\frac{\sin\frac{\varkappa_l nd}{2}}{\sin\frac{\varkappa_l d}{2}}\cdot\sin\varkappa_l\left(Vt - \frac{L+(n-1)d}{2}\right) -\right.$$

$$- \frac{\lambda^2_{0m}}{\omega_{ml}} \frac{\sin \frac{\omega_{ml} L}{2V}}{\frac{\omega_{ml} L}{2V}} \frac{\sin \frac{n\omega_{ml} d}{2V}}{\sin \frac{\omega_{ml} d}{2V}} \sin \omega_{ml} \left[t - \frac{L+(n-1)d}{2V} \right] \Big] \qquad (7)$$

at

$$0 < t < \frac{a+L+(n-1)d}{V}$$

When deriving the formulae (6) and (7), we assumed that the charge was distributed uniformly in the bunches, and $f(\xi)=1$. Equation (6) describes the radiation field in the case, when all N bunches have passed through the resonator, and eq.(7) - in the case, when first n bunches are in the resonator. At the moment of time $t=\frac{a+L+(N-1)d}{V}$ both equations conform. As expected, the radiation spectrum is a discrete one which lacks frequencies that meet the conditions

$$\omega_{ml} = \frac{2\pi Vs}{L} \quad \text{and} \quad \omega_{ml} = \frac{2\pi Vs}{Nd}, \qquad s = 0,1,2,... \qquad (8)$$

for $t > \frac{(N-1)d+L+a}{V}$.

When the condition

$$\omega_{ml} = \frac{2\pi kV}{d}, \qquad k = 1,2,3... \qquad (9)$$

is met, the $f_{ml}=\omega_{ml}/2\pi$ frequency of resonator's free oscillations turn out equal to the k-th harmonic of the bunch repetition rate $f=V/d$, and in the resonator mode, the indices of which meet the condition (9), the field strength increases sharply, and in the case when all N bunches have passed through the resonator, it is equal to

$$E_{zml}(r,z,t) = NE^0_{zml}(r,z,t), \qquad (10)$$

where $E^0_{zml}(r,z,t)$ is radiation field of a single bunch.

So, when the condition (9) is met at a corresponding eigen-

frequency of resonator, the resonance condition is also satisfied, and all the bunches of a beam excite a field in the resonator, which will be proportional to the number of bunches.

The condition (9) is satisfied for a variety of cylindrical resonators with radius

$$R_{m,l,k} = \frac{\mu_{0m}}{\pi\sqrt{\frac{4\beta^2\varepsilon k^2}{d^2} - \frac{l^2}{a^2}}}, \qquad (11)$$

whence follows a restriction on the number of modes excited in a resonator:

$$l < \frac{2\beta\sqrt{\varepsilon}ak}{d} \qquad (12)$$

When the condition

$$\frac{2\beta\sqrt{\varepsilon}ak}{d} < 1 \qquad (13)$$

is satisfied, there can be excited only the modes the l index of which is equal to zero.

In the case when the resonator height is chosen under the condition (13) and the radius is

$$R_{m,0,k} = \frac{\mu_{0m} d}{2\pi\beta\sqrt{\varepsilon}k}, \qquad (14)$$

then in such a resonator the beam radiation will occur in a single mode. In this case, only $E_z(r,t)$ and $H_\varphi(r,t)$ field components will be other than zero. The field strength in this case is expressed by:

$$E_{z,m}(r,t) = - \frac{16\pi qN\beta\sqrt{\varepsilon}k}{\sqrt{\varepsilon}bd} \frac{J_1(\frac{2\pi k\beta\sqrt{\varepsilon}}{d} b) J_0(\frac{2\pi k\beta\sqrt{\varepsilon}}{d} r)}{\mu^2_{0m} J^2_1(\mu_{0m})} \frac{\sin \frac{\pi kL}{d}}{\frac{\pi kL}{d}} \times$$

$$\times \frac{\sin \frac{\pi k a}{d}}{\frac{\pi k a}{d}} \cos \frac{2\pi k V}{d} \left(t - \frac{L+a}{2V}\right) \quad \text{at} \quad t > \frac{(N-1)d+L+a}{V} . \qquad (15)$$

The choice of the resonance excitation condition and optimization of resonator parameters allows us in case of a loss-free resonator to obtain a field of the order of GV/m. For a beam with parameters given in [3], the account of finite Q-factor affects the results essentially.

2. The Finite Q-Factor Resonator

When deriving the formulae (6) and (7) the Q-factor of resonator is assumed to be infinitely large. For a real resonator the Q-factor is a finite quantity and its account changes the results of integration over ω in eq.(3). In this case the eq.(6) takes the form

$$E_z(r,z,t) = - \frac{8qV}{\varepsilon b a R^2} \sum_{m,l} \frac{\varepsilon_l J_1(\lambda_{0m} b) J_0(\lambda_{0m} r) \cos \varkappa_l z}{\omega_{ml} \lambda_{0m} J_1^2(\mu_{0m})} \times$$

$$\mathrm{Re}\left[\frac{\left(\lambda_{0m}^2 \left(i - \frac{1}{Q}\right) - \frac{\varkappa_l^2}{Q}\right)}{\lambda_{0m}^2 \left(1 + \frac{i}{Q}\right) + \varkappa_l^2 \left(1 - \beta^2 \varepsilon + \frac{i}{Q}\right)} \frac{\sin \frac{\omega_{ml} dN}{2V}\left(1 + \frac{i}{2Q}\right)}{\sin \frac{\omega_{ml} d}{2V}\left(1 + \frac{i}{2Q}\right)} \frac{\sin \frac{\omega_{ml} L}{2V}\left(1 + \frac{i}{2Q}\right)}{\frac{\omega_{ml} L}{2V}\left(1 + \frac{i}{2Q}\right)} \times \right.$$

$$\left. \left(e^{i\left[\varkappa_l - \frac{\omega_{ml}}{V}\left(1 + \frac{i}{2Q}\right)\right] a} - 1 \right) e^{i\omega_{ml}\left(1 + \frac{i}{2Q}\right)\left(t - \frac{(N-1)d+L}{2V}\right)} \right] \qquad (16)$$

If $Q \to \infty$, then (16) turns into (6). For resonators with sizes satisfying the condition (13), the formula (16) is simplified, and as mentioned above, there can exist only the $E_z(r,t)$ and $H_\varphi(r,t)$ components of the electric and magnetic fields:

$$E_z(r,t) = - \frac{8qV}{\varepsilon b R^2} \frac{J_1(\lambda_{0m} b) J_0(\lambda_{0m} r)}{\lambda_{0m} J_1^2(\mu_{0m})} \frac{\sin \frac{\omega_{ml} L}{2V}}{\frac{\omega_{ml} L}{2V}} \frac{\sin \frac{\omega_{ml} a}{2V}}{\frac{\omega_{ml} a}{2V}} \times$$

$$\times \frac{e^{-\frac{\omega_m}{2Q}\left(t - \frac{L+a+(N-1)d}{V}\right)}}{\left(1 - 2\cos \frac{\omega_m d}{V} e^{-\frac{\omega_m d}{2QV}} + e^{-\frac{\omega_m d}{2QV}}\right)} \left[\left(\cos \frac{\omega_m d}{V} + \sin \frac{\omega_m d}{V} - e^{-\frac{\omega_m d}{2QV}}\right) \times \right.$$

$$\times \left\{\cos \omega_m \left(t - \frac{L+a}{2V} - \frac{Nd}{V}\right) - \cos \omega_m \left(t - \frac{L+a}{2V}\right) e^{-\frac{\omega_m d}{2QV}}\right\} - \qquad (17)$$

$$- \left(\sin \frac{\omega_m d}{V} - \frac{1}{2Q} \cos \frac{\omega_m d}{V} + \frac{1}{2Q} e^{-\frac{\omega_m d}{2QV}}\right) \times$$

$$\left. \times \left\{\sin \omega_m \left(t - \frac{L+a}{2V} - \frac{Nd}{V}\right) - \sin \omega_m \left(t - \frac{L+a}{2V}\right) e^{-\frac{\omega_m dN}{2QV}}\right\}\right] .$$

As it follows from eqs.(16) and (17), the account of finite Q leads to appearance of transient terms in the expressions of radiation fields, which essentially lower the field amplitudes. This lowering is due to energy dissipation in resonator walls. Nevertheless, when the resonator conditions of beam energy extraction are met, there can be created strong fields in a resonator. So, when $\omega_m = 2\pi k V/d$ condition is satisfied, for radiation fields we obtain:

$$E_{z,m}(r,t) = -\frac{16qQ\beta}{\sqrt{\varepsilon}bd}\,\frac{J_1\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}b\right)J_0\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}r\right)}{\mu_{0m}^2\,J_1^2(\mu_{0m})}\,\frac{\sin\frac{\pi kL}{d}}{\frac{\pi kL}{d}}\times \tag{18}$$

$$\times\frac{\sin\frac{\pi ka}{d}}{\frac{\pi ka}{d}}\left(1-e^{-\frac{\pi kN}{Q}}\right)\cos\pi k\left(\frac{2Vt}{d}-\frac{L+a}{d}\right)e^{-\frac{\pi kV}{Qd}\left(t-\frac{L+a+(N-1)d}{V}\right)}.$$

Figs.1-3 show the radiation field dependences $E_z(r,t)$ of N bunches on the axis of a cavity (ε=1) resonator as a function of time, for various m and k. The field is induced by a linac beam [5] that has the following parameters: bunches repetition rate f=3GHz, b=0.5cm, L=1cm, number of particles in a bunch $n=5\cdot10^9$, number of bunches in a pulse $N=3\cdot10^4$. The L-length transit time of a bunch is laid off as abscissa. The initial time is chosen from the condition that the accelerating field is optimal when the test (accelerated) bunch is in the resonator. As is seen, the field decreases with increasing resonator mode index m. This is due to increasing the corresponding resonator radius defined from eq.(14). The plots correspond to the optimal resonator height, at which the field in resonator is maximal. Fig.1 illustrates the case when the resonator frequency is equal to the bunch repetition rate in the beam, and in Figs.2-3 - to a double and a triple bunch repetition rate. Though the Q-factor in the first case is 1.5 times higher, nevertheless, due to the fact that the resonator radius at k=1 is twice as large as when k=2, the radiation field value average during the transit time, in case of k=2 is somewhat larger than when k=1. Thus, to generate accelerating fields in resonators by a train of bunches, the optimal resonator parameters should be: at k=1 R=3.83cm, a=2.75cm; at k=2 R=1.91cm,

a=1.275cm; at k=3 R=1.275cm, a=0.7cm. In case of the beam considered ($\rho=10^{-3}$k/m^3), for these resonator parameters the resonator field value average during the test bunch transit time is equal to:

at k=1 m=1 $\langle E_z\rangle$ = 205 MV/m

at k=2 m=1 $\langle E_z\rangle$ = 228 MV/m

at k=3 m=1 $\langle E_z\rangle$ = 198 MV/m

The fields can be increased using narrower beams that consist of denser bunches.

As follows from (18), the account of finite Q limits N number of efficiently radiating bunches. Thus, at $N>Q/\pi k$ the value of the resonator field practically is independent of the number of bunches passing through the resonator.

The role of the next bunches is to support the field induced by the preceding $N\sim Q/\pi k$ bunches. Figs.4-6 show the dependence of radiation field values average during the test bunch transit time as a function of N number of bunches that pass through the resonator, for various m and k. At $N=3q/\pi k$ the field is more dependent on N. As the Q-factor increases with m number of harmonics, then the value of N, at which saturation occurs, also increases. As indicated above, the number of bunches at which saturation occurs, decreases with increasing k (c.f. Figs.4-6). The number of efficiently radiating bunches can be increased by improving the Q-factor. Thus, when $Q=1/3\pi kN$, η=100%, all N bunches radiate effectively, and the storage energy increases in the resonator.

In a beam consisting of N bunches, efficiently radiate only

$$\eta = 3Q/\pi kN\cdot 100\% \tag{19}$$

percents of bunches. At k=1, m=1 it makes η_1=42%; at k=2, m=1

η_2=14%; at k=3, m=1 η_3=11%.

Thus, the taking of resonator Q-factor into account allows us to optimize the number of efficiently radiating bunches and decrease essentially the pulse width of the radiating bunch.

3. Energy Relations and Some Problems of Acceleration

As it is shown above, when the resonator conditions are met, a train of N bunches passing through a resonator create in it fields of $\langle E_z \rangle \sim 200$ mV/m. This allows to use a structure combined of a sequence of resonators as an accelerating system. The average acceleration of relativistically moving particles is defined as

$$\langle w_z \rangle = q \langle E_z \rangle / m_0 \ . \tag{20}$$

For the γ-factor change at unit length we have:

$$d\gamma/dz = \langle w_z \rangle / c^2 = q \langle E_z \rangle / m_0 c^2 \ , \tag{21}$$

which corresponds to the following acceleration rate

$$d\mathcal{E}/dz = m_0 \langle w_z \rangle \ . \tag{22}$$

For the beam parameters presented above, the values of w_z and $d\varepsilon/dz$ are:

$$\begin{array}{lll} k=1,\ m=1 & \langle w_z \rangle = 3.6\ 10^{19}\ m/c^2 & d\mathcal{E}/dz = 205\ \text{MeV/m} \\ k=2,\ m=1 & \langle w_z \rangle = 4.0\ 10^{19}\ m/c^2 & d\mathcal{E}/dz = 228\ \text{MeV/m} \\ k=3,\ m=1 & \langle w_z \rangle = 3.47\ 10^{19}\ m/c^2 & d\mathcal{E}/dz = 197\ \text{MeV/m} \end{array} \tag{23}$$

These estimates confirm the possibility of using resonators assembled from a sequence of resonators as an accelerating system.

Let us find the energy accumulated in a resonator after N bunches having passed through it, when the conditions (9) and (14) of resonance energy extraction from beam are met. Now calculate the energy loss of each bunch element in the form of radiation in resonator, then sum up these losses over all bunches:

$$\mathcal{E} = \int_0^L d\xi \int_0^{\frac{(N-1)d+a+L}{V}} dt \int_0^b j(\xi) E_z(r,t) r dr \ .$$

With account of (1) and (18) we obtain:

$$\mathcal{E} = - \frac{8q^2 Q^2 a}{\pi^2 \varepsilon^2 b^2 k^2} \frac{I_1^2\left(\frac{2\pi k \beta \sqrt{\varepsilon}}{d} b\right)}{\mu_{0m}^2 \, I_1^2(\mu_{0m})} \frac{\sin^2 \frac{\pi k L}{d}}{\left(\frac{\pi k L}{d}\right)^2} \frac{\sin^2 \frac{\pi k a}{d}}{\left(\frac{\pi k a}{d}\right)^2} \times$$

$$\times \left[1 - e^{-\frac{\pi k N}{Q}}\right]^2 e^{-\frac{2\pi k V}{Q d}\left(t - \frac{(N-1)d+L+a}{V}\right)} \ . \tag{24}$$

The work of bunch trains is negative, i.e. they give energy to the resonator. Thus, when the resonance conditions are satisfied, each next bunch increases the energy accumulated in the resonator.

It should be noted, that despite of large $\langle w_z \rangle$, the velocity of bunches at passage through the resonator is low.

Really, it follows from relativistic motion equations that

$$dV/dz = w_z / \gamma^3 c \ , \tag{25}$$

which for the cases considered makes less than 10^{-5}% of bunch entrance velocity. When calculating the radiation fields and resonator's work on bunches, this circumstance allows us to use expressions of current for the case of uniform motion of bunches. Taking of velocity variation into account when deriving the formula (18), leads to appearance of negligible terms of order of $c^2/2w_z\gamma_0 \sim 10^{-5}$ in the cosine argument.

Let after a train of N bunches a test bunch of radius b_1, linear density σ_{01}, in $t=d_1/V$ enter the resonator, just after the last N-th bunch. The resonator field work A on this bunch is defined as

$$A = -\frac{16qq_1Qa}{\pi\varepsilon bb_1k}\,\frac{I_1\!\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}\,b\right) I_1\!\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}\,b_1\right)}{\mu_{0m}^2\, I_1^2(\mu_{0m})}\,\frac{\sin^2\frac{\pi ka}{d}}{\left(\frac{\pi ka}{d}\right)^2}\,\frac{\sin\frac{\pi kL}{d}}{\frac{\pi kL}{d}} \times \tag{26}$$

$$\times\left[1-e^{-\frac{\pi kN}{\Omega}}\right]\cos\frac{2\pi k}{d}\,d_1 ,$$

where d_1 is distance between the centres of the N-th and the test bunches, $q_1=\sigma_0 L$, $L_1 \ll d/\pi k$ (the bunch length is negligible).

When the condition

$$d_1 = \frac{(2s+1)}{4k}\,d , \qquad s = 0,1,2... \tag{27}$$

is satisfied, the work on that bunch is equal to zero.

When d_1 meets the condition

$$3/4 + s \le kd_1/d \le 1/4+s , \tag{28}$$

the work of resonator field on the test bunch is negative, and it gives energy to the resonator. When the condition

$$1/4 + s < kd_1/d < 3/4+s , \tag{29}$$

is satisfied, and if $\cos 2\pi kd_1/d = -1$ is substituted into (26), the work of resonator field on the test bunch becomes positive,

and it receives energy in the resonator, the maximum value of which is defined by (26).

The rate of acceleration of the test bunch, the radius of which is small,

$$b_1 \ll d/2\pi k\beta_0\sqrt{\varepsilon} , \tag{30}$$

is

$$\frac{d\mathcal{W}}{dz} = \frac{16qQ\beta\sqrt{\varepsilon}}{\varepsilon bd}\,\frac{J_1\!\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}\,b\right)}{\mu_{0m}^2\, I_1(\mu_{0m})}\,\frac{\sin^2\frac{\pi ka}{d}}{\left(\frac{\pi ka}{d}\right)^2} \times \tag{31}$$

$$\times\frac{\sin\frac{\pi kL}{d}}{\frac{\pi kL}{d}}\left[1-e^{-\frac{\pi kN}{\Omega}}\right] .$$

The numerical calculations of the beam with parameters mentioned above yield

k=1,	m=1,	a = 2.75 cm,	$d\mathcal{W}/dz$ = 224.18 MeV/m	
k=2,	m=1,	a = 1.275 cm,	$d\mathcal{W}/dz$ = 299.1 MeV/m	(32)
k=3,	m=1,	a = 0.7cm,	$d\mathcal{W}/dz$ = 298.6 MeV/m	

The test charge acceleration rates given are a little higher than those presented in (23). This is due to the fact that in the calculations the test bunch size is assumed negligible, $L \ll d/\pi k$, which correspondingly shortens the bunch transit time. Here the origins and the tails of the plots 1-3 are cut off, and the mean value of accelerating fields appears higher than in (23).

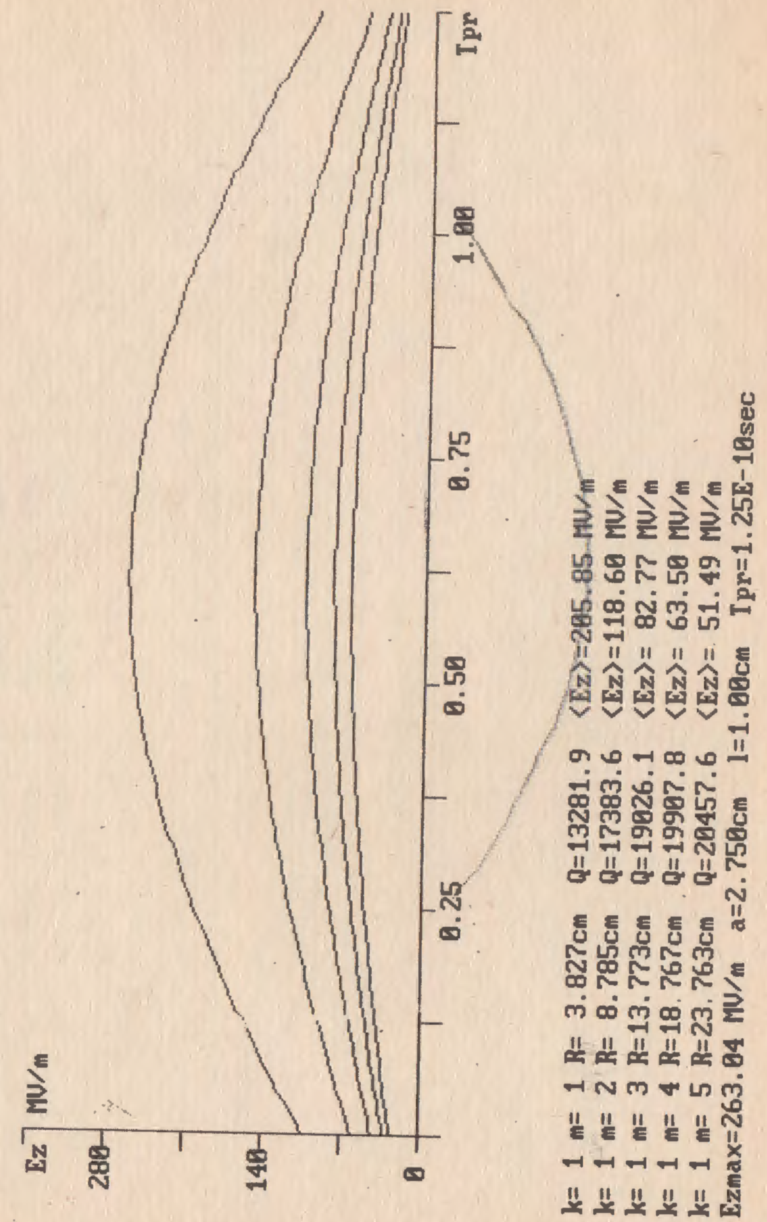
Being accelerated in a resonator system, a bunch carries part of the energy accumulated in the resonator.

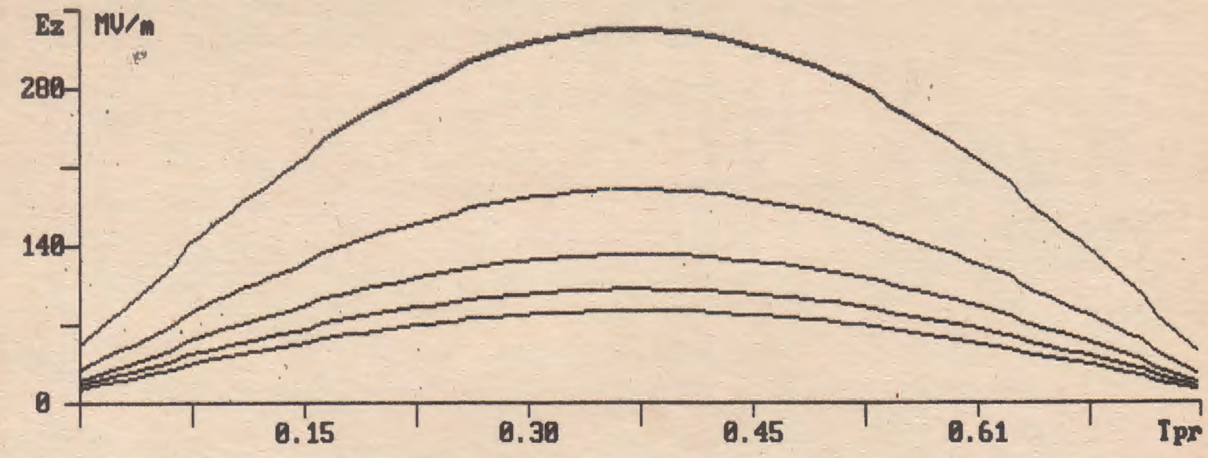
The expressions (24) and (31) allow to estimate the maximum charge of the bunch ($L_1 \ll \pi k/d$), which being accelerated, takes away the whole energy accumulated in the resonator. The charge of that bunch is:

$$q_1 = \frac{qQ}{2\pi k}\,\frac{b_1}{b}\,\frac{J_1\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}\,b\right)}{J_1\left(\frac{2\pi k\beta\sqrt{\varepsilon}}{d}\,b_1\right)}\,\frac{\sin\frac{\pi kL}{d}}{\frac{\pi kL}{d}}\left(1-e^{-\frac{\pi kN}{Q}}\right). \qquad (33)$$

This result allows us to affirm, that the method proposed makes it possible to obtain high-current accelerated beams.

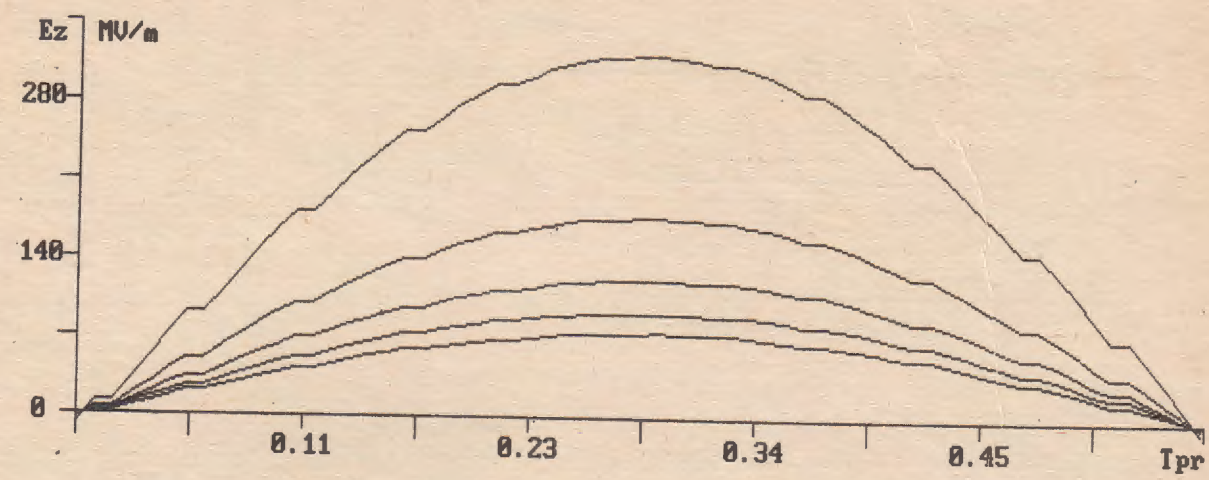
It should be noted in conclusion, that the values of the resonator electric field longitudinal component presented above give reason to hope that further investigations (self-consistent problem, bunch stability, etc.) will offer an experiment with a highly efficient two-beam acceleration circuit.





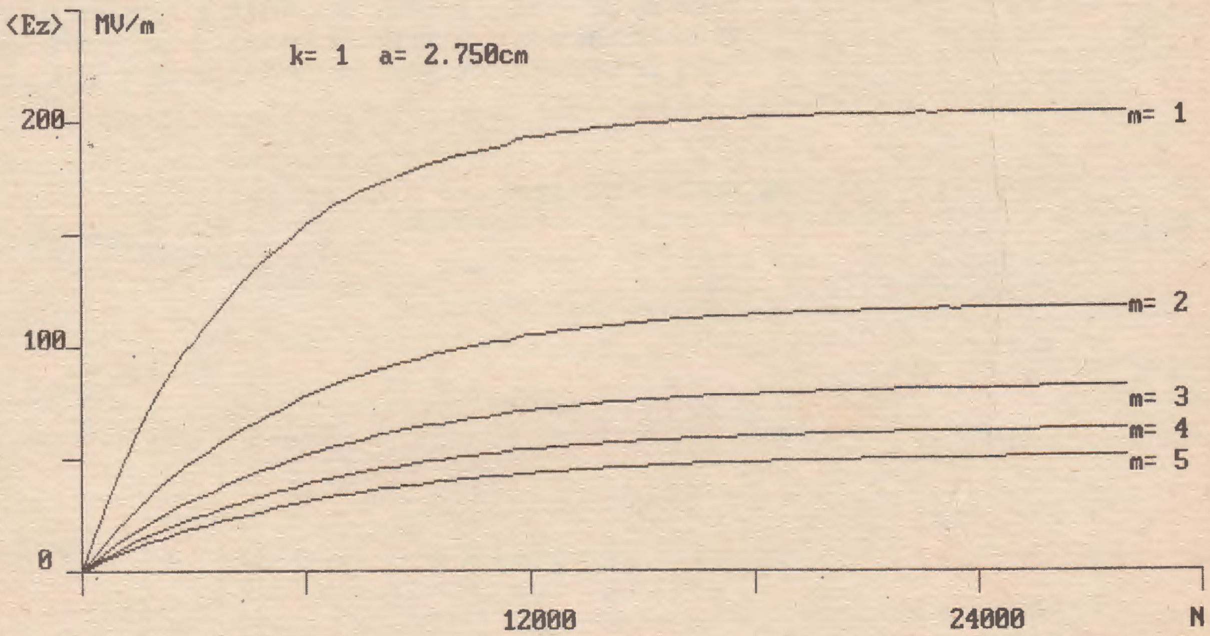
k= 2 m= 1 R= 1.914cm Q= 8981.8 <Ez>=228.74 MV/m
k= 2 m= 2 R= 4.393cm Q=11599.2 <Ez>=130.51 MV/m
k= 2 m= 3 R= 6.886cm Q=12627.9 <Ez>= 90.84 MV/m
k= 2 m= 4 R= 9.383cm Q=13175.6 <Ez>= 69.61 MV/m
k= 2 m= 5 R=11.882cm Q=13515.6 <Ez>= 56.41 MV/m
Ezmax=332.22 MV/m a=1.275cm l=1.00cm Tpr=7.58E-11sec

Fig.2



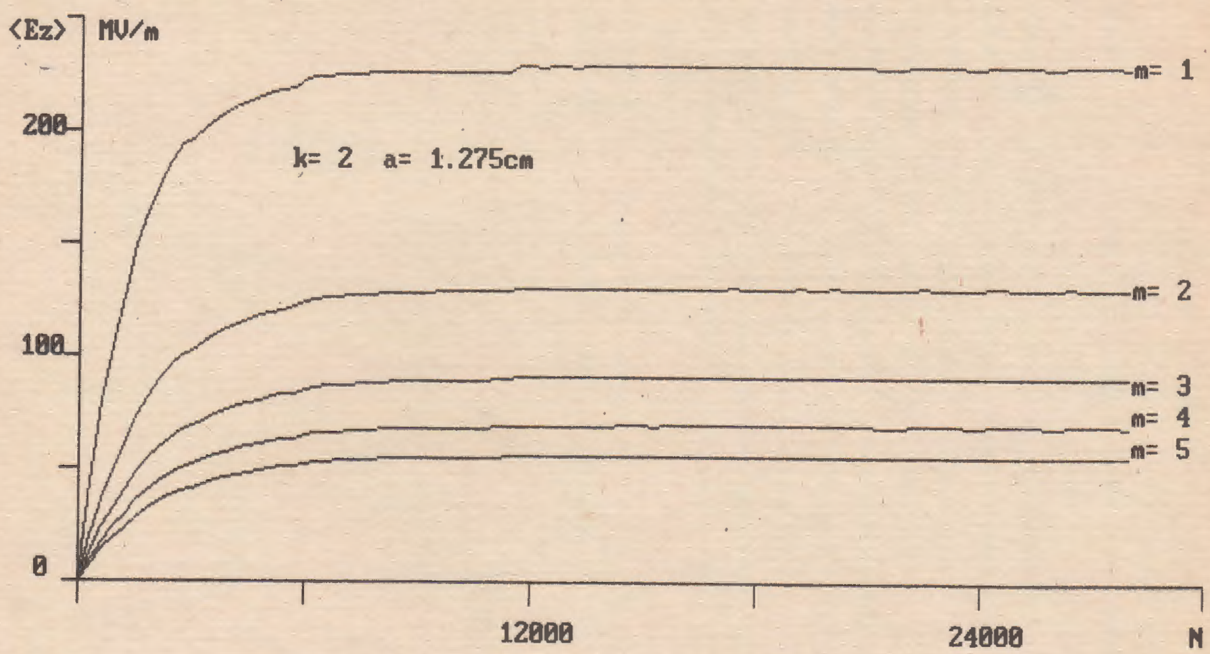
k= 3 m= 1 R= 1.276cm Q= 6497.9 <Ez>=198.27 MV/m
k= 3 m= 2 R= 2.928cm Q= 8121.8 <Ez>=109.49 MV/m
k= 3 m= 3 R= 4.591cm Q= 8731.8 <Ez>= 75.26 MV/m
k= 3 m= 4 R= 6.256cm Q= 9050.5 <Ez>= 57.29 MV/m
k= 3 m= 5 R= 7.921cm Q= 9246.1 <Ez>= 46.24 MV/m
Ezmax=321.63 MV/m a=0.700cm l=1.00cm Tpr=5.67E-11sec

Fig.3



<Ez> MV/m
k= 1 a= 2.750cm
200
100
0
12000
24000
N
m= 1
m= 2
m= 3
m= 4
m= 5

Fig.4



<Ez> MV/m
k= 2 a= 1.275cm
200
100
0
12000
24000
N
m= 1
m= 2
m= 3
m= 4
m= 5

Fig.5

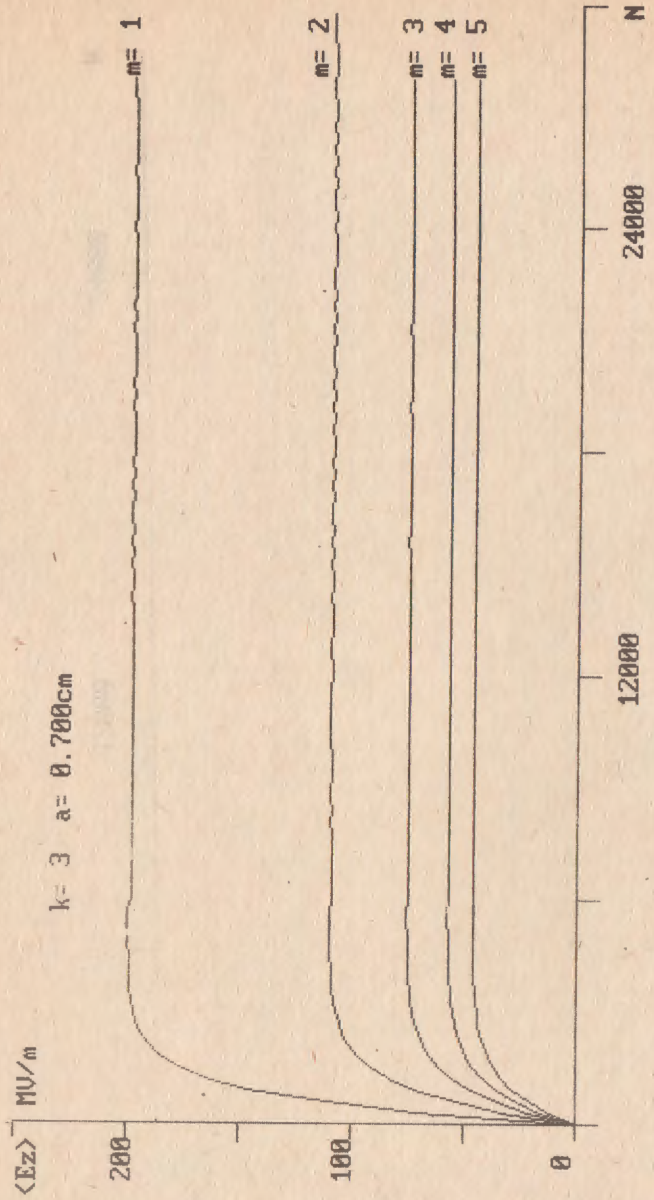


Fig.6

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