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SUPERFIELD FORMULATION OF STOCHASTIC QUANTIZATION FOR GAUGE THEORIES

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ՍՏՈԽԱՍՏԻԿ ՔՎԱՆՏԱՑՄԱՆ ԳԵՐԴԱՇՏԱՅԻՆ ՁԵՎԱԿԵՐՊՈՒՄԸ ՏՐԱՄԱՉԱՓԱՅԻՆ ՏԵՍՈՒԹՅՈՒՆՆԵՐԻ ՀԱՄԱՐ

Գերտարածական կոորդինատների համեմատ, տրամաչափային համաչափության տեղայնացման օգնությամբ ստացվել է ընդլայնված ստոխաստիկ գործողություն՝ գերտրամաչափային ինվարիանտություն ունեցող՝ Յանգ-Միլսի դաշտի համար։ Դա թույլ է տալիս ստեղծել տրամաչափային տեսությունների համար ստոխաստիկ ռեդուկցիայի մեխանիզմ՝ խոտորման տեսությունների շրջանակներից դուրս։

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SUPERFIELD FORMULATION OF STOCHASTIC QUANTIZATION FOR GAUGE THEORIES

Using gauge symmetry localization relative to superspace coordinates we have obtained an extended stochastic action for the Yang-Mills field possessing supergauge invariance. This allows us to formulate correctly a mechanism of stochastic reduction for gauge theories beyond the framework of perturbation theory.

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Эд.Ш.ЕГОРЯН, Р.П.МАНВЕЛЯН

СУПЕРПОЛЕВАЯ ФОРМУЛИРОВКА СТОХАСТИЧЕСКОГО КВАНТОВАНИЯ ДЛЯ КАЛИБРОВОЧНЫХ ТЕОРИЙ

С помощью локализации калибровочной симметрии по отношению к суперпространственным координатам получено расширенное стохастическое действие для поля Янга-Миллса обладающее суперкалибровочной инвариантностью. Это позволяет корректно сформировать механизм стохастической редукции для калибровочных теорий вне рамок теории возмущений.

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1. Introduction

In the present paper we consider a superfield formulation of stochastic quantization (SQ) of gauge models [1-4] . By means of gauge symmetry localization relative to superspace coordinates we have obtained an extended action written in superfields and possessing supergauge invariance. This enables us to formulate correctly a stochastic reduction mechanism for gauge theories and hence to carry out proof of equivalence of ghostless stochastic quantization of Yang-Mills field and the usual scheme of quantization in all the loops (for perturbative proof see Refs [5-7]). Besides, the gauge symmetry localization over superspace coordinates allows one to formulate correctly dimensional reduction of Witten's topologic 4-dimensional quantum field theory to Chern-Simons 3-dimensional quantum field theory [8] .

2. SQ for Yang-Mills Field

The SQ scheme for Yang-Mills field is formulated as follows [1-3] :

1) A naive Langevin equation is written:

$$\partial_z A_\mu^a(x_\mu,t) + \frac{\delta S'_{YM}(A_\mu(x_\mu,t))}{\delta A_\mu^a(x_\mu,t)} = J_\mu^a(x_\mu,t) \tag{1}$$

with a boundary condition

$$A_\mu^a(x_\mu,0) = 0 \tag{2}$$

where t is additional time, and x_μ are usual d-dimensional coordinates;

2) A Gaussian distribution of stochastic source is introduced:

$$< J_\mu^a(x_\mu,t)\, J_\mu^b(x'_\mu,t') >_{SQ} = 2\delta_{\mu\nu}\,\delta^{ab}\,\delta(x-x')\,\delta(t-t') \tag{3}$$

3) By means of (3) stochastic averages for solution of Eq.(1) A_μ^J are calculated. Equivalence to the usual quantization scheme is established from the following statement [10] :

$$\lim_{t\to\infty} < F_{GI}(A_\mu^J(x,t)) >_{SQ} = < F_{GI}(A_\mu) >_{FP} \tag{4}$$

where F_{GI} is gauge-invariant functional, and $< \ >_{FP}$ is the usual quantum average with respect to Faddeev-Popov ghosts. The main advantage of the SQ scheme consists in the absence of ghosts and Gribov's problem [2-4] .

Ref. [4] shows that in one-loop approximation the naive SQ scheme gives correct quantum averages for gauge-invariant functionals. The general proof of equivalence of SQ and the usual quantization scheme is based on superfield formulation

[9-11] . Consider now the peculiarities of this scheme in the case of Yang-Mills field.

Following [9] the SQ scheme can be realized in the form of a generating functional for the next d+1-dimensional supertheory:

$$S_{ss} = \int d^d x\, dt\, d\bar{\theta}\, d\theta\, \mathcal{L}_{ss}\, (\Phi^a_\mu) \tag{5}$$

$$\mathcal{L}_{ss}\,(\Phi^a_\mu) = \mathcal{L}_{YM}\,(\Phi^a_\mu) + \bar{D}\,\Phi^a_\mu\, D\Phi^a_\mu \tag{6}$$

where $\Phi^a_\mu = A^a_\mu(x,t) + \bar{\theta}\,\Psi^a_\mu(x,t) + \Psi^a_\mu(x,t)\theta + \bar{\theta}\theta C^a_\mu\,(x,t)$,

and D and $\bar{D}$ are supercovariant derivatives:

$$D = \frac{\partial}{\partial\bar{\theta}} - \theta\partial_t \;, \qquad \bar{D} = \frac{\partial}{\partial\theta}$$

Action (5) possesses one-dimensional supersymmetry:

$$\delta\Phi^a_\mu = (\varepsilon\bar{Q} + \bar{\varepsilon}Q)\,\Phi^a_\mu \tag{7}$$

where $\bar{Q} = \frac{\partial}{\partial\theta} + \bar{\theta}\partial_t$, $\bar{Q} = \frac{\partial}{\partial\bar{\theta}}$, and has gauge invariance with a parameter dependent on x_μ only:

$$\delta\,\Phi_\mu = D_\mu\,(\Phi_\mu)\,\alpha\,(x_\mu)$$

$$D_\mu\,(\Phi_\mu) = \partial_\mu + \Phi_\mu\,(x, t, \theta, \bar{\theta}) \tag{8}$$

The known stochastic reduction procedure [10] is based on the presence of supersymmetry (7) and consists in the fact that the equal-time (over additional time t) correlators calculated using action (5) coincide with theory correlators with an action:

$$S = \int d^d x \, dt \, d\theta \, d\bar{\theta} \, \delta(t)\delta(\theta)\delta(\bar{\theta}) \mathcal{L}_{SS} \tag{9}$$

However it is impossible to use this procedure in the given case, since formula (9) is correct in the case when action (5) is nondegenerated (e.g. in the scalar field case [10]). In our case action (5) possesses invariance (8) which does not allow to fix gauge supersymmetrically. Besides, due to the fact that boundary condition (2) breaks translational invariance over t, it turns out also impossible to apply the renormalizability proof [12] in all the orders of perturbation theory in the naive scheme. At the same time the introduction of Zwanziger's term [2,3] which allows to send boundary condition (2) to $-\infty$ and restore translational invariance breaks supersymmetry (7) which is necessary in proof of equivalence and renormalizability of SQ in all the loops.

3. Supergauge-Invariant Formulation

We try to avoid all above-cited troubles as follows. We construct a formulation in which supersymmetry (7) is preserved, and gauge invariance (8) is extended up to supergauge invariance:

$$\delta \Phi_\mu = D_\mu(\Phi^a_\mu)\, x(x,t,\theta,\bar{\theta}) \tag{10}$$

This will allow us to remove degeneration by means of gauge fixing without breaking supersymmetry, and realize reduction (9).

For that, we apply for action (5) Noether's procedure localizing symmetry (8) over $t, \theta, \bar{\theta}$. As a result we have:

$$\mathcal{L}_{SGI} = \mathcal{L}_{YM}(\phi_\mu^a) + \bar{D}\phi_\mu^a D\phi_\mu^a -$$
$$- \bar{D}\phi_\mu^a D_\mu \Psi^a - D_\mu \bar{\Psi}^a D\phi_\mu^a + D_\mu \bar{\Psi}^a D_\mu \bar{\Psi}^a = \tag{11}$$
$$= \mathcal{L}_{YM}(\phi_\mu^a) + [\bar{\mathcal{D}}, D_\mu][\mathcal{D}, D_\mu]$$

where

$$\mathcal{D} = D + \Psi = e^{-V} D + \bar{e}^{V}(D e^{V})$$
$$\bar{\mathcal{D}} = \bar{D} + \bar{\Psi} = \bar{D} + e^{-V}(\bar{D} e^{V}) \tag{12}$$

and $V = V_0(x,t) + \bar{\theta} V_1(x,t) + \bar{V}_1(x,t)\theta + \bar{\theta}\theta V_2(x,t)$
is a scalar superfield.

Action (11) possesses supersymmetry (7) and supergauge invariance (10) supplemented by transformation for field V :

$$e^{V'} = e^{V} e^{\alpha(x,t\theta,\bar{\theta})} \tag{11'}$$

One can see from (10) that for theory (11) there exists gauge $V = 0$ where (11) turns into (5). On the other hand, supergauge invariance (10) allows to fix another gauge, for example

$$\partial_\mu \phi_\mu^a (x, t, \theta, \bar{\theta}) = 0 \tag{13}$$

Here together with ghosts we have:

$$\mathcal{L}_{SGF} = \mathcal{L}_{CGI} + \frac{1}{2\alpha}(\partial_\mu \Phi_\mu^a)^2 + \bar{C}\partial_\mu D_\mu C \tag{14}$$

where C and $\bar{C}$ are Grassmann's superfields.

Action (14) is nondegenerated (hence it can be defined with boundary condition over t on -) and possesses supersymmetry (7); therefore, the formula of reduction (9) is true for it. One can readily see that

$$\mathcal{L}_{SGF}\Big|_{t=\bar\theta=\theta=0} = \mathcal{L}_{SGI}\Big|_{t=\theta=\bar\theta=0} + \frac{1}{2\alpha}(\partial_\mu A_\mu^a)^2 + \bar{c}\,\partial_\mu D_\mu (A_\mu^a)\,c \tag{15}$$

where

$$A_\mu^a - \Phi_\mu^o\Big|_{t=\theta=\bar\theta=0}, \quad c = C\Big|_{t=\theta=\bar\theta=0}, \qquad \bar{c} = \bar{C}\Big|_{t=\theta=\bar\theta=0}$$

Consider now a first term in expression (15):

$$\mathcal{L}_{SGI}\Big|_{t=\theta=\bar\theta=0} = \mathcal{L}_{YM}(\Phi_\mu^a)\Big|_{t=\theta=\bar\theta=0} + [\bar{\mathcal{D}} D_\mu][\mathcal{D}, D_\mu]\, t=\theta=\bar\theta=0$$

$$= \mathcal{L}_{YM}(A_\mu^a) - \bar\Psi_\mu(x)\Psi_\mu(x) + \bar\Psi_\mu(x) D_\mu(A_\mu)\Psi - D_\mu(A_\mu)\bar\Psi(x)\Psi_\mu(x) - D_\mu(A_\mu)\bar\Psi(x) D_\mu(A_\mu)\Psi(x) \tag{16}$$

where

$$\Psi(x) = e^{-V}\mathcal{D}e^{V}\Big|_{t=\theta=\bar\theta=0}, \quad \bar\Psi(x) = e^{-V}\bar{\mathcal{D}}e^{V}\Big|_{t=\theta=\bar\theta=0}$$

One can readily see that after integration over

$$\mathcal{L}_{SGI}\Big|_{t=\theta=\bar{\theta}=0} = \mathcal{L}_{YM}\,(A_\mu) \tag{17}$$

Thus we have found that

$$\mathcal{L}_{SGF}\Big|_{t=\theta=\bar{\theta}=0} = \mathcal{L}_{FP}\,(A_\mu) \tag{18}$$

where $\mathcal{L}_{FP}$ is a usual d-dimensional Faddeev-Popov lagrangian. Hence gauge invariance localization brings us to action (11) which in gauge $V = 0$ coincides with stochastic action (5) derived from Langevin equation and in gauge (14) it reduces to a usual Faddeev-Popov action. This proves equivalence of SQ and the usual quantization scheme for gauge-invariant functionals in all the orders of perturbation theory.

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