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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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SOLUTION OF ISING MODEL ON RANDOM LATTICES WITH LARGE NUMBER OF SURFACE GENUS

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We regret that some of the pages in this report may not be up to the proper legibility standards, even though the best possible copy was used for scanning

Նախնատիպ ԵրՖԻ-1318(13)-91

Դ.Բ.ՍԱՀԱԿՅԱՆ

ԻԶԻՆԳԻ ՄՈԴԵԼԻ ԼՈՒԾՈՒՄԸ ՄԱԿԵՐԵՎՈՒՅԹԻ ՍԵՌԻ ՄԵԾ ԹՎՈՎ ՊԱՏԱՀԱԿԱՆ ՑԱՆՑԵՐԻ ՎՐԱ

Հաշված են երկմատրիցային ինտեգրալի՝ ըստ $1/N^2$ աստիճանների շարքի հիմնական սինգուլյար անդամները՝ մինչև իններորդ կարգը։ Ըստացվել է վիճակագրական գումարի ասիմպտոտիկի (ըստ հանգույցների թըվի) խզումը, երբ սիստեմն անցնում է փուլային փոխարկման կետով։

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Last years the theory of phase transition scored up striking successes owing to the utilization of conformal symmetry (in $d=2$).

In Ref./1/ an accent was given to the ability of a system to make a decision during the phase transitions irrespective of the type of transition (when the symmetry is violated).

If we consider an asymptotical expansion of the logarithm of statistical sum $\ln Z$ in the number of sites N,

$$\ln Z(N,B) = NF_1(B) + \ldots + F_0(B) + O(1) \tag{1}$$

then, generally speaking, a discontinuity of $F_0(B)$ term is expected to occur in the transition point, when the phase transition point is passed (with the violation of symmetry). Let $Z_n(B)$ be the statistical sum Ising model on a random lattice. Its generating functional $Z(B,g) = \sum_n [-4gc/(1-c)^2]^n Z_n(B)$ and is expressed in terms of an integral over two Hermitean matrices /2,3/

$$Z(B,C) = \frac{1}{N^2} \ln \int du\,dv \exp\{ \mathrm{tr}(-u^2 - v^2 + \frac{g}{N}(u^4 + v^4)) \} \tag{2}$$

The value of (2) is expressed in recurrent constants f_K, z_K, S_K, for which the following equations were obtained /2,3/

$$c z_n = f_n [1 + 2g/N (z_{n-1} + z_n + z_{n+1})]$$

$$c f_n = -\frac{n}{2} + z_n [1 + 2g/N (z_{n-1} + z_n + z_{n+1})] + 2g/N (S_n + S_{n+1} + S_{n+2}) \quad (3)$$

$$c S_{n+1} = 2g/N \, f_n f_{n-1} f_{n-1}$$

$$\ln \int du\,dv \exp[t_2(-u^2 - v^2 + 2cuv - g/N (u^4 + v^4)] = \sum_{i=1}^{N} (N-i) \ln f_i \quad (4)$$

In the limit of large N we can write f_i as $1/N f_i \sim f(x) + f_1(x) + f_2(x) + \dots$, where $x = i/N$, and analogously for $1/N\, z_i$ and $1/N^2 S_i$. Here $f_K(x)$, $z_K(x)$, $S_K(x) \sim 1/N^{2K}$

For the first order corrections in $1/N^2$ we have

$$-(1+6gz) f_1 + (c - 6gf) z_1 = 2g f z'' 1/N^2$$

$$c f_1 - (1 + 12g \quad) z_1 - 6g S_1 = 2g (z z'' + S'') 1/N^2 \quad (5)$$

$$-6g f^2 f_1 \qquad + c S_1 = 2g (f f'' - f'^2) 1/N^2$$

One should remember, that for the solution of (5), the most singular term in the powers of $G'(z)$ is required, where

$$z = 2gf/c, \quad G(z) = z[1/(1-3z)^2 - c^2] + 3c^2 z^3, \; xg = G(z) \quad (6)$$

The allowance for this fact greatly simplifies the calculations.

From (4) we find for the part $Z(B,g)$ of interest to us

$$N^2 \int_0^1 (1-x) f_K / f \, dx \quad (7)$$

At the integration of (7) we have for the upper limit $g = G(z)$. The range of z in the vicinity of z_*, where $G'(z_K) = 0$ is important for the calculation of

When $C < 1/4$

$$(1 - 3z_*) = 2 \tag{8}$$

and when $C > 1/4$

$$(1 - 3z_*) = 1/\sqrt{C} \tag{9}$$

Near the transition point $2\sqrt{C} = 1/(1+\varepsilon)$, and we have for $\varepsilon \to 0$ (8) and $C = 1/4$

$$N^2 \int_0^1 (1-x) f_1/f \, dx = -\frac{1}{12} \ln (g - g_c) \tag{10}$$

and for (9)

$$N^2 \int_0^1 (1-x) f_1/f \, dx = -\frac{1}{24} \ln (g - g_c) \tag{11}$$

The equality in (10) and (11) is implied for the main singular terms. Here $f_1, z_1, S_1 \sim \varepsilon^2 / G'(z)^4$.

Included in the consideration were corrections up to the 9th order. For the K th order correction up to $K = 9$ we have

$$f_K, z_K, S_K \sim \varepsilon^{2K} / G'(z)^{5K-1} \tag{12}$$

The primary singularity $N^2 \int_0^1 (1-x) f_K/f \, dx$ is discontinuous by the factor

$$2^K \tag{13}$$

If this occurs for any K (and not only for the calculated $K \leq 9$), then an extraordinary possibility will result. If the Ising model is defined on a random surface of K genus and K along with N tends to infinity, then there arise an infinite discontinuity of the asymptotic limit

$$\lim_{\varepsilon\to 0} \lim_{N\to\infty} \ln Z(N, B_c+\varepsilon) - \ln Z(N, B_c-\varepsilon) \sim K \qquad (14)$$

If (14) is valid up to

$$K \sim N \qquad (15)$$

then (14) would lead to a jump in free energy (as if the "0"-order transition took place). But there is no contradiction, as (14) was observed only in the $N\to\infty$ limit at fixed K.

The expression (14) implies that in this case the problem consists rather in the change of transition nature. The limit presumably corresponds to the Ising model on lattices of $d > 2$ (d =3?) dimensions. The limit (15) should not necessarily coincide with the non-perturbative solution of Ising model connected with 2d gravitation /4,5/, in which the contribution of the surfaces of all the genuses was taken into account. But the required lattice (the non-trivial one) with finite number of sites N may be on the surface of K genus only for $|2K-1| \leqslant N$ and not for any K. Of great physical interest is the case when the surface with genus K in the vicinity of maximum possible $K_M = \frac{N+1}{2}$ is considered. It will be a new limit for strings. At last, let us consider the obtained results from the point of view of setting the macroscopic order ("stable" information) /6/. To establish this order in a system with complex topology of the phase space with large topological number K, a waste of free energy of the order

$\sim K$ was required.

It is interesting to seek after an analogous effect in ordinary models of phase transitions on hard lattices.

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РЕШЕНИН МОДЕЛИ ИЗИНГА НА СЛУЧАЙНЫХ РЕШЕТКАХ С БОЛЬШИМ ЧИСЛОМ РОДА ПОВЕРХНОСТИ

(на английском языке, перевод Исраеляна М.Х.)

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