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ON THE BEHAVIOUR OF STATISTICAL SUM OF A MODEL WITH YANG-LEE SINGULARITY NEAR CRITICAL POINT

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ՅԱՆԳ-ԼԻԻ ՍԻՆԳՈՒԼՅԱՐՈՒԹՅԱՄԲ ՄՈԴԵԼԻ ՎԻՃԱԿԱԳՐԱԿԱՆ ԳՈՒՄԱՐԻ ՎԱՐՔԻ՝ ՄԱՍԻՆ ՓՈՒԼԱՅԻՆ ՓՈԽԱՐԿՄԱՆ ԿԵՏԻ ՄՈՏ

Դիտարկվում է սինգուլյարության մեկ մատրիցայով մոդելը փուլային փոխարկման կետի մոտ։ Հաշվված է ազատ էներգիայի ասիմպտոտիկի (ըստ անսահմանություն ձգտող ցանցերի թվի) խզումն այդ կետում։

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The Yang-Lee singularity /1/ in matrix statistical models has been considered in a number of works /2,3/. In Ref./4/ a test for the symmetry violation in phase transitions was proposed.

If the asymptotical expansion of the logarithm of statistical sum on the lattice with M sites has a form

$$\ln Z(M, B) = MF_0(B) + \ldots + F_1(B) + O(1) \tag{1}$$

then we expect that the value of F_1 will be discontinuous in the symmetry violation point

$$\lim_{\varepsilon \to 0} F_1(B_c + \varepsilon) - F_1(B_c - \varepsilon) = \text{const} \ln(Q_2/Q_1) \tag{2}$$

where Q_2/Q_1 is the order of violated symmetry group. Numerical calculations show /4/, that in case of torus topology on rigid lattices the const=1 for $d2$, $Q=3$, $Q=5$ Potts models and $d3$ Ising model. Analogous result is obtained for $d2$ Ising model on a rigid or dynamical lattice with torus topology. In case of torus topology on a dynamical lattice, it is also interesting to study the lattices with $g \neq 1$.

Then $F_1(B)$ is singular at B_c and instead of (2) we have

$$\lim_{\varepsilon \to 0} F_1(B_c+\varepsilon) - F_1(B_c-\varepsilon) = g \ln 2 \qquad (3)$$

As we are interested in the problem of symmetry violation, we restrict our consideration of the model of Yang-Lee singularities to the case of torus topology, where F_1 has no singularities when the point B_c is approached.

For the generating function of statistical sum we have

$$Z(g,\gamma) = \frac{1}{N^2} \ln \int du \exp\left[-t_2\left(u^2 + \frac{g}{N} u^4 + \frac{g^2\gamma}{N^2} u^6\right)\right] \qquad (4)$$

Z is calculated using the method of orthogonal polynomials /5/.

In the limit $N \to \infty$, we have $\frac{i}{N} \to x$, $f_i \equiv h_i/h_{i-1} \to 6gf(i/N), z=6gf$

In the main planar approximation

$$gx = W(t) \equiv \frac{1}{3}\left(t + t^2 + \frac{5}{6}\gamma t^3\right) \qquad (5)$$

For the statistical sum on higher genus surfaces, one needs to solve the equation

$$gx = W(z) + \varphi(z) \frac{d^2 z}{dx^2} \cdot \frac{1}{N^2} \qquad (6)$$

We can represent z as $z = t + \sum_n z_n$, where $z_n \sim 1/N^{2n}$. We have

$$z_1 = -\varphi(t)/W'(t)\varphi'' \cdot 1/N^2 \qquad (7)$$

To calculate the statistical sum on surfaces of K genus with a lattice of M sites (taking the vertex with six lines for two sites), it is necessary to calculate the contour integral

$$Z_M(\gamma) = \frac{1}{2\pi i} \oint \frac{Z_K(g,\gamma) W'}{W^{M+1}} dz \qquad (8)$$

where

$$Z_K = \int_0^1 (1-x) \frac{Z_K}{t} dx \qquad (9)$$

In the limit $M \to \infty$, the main contribution to (8) is made by the vicinity of saddle points of the solution of $W'(z) = 0$ equation.

In case of $\varepsilon = \gamma - 2/5 < 0$ we have

$$z_1 = -1 - \sqrt{|\varepsilon|}, \quad z_2 = -1 + \sqrt{|\varepsilon|} \qquad (10)$$

and when $\varepsilon > 0$ we have two complex conjugate solutions

$$z_3 = -1 + i\sqrt{|\varepsilon|}, \quad z_4 = -1 - i\sqrt{|\varepsilon|} \qquad (11)$$

In case of $\varepsilon < 0$ only the solution z_1 is taken as $|W(z_1)| < |W(z_2)|$, while at $\varepsilon > 0$ one should take the integral around both the points z_3, z_4 as $|W(z_3)| = |W(z_4)|$. Hence, we obtain as a result that when $\varepsilon > 0$, then the asymptotics $Z_M(\gamma_c \pm \varepsilon)$ is discontinuous by the multiplier of 2.

It is easy to calculate (9) by expanding it at a selected singular point while keeping everywhere (just as in (6), (7)) only the most singular term in powers of $1/W'$

We obtain, that when $\varepsilon < 0$

$$Z_1 = \frac{1}{12} \ln Z = \frac{1}{36} \ln(g - g_c) \qquad (12)$$

and when $\varepsilon > 0$

$$Z_1 = \frac{1}{6} \ln Z = \frac{1}{18} \ln(g - g_c) \qquad (13)$$

It is worthwhile to mention that in the critical point

$$Z_1 = \frac{1}{18} \ln (g - g_c) \tag{14}$$

Thus, the situation with the discontinuity of statistical sum is analogous to that of the Ising model. In the violated symmetry phase the statistical sum coincides with the one in the critical point, while in the symmetrical phase it is two times as less.

One can treat other multicritical models in an analogous manner.

In conclusion I should like to express my gratitude to Sedrakyan A for useful discussions.

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Д.Б.СААКЯН

О ХАРАКТЕРЕ СТАТСУММЫ МОДЕЛИ С СИНГУЛЯРНОСТЬЮ ЯНГА-ЛИ ОКОЛО КРИТИЧЕСКОЙ ТОЧКИ

(на английском языке, перевод Исраеляна М.Х.)

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