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EQUIVALENCE OF LAGRANGIAN AND HAMILTONIAN BRST QUANTIZATIONS: SYSTEMS WITH FIRST-CLASS CONSTRAINTS

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ԼԱԳՐԱՆԺԻ ԵՎ ՀԱՄԻԼՏՈՆԻ ԲՌՍՏ-ՔՎԱՆՏԱՑՄԱՆ ՀԱՄԱՐԺԵՔՈՒԹՅՈՒՆԸ. ԱՌԱՋԻՆ ԿԱՐԳԻ ԿԱՊԵՐՈՎ ՀԱՄԱԿԱՐԳԵՐԸ

Ապացուցված է Լագրանժի և Համիլտոնի ԲՌՍՏ-քվանտացման համարժեքությունը՝ միայն առաջին կարգի կապերով համակարգերի համար։

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Two approaches to the quantization of gauge theories using BRST symmetry are widely used nowadays: the Lagrangian quantization, developed in [1-3] (BV-quantization) and Hamiltonian quantization, formulated in [4-8] (BFV-quantization). For all known examples of field theory (Yang-Mills theory, gravitation etc) both schemes give equivalent results. However the equivalence of these approaches in general wasn't proved. The main obstacle in comparing of these formulations consists in the fact, that in Hamiltonian approach the number of ghost fields is equal to the number of all first-class constraints, while in the Lagrangian approach the number of ghosts is equal to the number of independent gauge symmetries, which is equal to the number of primary first-class constraints only.

It should be mentioned, that paper [9] was devoted to the verification of the equivalence of the Lagrangian and Hamiltonian quantizations of the classical action, which in both cases coincided with the Hamiltonian action in the phase space. However the relation of these quantizations to the quantization of the initial classical Lagrangian action (which depends only on the coordinates) still remained unclear.

This paper is devoted to the proof of the equivalence of Lagrangian and Hamiltonian quantizations for the systems with first-class constraints only. This is achieved by a choice of special gauge in the Hamiltonian approach. It's shown, that after integration over "redundant" variables on the functional integral we come to effective action which is constructed according to rules for construction of the effective action in Lagrangian quantization scheme.

In sect.2 the rules of constructing effective actions in Lagrangian [1-3] and Hamiltonian [4-8] approaches are briefly reviewed. An effective Lagrangian action resulting from the Hamiltonian approach after the integration over redundant variables in the functional integral is introduced. In section 3 it is demonstrated that the effective action introduced in sect.2. satisfies the master equation [1-3] . Apart from that it's shown that the gauge fermion is incorporated in the above mentioned effective action according to the rules of Lagrangian quantization. In sect.4 the facts that the constructed effective action is local functional and has correct boundary values are established. This completes the proof of the equivalence of the quantization in both approaches.

In sect. 5 for methodical reasons we demonstrate explicitly that the summands of the first order over ghost fields in the effective action, introduced in sect. 2, at $\hbar=0$ are in agreement with the general structure of the master equation solution.

2. Here we will briefly state the rules of BRST quantizations in Lagrangian and Hamiltonian approaches to the quantization of gauge theories.

In the Lagrangian BRST-quantization approach the extended action of the theory is constructed by the rule:

$$\overset{I}{S}_{eff}(Q,I_Q,\Psi_L) = \left[S_{min}(Q_{min},I_{Q_{min}})+I_{\bar{c}}B\right]_{I_Q\to I_Q+\frac{\delta_r\Psi_L}{\delta Q}} \qquad (1)$$

Here $Q^A\equiv(q,c,\bar{c},B)$, where q is the set of the fields of the initial gauge theory, B^α, C^α, $\bar{C}^\alpha$ - additional fields, $\alpha=1,\dots,m$, m is equal to the number of parameters of gauge transformations, $Q^a_{min}\equiv(q,c)$, I_Q - sources of the fields Q (antifields), Ψ_L - an arbitrary local functional of the fields Q , which is fixing the gauge (gauge fermion), while $S_{min}(Q_{min},I_{Q_{min}})$ satisfies the master equation:

$$(-1)^{\varepsilon_{Q^a}}\frac{\delta_l}{\delta Q^a_{min}}\frac{\delta_l}{\delta I^a_{Q_{min}}}\exp\left\{\frac{i}{\hbar}S_{min}(Q_{min},I_{Q_{min}})\right\}=0 \qquad (2)$$

(the summation over repeated indices is assumed), with initial conditions:

$$S_{min}\Big|_{c=I_Q=\hbar=0}=S_{cl}\ ,\ \ \varepsilon(S_{min})=0,\ gh(S_{min})=0, \qquad (3)$$

where $\varepsilon(A)$ and $gh(A)$ denote as usual the grassmann parity and ghost number of the A quantity, respectively; "r" and "l" indices of functional derivatives denote right and left derivatives. It's important to stress here that the sources I_Q and the gauge fermion Ψ_L enter $\overset{I}{S}_{eff}$ only in the combination $\tilde{I}_Q=I_Q+\frac{\delta_r\Psi_L}{\delta Q}$. Note also that $\overset{I}{S}_{eff}$ too satisfies the master equation:

$$(-1)^{\varepsilon_{Q^A}}\frac{\delta_l}{\delta Q^A}\frac{\delta_l}{\delta I^A_Q}\exp\left\{\frac{i}{\hbar}\overset{I}{S}_{eff}(Q,I_Q,\Psi_L)\right\}=0 \qquad (4)$$

Within the Hamiltonian approach to BRST quantization the effective action is obtained in the following way. Let the dynamical system of the canonical variables q , p be described by a canonical Hamiltonian $H_0(q,p)$ and a system of linearly independent constraints $\Phi_{\bar{\alpha}}(q,p), \bar{\alpha}=1,\dots,\bar{m}$ (as it was mentinned in the introduction, only first-class constraints are considered in this paper). Then we have

$$\{\Phi_{\bar{\alpha}},\Phi_{\bar{\beta}}\} = U_{\bar{\alpha}\bar{\beta}}^{\bar{\gamma}}\,\Phi_{\bar{\gamma}}\,, \quad \{H_0,\Phi_{\bar{\alpha}}\} = V_{\bar{\alpha}}^{\ \bar{\beta}}\,\Phi_{\bar{\beta}}\,, \tag{5}$$

where $U_{\bar{\alpha}\bar{\beta}}^{\bar{\gamma}}, V_{\bar{\alpha}}^{\bar{\beta}}$ - are first-order structure functions. The set of constraints $\Phi_{\bar{\alpha}}$ is naturally divided into subsets Φ_{α_k} , $k=1,\dots,L$

$$\Phi_{\bar{\alpha}} = \{\Phi_{\alpha_1},\dots\Phi_{\alpha_L}\} \; ; \; \bar{\alpha} = \{\alpha_1,\dots\alpha_L\}\,,$$

where Φ_{α_k} - are the constraints of the K -th order of Dirac procedure (Φ_{α_1} are primary constraints). For simplicity we consider the case of equal number of constraints in all orders of Dirac's procedure.

To construct $S_{eff}^{(H),I}$ one has to enlarge the phase space by introducing new pairs of canonical variables: $(\lambda^{\bar{\alpha}},\pi^{\bar{\alpha}}), (C^{\bar{\alpha}},\bar{\mathcal{P}}^{\bar{\alpha}})$ $(\bar{C}^{\bar{\alpha}},\mathcal{P}^{\bar{\alpha}})$, where $\lambda^{\bar{\alpha}}$ (Lagrange multipliers of constraints $\Phi_{\bar{\alpha}}$) $C^{\bar{\alpha}}$, $\bar{C}^{\bar{\alpha}}$ (ghost fields) - are coordinates, while $\pi^{\bar{\alpha}}$, $\bar{\mathcal{P}}^{\bar{\alpha}}$, $\mathcal{P}^{\bar{\alpha}}$ are the corresponding canonical momenta. The BRST quantization in the extended phase space then consists of constructing of generating function $\Omega = \Omega_{min} + \mathcal{P}_{\bar{\alpha}}\pi^{\bar{\alpha}}$ of the BRST transformation and of Hamiltonian $\mathcal{H}$, which satisfy the equations

$$\{\Omega_{min},\Omega_{min}\} = 0\,, \quad \{\mathcal{H},\Omega_{min}\} = 0 \tag{6}$$

with the initial conditions:

$$\Omega_{min}\Big|_{\bar{\mathcal{P}}=0} = C^{\bar{\alpha}}\Phi_{\bar{\alpha}}\,, \quad \mathcal{H}\Big|_{\bar{\mathcal{P}}=0} = H_0\,, \tag{7}$$

where Ω_{min} and $\mathcal{H}$ depend only on the fields of the minimal sector $q,p,C^{\bar{\alpha}},\bar{\mathcal{P}}_{\bar{\alpha}}$. The solutions of these equations exist being as follows:

$$\begin{gathered}\Omega_{min} = C^{\bar{\alpha}}\Phi_{\bar{\alpha}} + C^{\bar{\alpha}}C^{\bar{\beta}}\,\overset{(1)}{U}{}_{\bar{\alpha}\bar{\beta}}^{\bar{\gamma}}\,\bar{\mathcal{P}}_{\bar{\gamma}} + \Delta\Omega(gh^5)\,,\\ \mathcal{H} = H_0 + C^{\bar{\alpha}}V_{\bar{\alpha}}^{\bar{\beta}}\bar{\mathcal{P}}_{\bar{\beta}} + \Delta\mathcal{H}\,, \quad \overset{(1)}{U}{}_{\bar{\alpha}\bar{\beta}}^{\bar{\gamma}} = -\frac{1}{2}(-1)^{\varepsilon_{\bar{\beta}}}U_{\bar{\alpha}\bar{\beta}}^{\bar{\gamma}}\,,\end{gathered} \tag{8}$$

where, in general, $\Delta\Omega$ can be expanded in a power series over ghost fields $C, \bar{\mathcal{P}}$, starting from the fifth power and $\mathcal{H}$ is a series starting from the fourth power of the ghosts C , $\bar{\mathcal{P}}$.

Finally, the effective action $S_{eff}^{(H),I}$ is constructed by the rule:

$$S_{eff}^{(H),I} = \int dt\left[\frac{1}{2}\dot{\Gamma}\omega\Gamma - \mathcal{H} + \{\psi_H(\Gamma),\Omega\} + I_Q^A\{Q^A,\Omega\}\right], \tag{9}$$

where Γ denotes the set of all variables of the extended phase space, $\{\Gamma,\Gamma\} = \omega$, ψ_H is the gauge fermion, I_Q^A are sources of BRST transformations of the fields Q^A. The generating functional of the theory is

$$Z_{\Psi_H} = \int D\Gamma \exp\left\{\frac{i}{\hbar} S^{(H),I}_{eff}(\Gamma, I_Q, \Psi_H)\right\} . \qquad (10)$$

According to Fradkin - Vilkovsky theorem [5] Z_{Ψ_H} is independent of the gauge fermion Ψ_H .

Let us divide the set of variables Γ into two subsets. The first of these subsets consists of the variables $Q=(q, c, \bar{c}, B)$ $c^{\alpha} \equiv c^{\alpha_L}$, $\bar{c}^{\alpha} \equiv \bar{c}^{\alpha_L}$, $B^{\alpha} \equiv \pi^{\alpha_L}$ while the second includes all remaining variables, which we will denote by X. Thus $\Gamma = (Q, X)$. Let us now define a functional $\tilde{S}^{(H),I}_{eff,\Psi_H}(Q, I_Q)$ by a relation:

$$e^{\frac{i}{\hbar}\tilde{S}^{(H),I}_{eff,\Psi_H}(Q,I_Q)} = \int DX \exp\left\{\frac{i}{\hbar} S^{(H),I}_{eff}(\Gamma, I_Q; \Psi_H)\right\} . \qquad (11)$$

We will show below that the functional $\tilde{S}^{(H),I}_{eff,\Psi_H}$ defined by (11) has all the properties of the Lagrangian action in BV-quantization scheme: it is local, depends on sources I_Q and Ψ_L only through combination $I_Q + \frac{\delta_r \Psi_L}{\delta Q}$, also it satisfies equation (4) with the initial condition

$$\tilde{S}^{(H),I}_{eff,\Psi_H}(Q, I_Q)\Big|_{I_Q=\hbar=C=0} = S_{cl.} \qquad (12)$$

This will prove the equivalence of the BRST quantizations in Lagrangian and Hamiltonian approaches of theories with first-class constraints.

3. In this section $\tilde{S}^{(H),I}_{eff,\Psi_H}$ will be shown to satisfy the master equation (4). To prove that we will use the method used in [10]. Consider the functional integral

$$\mathcal{J} = \int DX \exp\left\{\frac{i}{\hbar} S^{(H),I}_{eff}(Q', X, I_Q; \Psi_H)\right\} , \qquad (13)$$

where $Q' = Q + \{Q, \Omega\}\mu$, μ - grassmann parameter. Expanding the exponent in series over powers of μ $(\mu^2 = 0)$ we obtain

$$\mathcal{J} = \int DX \left[e^{\frac{i}{\hbar} S^{(H),I}_{eff}} + \frac{\delta_r}{\delta Q^A} e^{\frac{i}{\hbar} S^{(H),I}_{eff}} . \{Q^A, \Omega\}\mu \right] =$$

$$= \int DX e^{\frac{i}{\hbar} S^{(H),I}_{eff}} + i\hbar \int DX \frac{\delta_r}{\delta Q^A} \frac{\delta_l}{\delta I^A_Q} e^{\frac{i}{\hbar} S^{(H),I}_{eff}} . \mu -$$

$$- \int DX e^{\frac{i}{\hbar} S^{(H),I}_{eff}} \mathrm{str}\left[\frac{\delta_r}{\delta Q^B}\left(\{Q^A, \Omega\}\mu\right)\right] , \qquad (14)$$

where $S^{(H),I}_{eff} = S^{(H),I}_{eff}(Q, X, I_Q; \Psi_H)$. On the other hand, after the canonical change of variables $X_i \to X'_i = X_i + \{X_i, \Omega\}\mu$ we have:

$$\mathcal{J} = \int DX \,\mathrm{sdet}\left|\frac{\delta X'_i}{\delta X_j}\right| \exp\left\{\frac{i}{\hbar} S^{(H),I}_{eff}\left(Q + \{Q, \Omega\}\mu, X + \{X, \Omega\}\mu, I_Q; \Psi_H\right)\right\} =$$

$$= \int DX \,\mathrm{sdet}\left|\frac{\delta X'_i}{\delta X_j}\right| \exp\left\{\frac{i}{\hbar} S^{(H),I}_{eff}(Q, X, I_Q; \Psi_H)\right\} = \qquad (15)$$

$$= \int DX e^{\frac{i}{\hbar} S^{(H),I}_{eff}} + \int DX e^{\frac{i}{\hbar} S^{(H),I}_{eff}} \mathrm{str}\left[\frac{\delta_r}{\delta X_j}\left(\{X_i, \Omega\}\mu\right)\right] .$$

To obtain (15) we had to use the BRST invariance of $S^{(H),I}_{eff}$

$$S^{(H),I}_{eff}(\Gamma + \{\Gamma, \Omega\}\mu, I_Q; \Psi_H) = S^{(H),I}_{eff}(\Gamma, I_Q; \Psi_H)$$

Equating the expressions (14) and (15) and taking into account

$$\mathrm{str}\left[\frac{\delta_r}{\delta Q^B}\left(\{Q^A, \Omega\}\mu\right)\right] + \mathrm{str}\left[\frac{\delta_r}{\delta X_i}\left(\{X^j, \Omega\}\mu\right)\right] = 0 \qquad (16)$$

we find

$$\int DX \frac{\delta_r}{\delta Q^A} \frac{\delta_e}{\delta I_Q^A} e^{\frac{i}{\hbar} S_{eff}^{(H),I}(Q,X,I_Q;\psi_H)} = (-1)^{\varepsilon_{Q^A}} \frac{\delta_e}{\delta Q^A} \frac{\delta_e}{\delta I_Q^A} e^{\frac{i}{\hbar}\tilde{S}_{eff,\psi_H}^{(H),I}(Q,I_Q)} = 0 \qquad (17)$$

Thus the functional $\tilde{S}_{eff,\psi_H}^{(H),I}(Q,I_Q)$ satisfies the master equation.

Describing the procedure of the construction of extended effective action $S_{eff}^{I}(Q,I_Q;\psi_L)$ in Lagrangian formalism we stressed that the sources I_Q and the gauge fermion ψ_L enter S_{eff}^{I} in combination $I_Q+\frac{\delta_r\psi_L}{\delta Q}$. In general $\tilde{S}_{eff,\psi_H}^{(H),I}(Q,I_Q)$ doesn't have this property. However this takes place for a certain class of ψ_H. Since the physical consequences of the theory do not depend on ψ_H, we will consider the class of gauges

$$\psi_H(\Gamma) = \psi_L(Q) + \psi_1(X) \qquad (18)$$

Substituting (18) into (9) and taking into account

$$\{\psi_H(\Gamma),\Omega\} = \frac{\delta_r\psi_L}{\delta Q^A}\{Q^A,\Omega\} + \{\psi_1(X),\Omega\} \qquad (19)$$

we ascertain that $\tilde{S}_{eff,\psi_H}^{(H),I}$ does depend on I_Q and $\psi_L(Q)$ only through combination $\tilde{I}_Q=I_Q+\frac{\delta_r\psi_H}{\delta Q}$. Hence

$$\tilde{S}_{eff,\psi_H}^{(H),I}(Q,I_Q) \equiv \mathcal{S}(Q,\tilde{I}_Q), \qquad (20)$$

where $\mathcal{S}(Q,\tilde{I}_Q)$ still satisfies equation (4).

4. We will now prove that the functional $\mathcal{S}(Q,I_Q)$ defined in (20) can be chosen local and satisfying the initial condition (12). To achieve this and we choose the gauge fermion ψ_H in the form:

$$\psi_H = \sum_{\{\alpha_L\}} \bar{C}^{\alpha_L}\chi_{\alpha_L}(q) + \beta\sum_{\{\alpha_1\}} \bar{\mathcal{P}}_{\alpha_1}\lambda^{\alpha_1} + \beta\sum_{i=1\{\alpha_i\}}^{L-1} \bar{C}^{\alpha_i}\pi_{\alpha_i} + \beta\sum_{i=1,\{\alpha_i\}}^{L-1} \bar{C}^{\alpha_i}\lambda_{\alpha_{i+1}}. \qquad (21)$$

Setting in (9) the expression $\{\psi_H,\Omega\}$ found by using (8), (21) and relations

$$\{C^{\alpha_i},\bar{\mathcal{P}}_{\alpha_k}\} = \{\bar{C}^{\alpha_i},\mathcal{P}_{\alpha_k}\} = \{\lambda^{\alpha_i},\pi_{\alpha_k}\} = \delta^{\alpha_i}_{\alpha_k} \qquad (22)$$

we obtain

$$\begin{aligned} S_{eff}^{(H),I} = {} & \sum \dot{q}p + \sum_{k=1,\{\alpha_k\}}^{L}\left(\dot{C}^{\alpha_k}\bar{\mathcal{P}}_{\alpha_k} + \dot{\bar{C}}^{\alpha_k}\mathcal{P}_{\alpha_k} + \dot{\lambda}^{\alpha_k}\pi_{\alpha_k}\right) - H_0 - \\ & - \sum_{i=1,\{\alpha_i\}}^{L}\sum_{k=1,\{\alpha_k\}}^{i+1} C^{\alpha_i}V_{\alpha_i}^{\alpha_k}\bar{\mathcal{P}}_{\alpha_k} + \sum_{k=1,\{\beta_k\}}^{L}\sum_{\{\alpha_L\}} \bar{C}^{\alpha_L}\{\chi_{\alpha_L}(q),\Phi_{\beta_k}\}C^{\beta_k} + \\ & + \sum_{\{\alpha_L\}}\chi_{\alpha_L}\pi^{\alpha_L} + \beta\sum_{\{\alpha_1\}}\Phi_{\alpha_1}\lambda^{\alpha_1} + \beta\sum_{\{\alpha_1\}}\bar{\mathcal{P}}^{\alpha_1}\mathcal{P}_{\alpha_1} + \\ & + \beta\sum_{i=1,\{\alpha_i\}}^{L-1}\pi^{\alpha_i}\pi^{\alpha_i} + \beta\sum_{k=1,\{\alpha_k\}}^{L-1}\lambda_{\alpha_{k+1}}\pi_{\alpha_k} + \beta\sum_{k=1,\{\alpha_k\}}^{L-1}\bar{C}^{\alpha_k}\mathcal{P}_{\alpha_{k+1}} + \\ & + 2\beta\sum_{k=1,\{\alpha_k\}}^{L}\sum_{\ell=1,\{\beta_\ell\}}^{L}\sum_{\{\alpha_1\}}(-1)^{\varepsilon_{\alpha_1}+1}\lambda^{\alpha_1}C^{\alpha_k}\overset{(1)}{U}{}^{\beta_\ell}_{\alpha_1\alpha_k}\bar{\mathcal{P}}_{\beta_\ell} + O(gh^4) + I_Q^A\{Q^A,\Omega\}. \end{aligned} \qquad (23)$$

It's clear that $\psi_H(\Gamma)$ has the form of (18). Since the physics doesn't depend on the choice of ψ_H, we consider $\mathcal{S}(Q,\tilde{I}_Q)$ in the limit $\beta\to\infty$ and we'll show, that it has the following form:

$$\mathcal{S} = S_{min} + \tilde{I}_{\bar{C}_{\alpha_L}}\pi_{\alpha_L}; \qquad (24)$$

where S_{min} satisfies (2) and the initial condition (3). Making in (11) (where $S_{eff}^{(H),I}$ is given by (23)) the change of variables

$$\bar{\mathcal{P}}_{\alpha_1} \to b^{-1/2} \cdot \bar{\mathcal{P}}_{\alpha_1} ;\ \bar{C}^{\alpha_i} \to b^{-1/2} \bar{C}^{\alpha_i} ;\ \pi_{\alpha_i} \to b^{-1/2} \pi_{\alpha_i}, (i = 1, \dots, L-1);$$
$$\mathcal{P}_{\alpha_k} \to b^{-1/2} \mathcal{P}_{\alpha_k} (k = 1, \dots, L);\ \lambda^{\alpha_j} \to b^{-1/2} \lambda^{\alpha_j} (j = 2, \dots, L);\ \lambda^{\alpha_1} \to b^{-1} \lambda^{\alpha_1}.$$

we obtain in the limit $b \to \infty$ (after integration over all above-cited variables, except the λ^{α_1}) the expression:

$$e^{\frac{i}{\hbar}\mathcal{S}(Q, I_Q)} = e^{\frac{i}{\hbar}\tilde{I}_{\bar{C}_{\alpha_L}}\pi_{\alpha_L}} \int DY D\lambda_{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum \dot{q}p + \sum_{k=2,\{\alpha_k\}}^{L} \dot{C}^{\alpha_k} \bar{\mathcal{P}}_{\alpha_k} - H_0 - \sum_{i=1,\{\alpha_i\}}^{L} \sum_{k=2,\{\alpha_k\}}^{i+1} C^{\alpha_i} V_{\alpha_i}^{\alpha_k} \bar{\mathcal{P}}_{\alpha_k} + \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1} +$$
$$+ \sum_{k=1,\{\alpha_k\}}^{L} \tilde{I}_q\{q, \phi_{\alpha_k}\} C^{\alpha_k} + \sum_{k=2\{\beta_k\}}^{L} \sum_{\{\alpha_1\}} \sum_{\{\alpha_L\}} \lambda^{\alpha_1} C^{\alpha_L} (-1)^{\varepsilon_{\alpha_1}+1} \overset{(1)}{U}{}^{\beta_k}_{\alpha_1\alpha_L}(q,p) \bar{\mathcal{P}}_{\beta_k} + \tag{25}$$
$$+ T(Y, q, C^{\alpha_L}, \lambda^{\alpha_1}, \tilde{I}_q, \tilde{I}_{C^{\alpha_L}})\Big]\Big\},$$

where $\{Y\} = \{p, \bar{\mathcal{P}}_{\alpha_2}, \dots, \bar{\mathcal{P}}_{\alpha_L}, C_{\alpha_1}, \dots, C_{\alpha_{L-1}}\}$, and the functional T is independent of π_{α_L} and $\tilde{I}_{\bar{C}_{\alpha_L}}$ and contributes to $\mathcal{S}$ either in loop approximations or in summands, containing powers of the fields C_{α_L} higher than unity. Thus $\mathcal{S}$ in this gauge indeed has the form (24), with S_{min} given by the expression:

$$e^{\frac{i}{\hbar}S_{min}(Q_{min}, I_{Q_{min}})} = \int DY D\lambda^{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum \dot{q}p + \sum_{k=2,\{\alpha_k\}}^{L} \dot{C}^{\alpha_k} \bar{\mathcal{P}}_{\alpha_k} - H_0 - \sum_{i=1,\{\alpha_i\}}^{L} \sum_{k=2,\{\alpha_k\}}^{i+1} C^{\alpha_i} V_{\alpha_i}^{\alpha_k} \bar{\mathcal{P}}_{\alpha_k} + \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1} +$$
$$+ \sum_{k=1,\{\alpha_k\}}^{L} I_q\{q, \phi_{\alpha_k}\} C^{\alpha_k} + \sum_{k=2,\{\beta_k\}}^{L} \sum_{\{\alpha_1\}} \sum_{\alpha_L} \lambda^{\alpha_1} C^{\alpha_L} (-1)^{\varepsilon_{\alpha_1}+1} \overset{(1)}{U}{}^{\beta_k}_{\alpha_1\alpha_L}(q,p) \bar{\mathcal{P}}_{\beta_k} + \tag{26}$$
$$+ T(Y, q, C^{\alpha_L}, \lambda^{\alpha_1}; I_q, I_{C_{\alpha_L}})\Big]\Big\}$$

Using this expression it's easy to demonstrate that S_{min} satisfies the initial condition (12) (or (3)). The contribution to $S_{min}\big|_{I=C_{\alpha_L}=\hbar=0}$ comes from the tree approximation of the integral

$$\int DY D\lambda^{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum \dot{q}p - H_0 + \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1} + \sum_{k=2,\{\alpha_k\}}^{L} \Big(\dot{C}^{\alpha_k} - \sum_{i=k-1,\{\alpha_i\}}^{L-1} C^{\alpha_i} V_{\alpha_i}^{\alpha_k}\Big) \bar{\mathcal{P}}_{\alpha_k}\Big]\Big\}. \tag{27}$$

Integrating in (27) over ghost fields, contained in $\{Y\}$, we get from (27)

$$\int Dp D\lambda^{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum \dot{q}p - H_0 + \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1}\Big]\Big\}. \tag{28}$$

In the latter expression we omitted the factor

$$\prod_{i=1}^{L-1} \Big(\mathrm{sdet}\, V_{\alpha_i}^{\alpha_{i+1}}\Big)^{-1} \tag{29}$$

which doesn't contribute to the tree approximation of the integral (27). The tree approximation of the integral (28) is equal to $\exp\{\frac{i}{\hbar} S_0\}$ where S_0 is given by the value of the action

$$S = \int dt \Big(\sum \dot{q}p - H_0 + \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1}\Big) \tag{30}$$

on the extremals defined by equations:

$$\frac{\delta S}{\delta p} = 0 \quad , \quad \frac{\delta S}{\delta \lambda^{\alpha_1}} = 0 \tag{31}$$

which coincides with the classical action S_{cl}, i.e. $S_0 = S_{cl}$

Note also that the calculation of S_{min} within the perturbation theory is performed using Feynman rules with propagators of the fields $\bar{\mathcal{P}}_{\alpha_2},\dots,\bar{\mathcal{P}}_{\alpha_L}, C_{\alpha_2},\dots,C_{\alpha_{L-1}}$ which are defined by a quadratic over these fields part of the effective action in the exponent of the integral (26). From (27) and (28) it's easy to see, that all these propagators are proportional to $\delta(x-y)$ from which follows the locality of $\hat{S}_{min}$.

This completes the proof of the equivalence of Lagrangian and Hamiltonian approaches of BRST quantization of gauge theories with first-class constraints.

5. In this section, for methodical reasons, we consider the structure of the summand in $\hat{S}_{min}$ of the form $\sim I_q c$ at $t=0$. It's well known [1-3] from the structure of general solution of the master equation that this summand has the form $I_{q^i} R^i_\alpha(q) c^\alpha$ where $R^i_\alpha(q)$ is the generator of gauge transformations, i.e. has the following property:

$$\frac{\delta_r S_{cl}(q)}{\delta q^i} R^i_\alpha(q) = 0. \tag{32}$$

We'll show explicitly that the coefficient functions of the structures $I_{q^i} c^\alpha$ $(c^\alpha \equiv c^{\alpha_1})$, obtained after calculation of the integral (26), do satisfy the condition (32).

For choice of Φ_α in general case the resulting expressions are rather cumbersome. However they are simplified significantly, when we consider a special representation for the constraints Φ_α, whose existence was proved in [11]. As it was shown in [11] the constraints Φ_α may be chosen in such a way that the conditions

$$\begin{aligned}
&\{\Phi_{\alpha_k}, \Phi_{\alpha_1}\} = \sum_{\{\beta_1\}} D^{\beta_1}_{\alpha_k \alpha_1} \Phi_{\beta_1}, \quad k = 1,\dots,L, \\
&\{H_0, \Phi_{\alpha_k}\} = \Phi_{\alpha_{k+1}} + \sum_{\{\alpha_1\}} \tilde{V}^{\alpha_1}_{\alpha_k} \Phi_{\alpha_1}, \quad k = 1,\dots,L-1, \\
&\{H_0, \Phi_{\alpha_L}\} = \sum_{\{\alpha_1\}} \tilde{V}^{\alpha_1}_{\alpha_L} \Phi_{\alpha_1}.
\end{aligned} \tag{33}$$

hold, where the matrices $C^{\alpha_{k+1}}_{\alpha_k}$, present in [11], without loss of generality are taken equal to unity. Comparing (33) with (5) and (8) we have

$$\begin{aligned}
&U^{\beta_\ell}_{\alpha_k \alpha_1} = 0, \quad \ell = 2,\dots,L \\
&V^{\alpha_k}_{\alpha_\ell} = 0, \quad k \neq 1,\ k \neq \ell+1 \\
&V^{\beta_{\ell+1}}_{\alpha_\ell} = \delta^{\beta_{\ell+1}}_{\alpha_\ell}
\end{aligned} \tag{34}$$

With respect to (34) the expression (26) defining S_{min} takes the form (with the accuracy we need)

$$\begin{aligned}
e^{\frac{i}{\hbar} S_{min}} &= \int DY D\lambda^{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum \dot{q}p - H_0 + \\
&+ \sum_{\{\alpha_1\}} \Phi_{\alpha_1}\lambda^{\alpha_1} + \sum_{k=1,\{\alpha_k\}}^{L} \sum_i I_{q^i}\{q^i, \Phi_{\alpha_k}\} c^{\alpha_k} + \\
&+ \sum_{k=2,\{\alpha_k\}}^{L} \left(\dot{c}^{\alpha_k} - c^{\alpha_{k-1}}\right) \bar{\mathcal{P}}_{\alpha_k} = \\
&= \int Dp D\lambda^{\alpha_1} \exp\Big\{\frac{i}{\hbar}\int dt \Big[\sum_i \dot{q}^i p_i - H_0 + \sum_{\{\alpha_1\}} \Phi_{\alpha_1}\lambda^{\alpha_1} + \\
&+ \sum_i I_{q^i} \sum_{k=1,\{\alpha\}}^{L} \{q^i, \Phi^{(k)}_\alpha\}\, \partial_t^{(L-k)} c^\alpha \Big]\Big\},
\end{aligned} \tag{35}$$

where $\phi^{(k)}_{\alpha} \equiv \phi_{\alpha_{k}}, \; c^{\alpha} \equiv c^{\alpha_{L}}$.

The tree approximation of this integral is given by a Lagrangian action, constructed for the"Hamiltonian":

$$H_0 - \sum_{\{\alpha_1\}} \phi_{\alpha_1} \lambda^{\alpha_1} + \Delta H \tag{36}$$

where

$$\Delta H = -\sum_{i} I_{q_i} \sum_{k=1\{\alpha\}} \{q^i, \phi^{(k)}_{\alpha}\} \partial_t^{(L-k)} c^{\alpha}$$

As known, for the first-order corrections the relation $\Delta L = -\Delta H$ holds. Hence in the first order over C we have

$$S_{min} = S_{cl} + \sum_{i} I_{q_i} R^i_{\alpha} C^{\alpha}, \tag{37}$$

where

$$R^i_{\alpha} = \sum_{k=1,\{\alpha\}}^{L} \{q^i, \phi^{(k)}_{\alpha}\} \partial_t^{(L-k)}. \tag{38}$$

In papers [11,12] it was shown that the expression (38), indeed, is a generator of gauge transformations of the action in the Lagrangian form.

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СИСТЕМЫ СО СВЯЗЯМИ ПЕРВОГО РОДА

(на английском языке,перевод авторов)

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