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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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RADIATIVE CORRECTION TO COINCIDENCE EXPERIMENTS

ЦНИИатоминформ

29 - 45

ЕРЕВАН - 1991

We regret that some of the pages in this report may not be up to the proper legibility standards, even though the best possible copy was used for scanning

Նախատիպ ԵՖԻ-1331(26)-91

Ս.Մ.ՍԱՐԳՍՅԱՆ

ՃԱՌԱԳԱՅԹԱՅԻՆ ՈՒՂՂՈՒՄՆԵՐԸ ՀԱՄԸՆԿՆՈՂԱԿԱՆ ՓՈՐՁԵՐԻ ԺԱՄԱՆԱԿ

Նկարագրված է ճառագայթային ուղղման մեթոդիկան համընկնողական $e + A \longrightarrow e' + B + C$ ռեակցիայի ժամանակ, երբ ցրված էլեկտրոնը և B մասնիկը, որը շատ անգամ ծանր է էլեկտրոնից, գրանցվում են միաժամանակ: Ստացված արտահայտությունները հնարավորություն են տալիս հաշվի առնել նաև, բազմաֆոտոնային ճառագայթումները և ճառագայթումները կապված ոչ միայն փոխազդող միջուկի դաշտում արգելակային ճառագայթման հետ, այլ նաև թիրախի նյութի միջով էլեկտրոնի անցման հետ: Դուրս է բերված նաև ֆորմուլաներ, որոնք թույլ են տալիս գնահատել ոչառաձգական պրոցեսների տիրույթում ավելի փոքր վերջնական հադրոնային W մասսա ունեցող պրոցեսների ճառագայթային աղավաղումները:

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1. Introduction

The electron-nuclear reactions are very successful tools for investigation of fundamental structure of nuclei. Although the weakness of electromagnetic interaction allows to resolve the electromagnetic property of nuclei by less distortion, the smallness of electron mass requires to carefully take into account effects of bremsstrahlung both: in the field of nucleus on which interactions take place (internal bremsstrahlung) and in the matter of target (external bremsstrahlung) through which electrons passed before and after interaction.

Since the available electron accelerators allow to effectively investigate mainly inclusive processes, there are more detailed considerations of the radiative corrections to inclusive electron spectra [1,2,3] and elaborations of the methods to restore the nonradiative cross sections from experimental data (unfolding procedure see chap.3 in [2]). The coincidence processes up to now were performed mainly at not so large value of Q^2 and in the kinematical range of quasielastic scattering (e.g. see Refs.[4,5,6]) where the mass of undetected final hadron system is known, which allows to fix the value of bremsstrahlung photon energy. For these conditions the radiative effects are small and can be sufficiently described in the framework of the one - photon approximation (at the moderate values of the bremsstrahlung photon energy).

However the designed facilities at CEBAF [7] and SLAC [8] significantly extend the possibilities of coincidence experiments, where apart from quasielastic processes the inelastic processes at the large range of Q^2 can be investigated. For such experiments the radiative correction for all kinematical ranges (quasielastic, inelastic, deep-inelastic) are needed. Moreover, it is essential to take into account the distortion of scattered electron spectra by radiation effect of processes with lower mass of final hadronic state (radiation tails).

In this work procedures of radiative corrections obtained by Mo and Tsai [1,3] for inclusive (ee') scattering in the continuum are modified for the coincidence processes and the radiative tail from quasielastic peak is obtained.

In Section 2 a brief description of the main process and the cross section of coincidence reactions in the Born approximation (in general form) are presented.

In Section 3 the coincidence process by bremsstrahlung photon accompanied are considered. At first we restricted ourselves to one photon emission and the corresponding diagrams for infrared singularity reducing are considered. And then similarly to [1,3] takes into account the corrections for multi-photon radiations.

In Section 4 the effects of radiation in the target matter (straggling effects) are considered. The coincidence cross sections by taking into account the internal and external bremsstrahlung are obtained. Besides these the radiative tail from discrete excitation levels is described. At the conclusion the same model calculations allowing to estimate the dependence of the size of radiative effects on kinematical conditions of coincidence processes and the restrictions and deficiency of radiative correction calculations are discussed.

The following designation is used throughout [3]:

$p_i=(E_i,\vec{p}_i)$ – four momentum of the incident electron

$p_f=(E_f,\vec{p}_f)$ – four momentum of the outgoing electron

$p_A=(M_A,0)$ – four momentum of the target particle

$k=(\omega,\vec{k})$ – four momentum of the photon emitted

$p_B=(E_B,\vec{p}_B)$ – four momentum of the detected particle (in coincidence to electron)

$p_C=p_i+p_A-p_f-k$ – four momentum of the undetected final hadron system

$u = (u_0,\vec{u}) = p_i + p_A - p_f - p_B = p_C + k$

$q = (q_0,\vec{q}) = p_i - p_f$

$q'^2=(p_i - p_f - k)^2 = (p_C + p_B - p_A)^2$

ϑ_e – electron scattering angle

ϑ_B – angle between $\vec{p}_i$ and $\vec{p}_B$

φ – angle between $(\vec{p}_i\vec{p}_f)$ and $(\vec{p}_i\vec{p}_B)$ planes

θ – angle between $\vec{p}_B$ and transferred momentum $q = (p_i - p_f)$

ϑ_k – angle between

$M_A, M_B, M_C, M_N, m_e, m_\pi$ – are the mass of target, detected particle, undetected hadron system, nucleon, electron and pion respectively.

Ze – target nucleus charge.

$\alpha = 1/137$ – fine structure constant

2.Cross section of Basic Process

In general we'll consider the following process (see Diagram 1a)

$$e + A \to e' + B + C \qquad (2.1)$$

where the scattered electron e' and particle B are registered in coincidence.

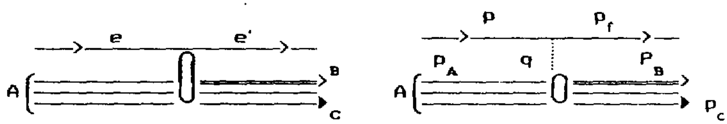


Diagram 1a Diagram 1b

In the case of one photon exchange approximation (Born approximation, Diagram 1b) the invariant cross section for the process (2.1) in general can by presented as follows (see e.g. Ref.[9]):

$$d\sigma = \frac{e^4 Z^2}{((p_i p_A)^2 - m_e^2 M_A^2)^{1/2}} |T_{if}|^2 \frac{d^3p_f}{2E_f} \frac{d^3p_B}{2E_B} \frac{1}{(2\pi)^6} \qquad (2.2)$$

where

$$|T_{if}|^2 = t^{\mu\nu} T_{\mu\nu} / q^4 \qquad (2.3)$$

in (2.3) $t^{\mu\nu}$ is the tensor composed by electron currents:

$$t^{\mu\nu} = \frac{1}{2}\cdot\sum_{s\,p_i} \bar{u}(p_f)\cdot\gamma^{\mu}u(p_i)\cdot\bar{u}(p_i)\cdot\gamma^{\nu}u(p_f) =$$
$$= 2\cdot(p_i^{\mu}p_f^{\nu} + p_i^{\nu}p_f^{\mu} - (p_i p_f)g^{\mu\nu})) \qquad (2.4)$$

where $T_{\mu\nu}$ is the tensor consisting of A,B,C particle electromagnetic currents. The tensor $T_{\mu\nu}$ in general can be expressed (using the gauge and Lorentz invariance) as follows (see Refs.[10,16]):

$$T_{\mu\nu} = g_{\mu\nu}\cdot F_0 + p_{A\mu}p_{A\nu}\cdot F_1 + p_{B\mu}p_{B\nu}\cdot F_2 + \frac{1}{2}\cdot(p_{A\mu}p_{B\nu} + p_{A\nu}p_{B\mu})\cdot F_3 \quad (2.5)$$

where $F_i(\equiv F_i(q,p_A,p_B,M_C^2)$, $i=0,1,2,3)$ are the invariant nuclear structure functions. In the (2.5) terms proportional to q^{μ} or q^{ν} are omitted (by taking into account the current conservation).

Note that for the case of discrete excitation state of final hadron system the structure functions are expressed as follows:

$$F_i(q,p_A,p_B,M_C^2) \equiv F_i(q,p_A,p_B)\cdot\delta((q+p_A-p_B)^2 - M_C^2) \qquad (2.6)$$

3. Bremsstrahlung Process in Coincidence Reactions

As mentioned above, the investigated process (2.1) is always accompanied by real photon emission and the following process actually takes place:

$$e + A \;\rightarrow\; e' + B + C + \gamma \qquad (3.1)$$

In the lowest order of the perturbative theory the bremsstrahlung process corresponds to the case of one - photon emission. In the calculation of the one - photon emission cross section, to avoid the infrared singularity, it is necessary to simultaneously consider the following diagrams (Diagram 2).

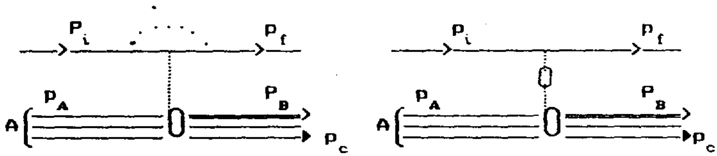


Diagram 2

The cross section of the corresponding diagrams was calculated by many authors (see e.g. Refs.[13,18] and can be expressed as follows:

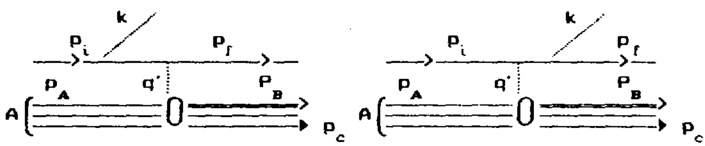
$$\frac{d^4\sigma}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B}\cdot\delta^{el} \qquad (3.2)$$

where

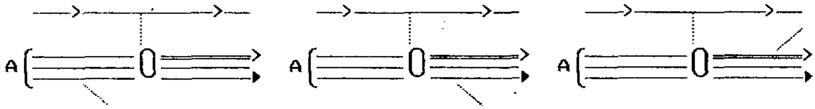
$$\delta^{el} = \alpha/\pi\cdot\{\ln(m_e^2/\lambda^2)\cdot(1-\ln(-q^2/m_e^2)) - \frac{1}{2}\cdot\ln^2(-q^2/m_e^2) + \frac{13}{6}\cdot\ln(-q^2/m_e^2) - \frac{28}{9} + \frac{\pi^2}{6}\} \qquad (3.3)$$

the $\frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B}$ is a cross section of coincidence process (2.1) in the Born approximation. The real photon mass is brought to artificially, which will be reduced at simultaneously considered one photon bremsstrahlung process (this is a general method to avoid the infrared singularities [13,18]).

It can be seen [12] that the main contribution to the bremsstrahlung cross section for process (3.1) is given by the following diagrams (Diagram 3)



a b



c d e

Diagram 3

Assuming that A,B,C particles have a mass much heavier than electron (that are true for the process of interest), only the diagrams 3a and 3b will be considered. The cross section corresponding to these diagrams, in general, can be presented similarly as eqs.(2.2) and (2.3) :

$$d\sigma^r = \frac{e^4 Z^2}{((p_i p_A)^2 - m_e^2 M_A^2)^{1/2}} \left| T^r_{if} \right|^2 \frac{d^3 P_f}{2E_f} \frac{d^3 P_B}{2E_B} \frac{d^3 k}{2E_k} \cdot \frac{1}{(2\pi)^8} \qquad (3.4)$$

where

$$\left| T^r_{if} \right|^2 = t_r^{\mu\nu} T^r_{\mu\nu} \; / \; q'^4 \qquad (3.5)$$

The tensor of $t_r^{\mu\nu}$ after summation over photon polarization and averaging over electron spins can be expressed in the following terms [16]:

$$t_r^{\mu\nu} = 2e^4 \cdot \Big\{ (p_i^\mu p_f^\nu + p_i^\nu p_f^\mu - (p_i p_f) g^{\mu\nu}) \times$$

$$\left(\frac{1}{kp_f} - \frac{1}{kp_i} - \frac{m_e^2}{(kp_f)^2} - \frac{m_e^2}{(kp_i)^2} + \frac{2 \cdot (p_i p_f)}{(kp_i)(kp_f)} \right) +$$

$$+ g^{\mu\nu} \left((p_i p_f) \cdot \left(\frac{1}{kp_i} - \frac{1}{kp_f} \right) + \frac{m_e^2 (kp_i)}{(kp_f)^2} - \frac{m_e^2 (kp_f)}{(kp_i)^2} - \frac{kp_i}{kp_f} - \frac{kp_f}{kp_i} \right) +$$

$$+ (k^\mu p_i^\nu + k^\nu p_i^\mu) \times \left(\frac{-m_e^2}{(kp_f)^2} + \frac{p_i p_f}{(kp_i)(kp_f)} + \frac{1}{kp_f} \right) - \frac{2 p_i^\mu p_f^\nu}{kp_i} +$$

$$+ (k^\mu p_f^\nu + k^\nu p_f^\mu) \times \left(\frac{m_e^2}{(kp_i)^2} - \frac{p_i p_f}{(kp_i)(kp_f)} + \frac{1}{kp_i} \right) + \frac{2 p_f^\mu p_f^\nu}{kp_f} \Big\} \qquad (3.6)$$

In (3.5) $q' = p_i - p_f - k$ and $T_r^{\mu\nu}$ have exactly the same form as in (2.5), only for structure functions F_i(i=0,1,2,3) one must use

q' instead of $q = p_i - p_f$.

Since in the investigated process of (2.1) the emitted real photon is unregistered, one must integrate over energy and angles of such photon. Therefore one must in fact calculate the following:

$$\frac{d^4\sigma^r}{dE_f d\Omega_f dp_B d\Omega_B} = \int_{\Omega_k}\int_{o}^{\omega_{kmax}} \left[\frac{e^2 Z^2 E_f P_B^2}{4E_i M_A (2\pi)^8}\right] \cdot |T^r_{if}|^2 \frac{k^2 dk\, d\Omega_k}{2\omega_k} \quad (3.7)$$

where ω_{kmax} is the maximal possible value of bremsstrahlung photon. To perform such an integration, one divides the energy integration range into two parts: $(0,\omega_{kma}) = (0,\Delta) + (\Delta,\omega_{kmax})$, where Δ are chosen by two following conditions: at first the quantity of Δ must by sufficiently small to neglect structure functions change in the $(0,\Delta)$ range, and secondly the integrals in $(0,\Delta)$ range are determined mainly by infrared part of $t_r^{\mu\nu}$

$$t_r^{\mu\nu}(\text{inf. red}) \simeq \left(-\frac{me^2}{(kp_f)^2} - \frac{me^2}{(kp_i)^2} + \frac{2\cdot(p_i p_f)}{(kp_i)(kp_f)}\right) \quad (3.8)$$

After such dividing of energy integration range and using the so called "peaking" approximation for the angular integration in the range (Δ,ω_{kmax}) (see appendix A), one obtains (simultaneously taking into account eqs. (2.2), (A.2), (A.4) and (3.2)) the following relations for the radiative cross section restricted by one – photon bremsstrahlung:

$$\frac{d^4\sigma^r}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B}\cdot(1+\delta) +$$

$$+ \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i - E_i')} \cdot \frac{d^4\sigma^0(E_i',E_f)}{dE_f d\Omega_f dp_B d\Omega_B} \cdot dE_i' + \int_{E_f-\Delta_f}^{E_{fMax}} \frac{\psi_f}{(E_f' - E_f)} \cdot \frac{d^4\sigma^0(E_i,E_f')}{dE_f' d\Omega_f dp_B d\Omega_B} \cdot dE_f' \quad (3.9)$$

where $\delta = \delta^{el} + \delta^{inel}$ and δ^{inel}, ψ_i, ψ_f are presented in appendix A. The E_{iMin} and E_{fmax} are determined under kinematical conditions of (2.1) process.

Note that the formulas in (3.9) come to agreement with the corresponding expressions obtained in [17], if the latter is integrated over emitted photon energy.

In order to generalize the (3.9) for the case of multiphoton emission, one needs to point out the following: the ratio of the first term to the two remained in (3.9) has an order $\simeq \ln(E_i/\Delta)$, and therefore the last two terms can be taken as smallest quantities of the first order with respect to first term, and that for each order of perturbative theory, there are corresponding diagrams (similar to diagrams in fig.2) to cancel the infrared singularity in the same order of bremsstrahlung [13]. In the framework of this assumption the radiative cross section generalized by the multiphoton emission (see appendix C) can be written as:

$$\frac{d^4\sigma^r}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B}\cdot \exp(\delta(\Delta_i,\Delta_f)) +$$

$$+ \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i-E_i')}\cdot\frac{d^4\sigma^0(E_i',E_f)}{dE_f d\Omega_f dp_B d\Omega_B}\cdot \exp(\delta(E_i-E_i'))dE_i' +$$

$$+ \int_{E_f-\Delta_f}^{E_{fMax}} \frac{\psi_f}{(E_f'-E_f)}\cdot\frac{d^4\sigma^0(E_i,E_f')}{dE_f' d\Omega_f dp_B d\Omega_B}\cdot \exp(\delta(E_f'-E_f))dE_f' \qquad (3.10)$$

4.Total Radiative Correction to the Continuum spectra and Radiative Tail From the Discrete Levels

Before and after the considered electron-nucleu scattering takes place (suppose that the scattered electron angle is much larger than m_e/E_i), the electron passed through a medium of target, which have a finite thickness and therefore lose the energy by ionization and bremsstrahlung (straggling). The simultaneous accounting of these effects similar to [3] and processes of internal bremsstrahlung (see eq.(3.10)), allowed to obtain the total radiative (and ionization) cross section for considered process (2.1)

$$\frac{d^4\sigma^{exp}}{dE_f d\Omega_f dp_b d\Omega_B} = \left(\frac{R\Delta}{E_i}\right)^{T'} \cdot \left(\frac{\Delta}{E_f}\right)^{T'} \cdot \left[1 - \frac{\xi}{(1-2T')\Delta}\right] \cdot \sigma^{eff}(E_i,E_f) +$$

$$+\int_{E_{iMin}(E_f)}^{E_i - R\Delta} \sigma^{eff}(E_i',E_f) \left(\frac{E_i - E_i'}{E_f R}\right)^{T'} \left(\frac{E_i - E_i'}{E_i}\right)^{T'} \left[\frac{T'}{E_i - E_i'} \cdot \varphi((E_i - E_i')/E_i) + \frac{\xi}{2(E_i - E_i')^2}\right] dE_i'$$

$$+\int_{E_f + \Delta}^{E_{fMax}(E_i)} \sigma^{eff}(E_i,E_f') \left(\frac{E_f' - E_f}{E_f'}\right)^{T'} \left(\frac{E_f' - E_f}{E_i} \cdot R\right)^{T'} \left[\frac{T'}{E_f' - E_f} \cdot \varphi((E_f' - E_f)/E_f') + \frac{\xi}{2(E_f' - E_f)^2}\right] dE_f'$$

(4.1)

where (all these quantities are determined in [1,3])

$\frac{d^4\sigma^{exp}}{dE_f d\Omega_f dp_b d\Omega_B}$ – experimentally measured cross section,

$\sigma^{eff}(E_i,E_f) = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_b d\Omega_B} \cdot F(q^2,T)$,

$$F(q^2,T) = (1 + 0.5772 \cdot b \cdot T) \times$$
$$\times \exp\left[-\frac{\alpha}{\pi} \cdot \left(\frac{28}{9} - \frac{13}{6} \cdot \ln(-q^2/m_e^2) - \Phi(-(E_i - E_f)/E_f) - \Phi((E_i - E_f)/E_i)\right)\right],$$

$\varphi(\Delta/E) = 1 - (\Delta/E) + 3/4 \cdot (\Delta/E)^2$ (ref.[3])

$T' = b \cdot (T/2 + t_r)$, $t_r = \frac{\alpha}{\pi \cdot b} \cdot (\ln(2p_i p_f/m_e^2) - 1)$

$\xi = 0.154 \cdot Z/A \cdot T \cdot x_0$ (MeV)

$b \simeq 4/3$ (for complete expression of b see [1])

T – is the target thickness in units of radiative length,

x_0 – is the unit radiation length in gm/cm^2

$\Phi(x)$ is the Spence function (see appendix B) and the

$$R = \left| \frac{M_A + 2E_i \sin^2(\vartheta_e/2) - E_B + p_B(\cos(\vartheta_e)\cos(\vartheta_B) + \sin(\vartheta_e)\sin(\vartheta_B)\cos(\varphi))}{M_A - 2E_f \sin^2(\vartheta_e/2) - E_B + p_B \cos(\vartheta_B)} \right|$$

(4.2)

The expression in (4.1) represents the formulas for radiative correction to continuum spectra. As mentioned above, the E_{iMin} and E_{fMax} were determined from kinematical conditions of investigated process and for the reaction (2.1) they are as follows:

$$E_{fMax} = \frac{M_A^2 + M_B^2 - (M_C^0 + m_\pi)^2 - 2M_A E_B + 2E_i(M_A - E_B + |\vec{p}_B|\cos(\vartheta_B))}{2M_A + 4E_i\sin^2(\theta_e/2) - 2E_B + 2(\vec{p}_B\vec{p}_f)/|\vec{p}_f|}$$

$$E_{iMin} = \frac{-M_A^2 - M_B^2 + (M_C^0 + m_\pi)^2 + 2M_A E_B + 2E_f(M_A - E_B + (\vec{p}_B\vec{p}_f)/|\vec{p}_f|)}{2M_A - 4E_f\sin^2(\theta_e/2) - 2E_B + 2|\vec{p}_B|\cos(\vartheta_B)} \tag{4.3}$$

The Δ is taken as a segment in the E_f value axis satisfied the conditions described in Section 3. As can be seen (from derivation of (4.1) by Δ), the experimentally measured cross section $\left(\frac{d^4\sigma^{exp}}{dE_f d\Omega_f dp_b d\Omega_B}\right)$ is independent of Δ (with precision up to $O((\Delta/E_f)^2)$), that confirms artificially brought of Δ.

Note that in (4.3) $M_C = M_C^0 + m_\pi$, where M_C^0 is the lowest possible final hadron system mass corresponding to quasielastic scattering.

The expression in (4.1) allows to obtain the radiative tail from discrete excitation level. In order to calculate such radiative tails, one must use the relation of (2.6), which permits to perform the integration in (4.1). For illustration we present here the radiative corrections and tails from quasielastic peak. After inserting (2.6) to (4.1) and integrating over E_i' and E_f' one obtains:

- for radiative corrections to the quasielastic scattering

$$\frac{d^3\sigma^{exp}}{dE_f d\Omega_f d\Omega_B} = \left(\frac{R\Delta}{E_i}\right)^{T'} \cdot \left(\frac{\Delta}{E_f}\right)^{T'} \cdot \left[1 - \frac{\xi}{(1-2T')\Delta}\right] \cdot \frac{d^3\sigma^{eff}(E_i, E_f)}{dE_f d\Omega_f d\Omega_B} \tag{4.4}$$

which agrees to the corresponding expression obtained in [16],

- and for the radiative tail from quasielastic peak

$$\frac{d^4\sigma^{tail}}{dE_f d\Omega_f dp_b d\Omega_B} =$$

$$= \left\{ \frac{d^3\sigma^{eff}(E_i - w_i, Ef)}{dE_f d\Omega_f d\Omega_B} \cdot \left| \frac{M_A \cdot |\vec{p}_B|/E_B + q_c^i \cdot |\vec{p}_B|/E_B - q_v^i \cdot \cos(\theta^i)}{-2E_f \sin^2(\theta_e/2) + M_A - E_B + |\vec{p}_B| \cdot \cos(\theta_B)} \right| \times \right.$$

$$\times \left(\frac{T'}{w_i} \cdot \varphi(w_i/E_i) + \frac{\xi}{2w_i^2} \right) \quad +$$

$$+ \frac{d^3\sigma^{eff}(Ei, Ef+wf)}{dE_f d\Omega_f d\Omega_B} \cdot \left| \frac{M_A \cdot |\vec{p}_B|/E_B + q_c^f \cdot |\vec{p}_B|/E_B - q_v^f \cdot \cos(\theta^f)}{-2E_i \sin^2(\theta_e/2) - M_A + E_B - (\vec{p}_B\vec{p}_f)/|\vec{p}_f|} \right| \times$$

$$\left. \times \left(\frac{T'}{w_f} \cdot \varphi(w_f/(E_f + w_f)) + \frac{\xi}{2w_f^2} \right) \right\} \cdot \left(\frac{wi}{E_i} \right)^{T'} \cdot \left(\frac{wf}{E_f + w_f} \right)^{T'} \qquad (4.5)$$

where

$$w_i = \frac{u_\mu^2 - M_c^{o\,2}}{2(M_A - E_f(1-\cos(\theta_e)) - E_B + |\vec{p}_B| \cdot \cos(\theta_B))}$$

$$w_f = \frac{u_\mu^2 - M_c^{o\,2}}{2(M_A + E_i(1-\cos(\theta_e)) - E_B + (\vec{p}_B\vec{p}_f)/E_f)}$$

(note that $w_f \cdot R = w_i$, where R is determined in eq.(4.2))

$$q_c^i = E_i - w_i - E_f; \quad -q_\mu^{i\,2} = 4(E_i - w_i)E_f \sin^2(\theta_e/2); \quad q_v^{i\,2} = q_c^{i\,2} - q_\mu^{i\,2}$$

$$\cos(\theta^i) = ((E_i - w_i) \cdot \cos(\theta_B) - (\vec{p}_B\vec{p}_f)/|\vec{p}_B|) \; / \; q_v^i$$

$$q_c^f = E_i - E_f - w_f; \quad -q_\mu^{f\,2} = 4E_i(E_f + w_f)\sin^2(\theta_e/2); \quad q_v^{f\,2} = q_c^{f\,2} - q_\mu^{f\,2}$$

$$\cos(\theta^f) = ((E_i \cdot \cos(\theta_B) - (E_f + w_f) \cdot (\vec{p}_B\vec{p}_f)/|\vec{p}_B|/E_f) \; / \; q_v^f \qquad (4.6)$$

and the labels of "eff" and other quantities (such as R, T', and $\varphi(x)$) are defined above.

The above - obtained expressions allow to estimate the dependence of radiative effects on kinematics of a concrete process to be investigated. As follows from (4.1) at the target thickness about $\Gamma \simeq 2\alpha/\pi b \cdot (\ln(-q_{\mu}^2/m_e^2)-1)$ the external bremsstrahlung is of the same order as internal one, however by increasing $-q_{\mu}^2$ (at the same target thickness) the relative contribution of internal bremsstrahlung will be dominant.

In fig.1 the events generated from quasielastic (e,e'p) process in quasifree region for the 10mm ^{12}C target (with (4b) and without (4a) radiative effects) are presented. As was expected in this case, when the one-body quasifree processes dominate, the radiative effects are small (they contain $\simeq$3-5% of the main process), since the quasielastic scattering kinematics restrict the emitted photon energy to about 30 - 40 Mev (the missing energy for the quasifree scattering).

However for the processes, whose description lies out of the framework of one-body formalism (the processes corresponded to high value of missing energy > 80 MeV), the large radiative effects are expected. In fig 2, for illustration, the radiative correction to the fast backward proton electroproduction cross section (calculated by the short-range nucleon correlation models [19]), is presented. As can be seen in fig.2, the radiative effects are significantly large ($\simeq$ 15-20% of the main process), that proves the necessity of careful performance of the radiative corrections in the investigations of many-body process (Meson Exchange Currents, Two-Three nucleon correlations, etc.).

In fig.3 the comparison of calculated (e,e'p) quasielastic cross section (where radiative effects are taken into account according of formulas (4.1) - (4.6)) with experimental data[20] is presented. This comparison shows the applicability of the considered formalism of radiative correction and allows to estimate the extent of internal and external radiative effects.

In conclusion note that expressions for radiative corrections have, in principle, the same deficiency as the corresponding ones for inclusive processes [1,3], which mainly is caused by the employment of the peaking approximation. One of these deficiencies is the unreliability of the radiative corrections at the ranges $\omega_{\gamma} \simeq E_i, E_f$, which requires certain caution in the use of radiative corrections in the deep inelastic region.

Besides that, the calculated radiative tails in eq.(4.5) are relevant to the assumption of $\delta(x)$ distribution for missing energy in nuclear spectral function, which is an acceptable approximation in the case of relatively small radiative tails. However these expressions can be safely modified for the other case of missing energy distribution.

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Appendix A. Integration over the Emitted Photon Phase Space

After dividing the range of integration described in Sec. 3 one must perform two following integrations:

$$\frac{d^4\sigma^r}{dE_f d\Omega_f dp_B d\Omega_B} = \int_{\Omega_k}\int_{0}^{\Delta}\left(\frac{e^2 Z^2 E_f P_B^2}{4E_i M_A (2\pi)^8}\right)\cdot|T^r_{if}|^2\,\frac{k^2 dk\, d\Omega_k}{2\omega_k} +$$

$$+\int_{\Omega_k}\int_{\Delta}^{\omega_{k\,max}}\left(\frac{e^2 Z^2 E_f P_B^2}{4E_i M_A (2\pi)^8}\right)\cdot|T^r_{if}|^2\,\frac{k^2 dk\, d\Omega_k}{2\omega_k} \qquad \text{(A.1)}$$

If the Δ satisfies the assumptions presented in Sec.3, then the first term of eq.(A.1) represents the contribution of soft photon emission and can be factorized in the following form (see e.g. Refs. [2,11,13]):

$$\int_{\Omega_k}\int_{0}^{\Delta}\left(\frac{e^2 Z^2 E_f P_B^2}{4E_i M_A (2\pi)^8}\right)\cdot|T^r_{if}|^2\,\frac{k^2 dk\, d\Omega_k}{2\omega_k} = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B}\cdot\delta^{inel} \qquad \text{(A.2)}$$

where

$$\sigma^{inel} = \alpha/\pi\cdot\left[\left(1+\ln(m_e^2/-q^2)\right)\cdot\left(-\ln(\Delta^2/\lambda^2)+\ln(E_i)\ -\ln(E_f)\ -\ln(m_e^2)\right)+\right.$$

$$+ 1/2\cdot\ln^2(m_e^2/-q^2) - \pi^2/6 + \Phi(-q_o/E_f) + \Phi(q_o/E_i)\Big] \qquad (A.3)$$

here $\Phi(x) = \int_0^x \frac{-\ln|1-y|}{y}\,dy$ is the Spence function.

For the integration of the second term in (A.1), *note that* the integrand has the peaking behavior (which followed from eq.(3.6)) at the both values of emitted photon angle: at $\vec{k} \parallel \vec{p}_f$ and $\vec{k} \parallel \vec{p}_i$. Such behavior of the integrand makes it possible to perform the angular integration using the "peaking" approximation [14] (according to which the integration of the function of $f(x) = h(x)\cdot g(x)$, where $h(x)$ has the evident peak at the x_o and the $g(x)$ relatively smooth, can be approximately estimated as follows: $\int_a^b h(x)g(x)dx \simeq h(x_o)\int_a^b g(x)dx$), which gives:

$$\int_{\Omega_k}\int_{\Delta}^{\omega_{k\,max}} \left(\frac{e^2Z^2E_fP_B^2}{4E_iM_A(2\pi)^8}\right)\cdot|T^r_{if}|^2\,\frac{k^2dk\,d\Omega_k}{2\omega_k} =$$

$$= \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i-E_i')}\cdot\frac{d^4\sigma^0(E_i',E_f)}{dE_f\,d\Omega_f\,dp_B\,d\Omega_B}\cdot dE_i' + \int_{E_f-\Delta_f}^{E_{fMax}} \frac{\psi_f}{(E_f'-E_f)}\cdot\frac{d^4\sigma^0(E_i,E_f')}{dE_f'\,d\Omega_f\,dp_B\,d\Omega_B}\cdot dE_f' \qquad (A.4)$$

where

$$\psi(E_i) = \frac{\alpha}{\pi}\cdot\left[\frac{1}{2}\cdot\ln(-q^2/4E_i^2) + \frac{1}{2}\cdot\ln(-q^2/m_e^2) + (E_i'/E_i)\cdot\left[(E_i'/E_i)\cdot\ln(2E_i/m_e) - 1\right]\right]$$

$$\psi(E_i) = \frac{\alpha}{\pi}\cdot(E_f^2/E_f'^2)\cdot\left[\frac{1}{2}\cdot\ln(-q^2/4E_f^2) + \frac{1}{2}\cdot\ln(-q^2/m_e^2) + (E_f'/E_f)\cdot\left[(E_f'/E_f)\cdot\ln(2E_f/m_e) - 1\right]\right] \qquad (A.5)$$

Appendix B. Multiphoton Emission

Similarly to eqs. (A.2) and (A.4) the cross section of the two - photon bremsstrahlung can be written as

$$\frac{d^4\sigma^{rr}}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^{r}}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \frac{\delta^{r}_{inel}}{2} +$$

$$+ \frac{1}{2}\int_{E_{iMin}}^{E_i-\Delta_i^r} \frac{\psi_i^{rr}}{(E_i - E_i')} \cdot \frac{d^4\sigma^r(E_i', E_f)}{dE_f d\Omega_f dp_B d\Omega_B} \cdot dE_i' + \frac{1}{2}\int_{E_f-\Delta_f^r}^{E_{fMax}} \frac{\psi_f^{rr}}{(E_f' - E_f)} \cdot \frac{d^4\sigma^r(E_i, E_f')}{dE_f' d\Omega_f dp_B d\Omega_B} \cdot dE_f' \qquad (B.1)$$

where $\frac{d^4\sigma^r}{dE_f d\Omega_f dp_B d\Omega_B}$ is the cross section in the case of one photon emission. By taking the value of Δ the same for one and two - photon bremsstrahlung (it always can be done, because of artificial appearance of Δ) and inserting the eqs. (A.2) and (A.4) into eq. (B.1) for the cross section in the case of two real photon emission (by taking into account the assumptions described in Sec.3) one obtains:

$$\frac{d^4\sigma^{rr}}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^{0}}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \frac{\delta^{2}_{inel}}{2} +$$

$$+ \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i - E_i')} \cdot \frac{d^4\sigma^0(E_i', E_f)}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \delta_{ine} \cdot dE_i' + \int_{E_f-\Delta_f}^{E_{fMax}} \frac{\psi_f}{(E_f' - E_f)} \cdot \frac{d^4\sigma^0(E_i, E_f')}{dE_f' d\Omega_f dp_B d\Omega_B} \cdot \delta_{ine} \cdot dE_f'$$

(B.2)

The similar expressions can be obtained for three and more photon bremsstrahlung, and for experimentally measured cross section, which represents the sum of the processes e + A -> e'+ B + C + $\sum_{n=0}^{\infty} n \cdot \gamma^n$, one can write

$$\frac{d^4\sigma^{exp}}{dE_f d\Omega_f dp_B d\Omega_B} = \frac{d^4\sigma^{0}}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \left[1 + \delta_{inel} + \frac{\delta^2_{inel}}{2!} + \cdots + \frac{\delta^n_{inel}}{n!}\right]_{n\to\infty}$$

$$+ \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i-E'_i)} \cdot \frac{d^4\sigma^0(E'_i,E_f)}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \left[1 + \delta_{in} + \frac{\delta^2_{inel}}{2!} + \cdot\cdot + \frac{\delta^{n-1}_{inel}}{n-1!}\right]_{n \to \infty} \cdot dE'_i +$$

$$+ \left[i \leftrightarrow f \right] = \frac{d^4\sigma^0}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \exp(\delta_{inel}) +$$

$$+ \int_{E_{iMin}}^{E_i-\Delta_i} \frac{\psi_i}{(E_i-E'_i)} \cdot \frac{d^4\sigma^0(E'_i,E_f)}{dE_f d\Omega_f dp_B d\Omega_B} \cdot \exp(\delta_{inel}) \cdot dE'_i + \left[i \leftrightarrow f \right] \qquad (B.3)$$

By taking into account the statement according to which for the each order of perturbative theory there are corresponding diagrams (like in fig.2) accounting of which reduces the infrared singularity, one can use in eq.(B.3) the $\delta = \delta^{el} + \delta^{inel}$ instead of δ^{inel}.

Note that the latter replacement means, that the cross sections of the diagram reduce the infrared singularity, summing to exponent the reliability of which is discussed in Ref. [1].

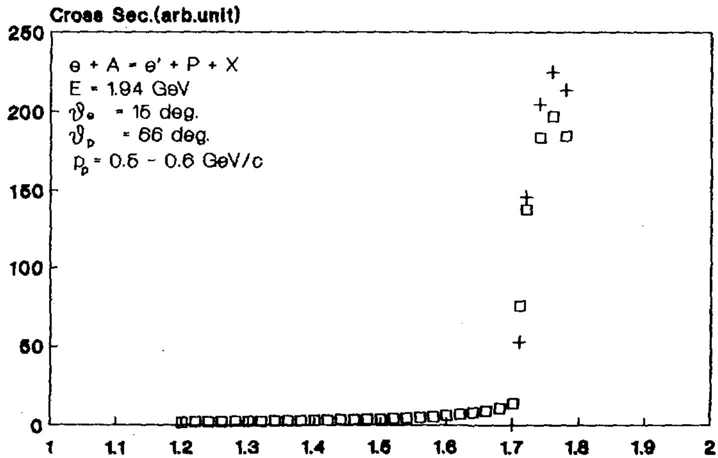
Cross Sec.(arb.unit)
e + A = e' + P + X
E = 1.94 GeV
ϑe = 15 deg.
ϑp = 66 deg.
pp = 0.5 - 0.6 GeV/c
250
200
150
100
50
0
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
E',GeV

Fig.1

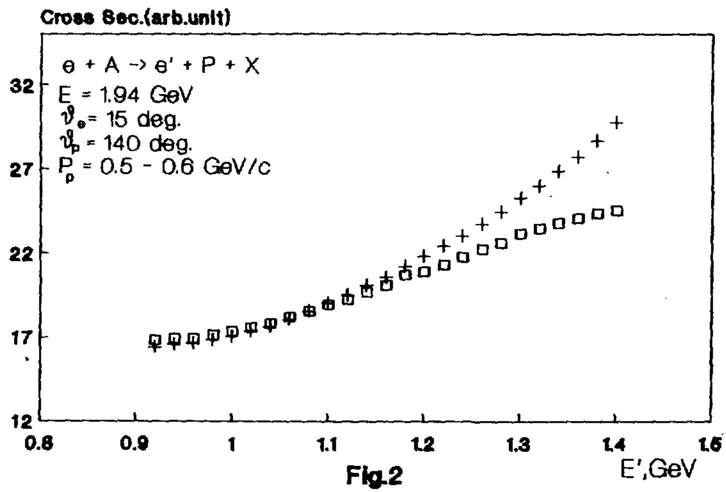
Cross Sec.(arb.unit)
e + A -> e' + P + X
E = 1.94 GeV
ϑe = 15 deg.
ϑp = 140 deg.
Pp = 0.5 - 0.6 GeV/c
32
27
22
17
12
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
E',GeV

Fig.2

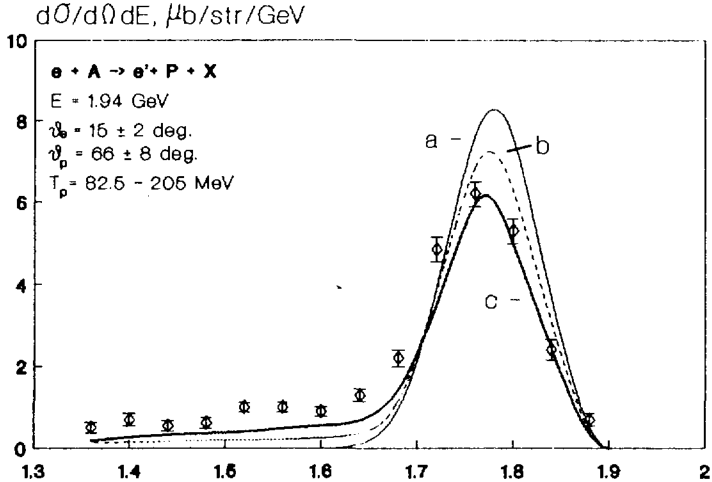
dσ/dΩ dE, μb/str/GeV
e + A -> e'+ P + X
E = 1.94 GeV
ϑ_e = 15 ± 2 deg.
ϑ_p = 66 ± 8 deg.
T_p= 82.5 - 205 MeV
a
b
c
E',GeV

Fig.3

Figure Captions

Fig.1 Spectra of events generated by quasielastic (e,e'P) processes on ^{12}C in the quasifree region, (+) - without radiative effects, (□) - radiative effects taken into account.

Fig.2 Spectra of generated events in the region kinematically forbidden for quasifree scattering, (+) - no radiative effects, (□) - radiative effects taken into account.

Fig.3 Comparison of calculated quasielastic cross section with experimental data [20] for a 10mm thick carbon target, (a) - nonradiative calculations, (b) radiative calculation without straggling effects, (c) - full radiative calculation. The calculation was made using the computer code from [21].

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РАДИАЦИОННЫЕ ПОПРАВКИ ПРИ СОВПАДАТЕЛЬНЫХ ЭКСПЕРИМЕНТАХ
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