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ON ONE SIMPLE LIMIT OF MATRIX MODELS

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After solving the Ising model on random planar dynamical lattices 1 other models on them were solved.

It turned out that these models are solved much easier than the relevant models on rigid lattices. Random lattices with M nodes come out as a set of connected M-th ordered diagrams in expansion $I(g,N)$

$$I_0(g,N)=\frac{1}{N^2}\int dU e^{-} \tag{1}$$

If we expand $I(g,N)$ in contributions of different orders of K

$$I_0(g,N)=\sum_M I\,(M,N)\frac{g^M}{M!}=\sum_M \frac{g^M}{M!}\sum_K I_2\,(M,K)\,\frac{1}{N^{2K}} \tag{2}$$

then for the given M the sum over K in (2) is taken up to (maximum possible) (1+M)/2. I(M,N) is an analytical function of N. We can continue to N=1.

Thus the simple integral

$$I(g)=\ln\int_{-\infty}^{\infty} e^{-\left(\frac{x^2}{2}+gx^4\right)}\,dx \tag{3}$$

contains the sum over random lattices of all orders.

The lattices are obtained due to different schemes of connection of vertices in the framework of Wick's theorem.

Find out the number of lattices with M nodes. It is proportional to $\frac{1}{M!}\frac{d^{\mu}}{dg^{M}}I(0)$. Consider the expansion I at large M:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(\frac{x^2}{2}+gx^4)} = \sum_{M}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}\frac{g^M}{M!}x^{4M}dx = \sum_{M}\frac{g^M(4M)!}{M!(2M)!4^M}. \qquad (4)$$

Expression (4) grows rapidly as $M! \sim M^M$. Therefore, with the accuracy to 1/M we obtain

$$\frac{1}{M!}\,\frac{d^M}{dg^M}\,\ln\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(\frac{x^2}{2}+gx^4)} = \frac{(4M)!}{(2M)!\,4^M}. \qquad (5)$$

Now let us place the Ising spins on the lattice, just like in [1] :

$$I(g,c) = \ln\int_{-\infty}^{\infty} e^{-(\frac{x^2}{2}+\frac{y^2}{2}-cxy)-g(x^4+y^4)} \qquad (6)$$

We are interested in $I_1(c,M)$ in the expansion

$$I(g,c) = \sum_{M} I_1(c,M)g^{\mu}/M! \qquad (7)$$

Similarly to (5), at $M \to \infty$ we have

$$I_1(c,M) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(x^2+y^2-2cxy)/2+M\ln(x^4+y^4)} \qquad (8)$$

Due to factorial increase in (5) we cannot obtain phase transition for any value of string tension for (1).

However, if we consider $I_2(c,M) = I_1(c,M)/I_1(0,M)$, the partition $I_2(c,M)$ will grow with M only as an exponent. Therefore, here we have the second-order phase transition over (instead of the third-order one in the matrix models with a fixed surface order).

The saddle-point equations give

$$-x+cy+4Mx^3/(x^4+y^4)=0 \tag{9}$$

$$-y+cx+4My^3/(x^4+y^4)=0 \tag{10}$$

At $1/2 < C < 1$ the symmetric solution of (9), (10) holds

$$x^2=y^2=2M/(1-c) \tag{11}$$

At $0 < C < 1/2$ we have

$$x=\sqrt{\frac{M}{1-2c^2}}\left(\sqrt{1+2c}+\sqrt{1-2c}\right), \qquad y=\sqrt{\frac{M}{1-2c^2}}\left(\sqrt{1+2c}-\sqrt{1-2c}\right) \tag{12}$$

At the saddle-point, $C_0 = 1/2$, a jump of the free-energy second derivative takes place.

At $C \to C_0$ the magnetization in nonsymmetric phase vanishes ~ (X-Y):

$$m \sim (C-C_0)^{1/2} \tag{13}$$

Such a law is characteristic of the mean field or $d \to \infty$.

Consider now the Potts model

$$Z(M,C)=\prod_\alpha \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx_\alpha\, e^{-\sum_\alpha \frac{x_\alpha^2}{2}+c\sum_{\alpha>\beta}x_\alpha x_\beta+M\ln\left(\sum_\alpha x_\alpha^4\right)} \tag{14}$$

Here the phase transition occurs at C=2/(I+3 (Q-1)). In addition, we find $f_1(C)$ function breaking in asymptotic expansion $\ln Z(M,C)$

$$\begin{gathered}\ln Z(M,C)=Mf_0(C)+\dots f_1(C)+O(1)\\ \lim_{\varepsilon\to 0} f_1(c_0+\varepsilon)-f_1(c_0-\varepsilon)=\ln Q\end{gathered} \tag{15}$$

which is in agreement with [2] .

Other phase transition models can be solved in the same way. Though, most probably, these solutions will correspond to infinite-dimensional lattice limit. However, there exist models (say, spin glasses) wherein this limit has a physical sense.

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