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THE INFLUENCE OF THE INSTANTON MEDIUM ON LIGHT QUARKS

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We regret that some of the pages in this report may not be up to the proper legibility standards, even though the best possible copy was used for scanning

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ԻՆՍՏԱՆՏՈՆԱՅԻՆ ՄԻՋԱՎԱՅՐԻ ԱԶԴԵՑՈՒԹՅՈՒՆԸ ԹԵԹԵՎ ՔՎԱՐԿՆԵՐԻ ՎՐԱ

Ինստանտոնային վակուումի մոդելում ($N_f=1$) շարունակական ինտեգրման եղանակով ուսումնասիրվել են զանգվածներով պայմանավորված ուղղումները: Քննարկվել են բազմաթյուրմանիկային վիճակագրական գումարի ընտրությունն ու նրա միջինացման հիմնահարցը: Ստացված գործողությունը պարունակում է բոզոնային կես սպինով դաշտերի հետ քվարկների արդյունարար փոխազդեցություն, որը թույլ է տալիս կիրառել խոտորումների տեսություն: Վերականգնվել են նախկինում դիագրամային մոտեցմամբ ստացված ինքնահամաձայնեցման պայմանն ու քվարկի պրոպագատորը:

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Recently there was an essential progress in understanding the mechanism of spontaneous breaking of chiral symmetry (SBCS) [1-5]. As a consequence of this, the physics of pseudoscalar octet particles becomes more transparent [5,6,7]. The instanton "fluid" vacuum model allowed us to describe the low-energy characteristics of (π,K) mesons in a good agreement with experimental data [5,6,8,9].

The general method for calculating the correlation functions in instanton medium proposed by Dyakonov and Petrov is the following. The instanton anti-instanton superpositions must be considered as an external classical field. The correlators in the presence of this field will depend on the characteristics of all pseudoparticles, i.e., their dimensions ρ, orientation U, and center z. Averaging over the statistical ensemble of instantons with distribution function finally give an exact correlator in instanton vacuum. This distribution function, which is the propability of arising of fluctuations with a given parameter, determines exceptionally by pseudoparticles interactions and not contain fermion-pseudoparticle interaction corrections, i.e. in quenched approximation $N_f/N_c \ll 1$. The averaging is substantially simplified due to the following approximations:

a. The instanton fluid packing parameter ρ/R, when R is the mean distance between pseudoparticles, is small $\rho/R \approx 1/3$ [4], and therefore the pseudoparticles may be considered as uncorrelated.

b. When the number of colors N_c is large, the instanton distribution as a function of instanton size ρ is very narrow and tends to the δ-shaped function at $N_c \to \infty$, so that in the leading order over $1/N_c$ sizes of instantons ρ_I may be replaced by their mean values ρ[4].

c.The Hilbert space of the fermions is truncated to the space of zero modes. This approximation is justified for the long wavelength properties of the vacuum (scales>0.3fm). The reason is that the spectrum of the Dirac operator in the field of an instanton is characterized by a gap of about $\rho^{-1} \approx 600MeV$ between the zero energy state and the continuum of scattering states[5].

A diagrammatic procedure for averaging was developed and two point correlators were calculated. However, three and higher point functions are not obtained by such a method. The procedure of ladder diagrams calculation in different directions does not exist. In general, the solution of the coupled Bethe-Salpeter type equations is technically difficult to obtain.

Therefore, there arises a problem of finding a reliable algorithm for calculation of another Green function. Beside that, it is interesting to find the effective action which is equivalent to the previous model. There are several attempts to write an action of such type[9-11]. The idea is the following: to start from the multipseudoparticle partition function, an equivalent of the previous model, and after averaging obtain the effective action. Nevertheless, all of these actions are not equivalent to the previous model, (see, for example, [11]). In fact, the QCD theory has a chiral limit, which means that quark mass may be tend to zero in the final stage of calculations and, obviously, no singular by m terms will arise. But, as will be shown in sect.1, exact calculations give such terms. In our point of view, this discrepancy is actually connected with the incorrect procedure of averaging. The influence of fermions on the distribution function (that implies extractions of zero modes and ultraviolet divergences from fermion determinant) was neglected, but this non-regularized determinant was included in the correlator definition.

In this paper we investigate this problem more carefully and in (c) approximation from QCD Lagrangean will obtained the multipseudoparticle action. Averaging over statistical ensambles using (a-b), finally gives effective action of quarks in instanton medium. Besides, we will obtain the propagator of quarks and investigate stability of our results in both chiral limit and with mass corrections.

1.Let us begin with the QCD partition function in Euclidean space.

$$Z=\int D\psi D\psi^{+} DA_{\mu} \exp[-\mathcal{S}(\psi,\psi^{+},A_{\mu})] \qquad (1)$$

The main points of the model are the following: integration on the gauge field A_{μ} is equivalent to averaging over the statistical ensemble of pseudoparticles with known distribution function of given configuration and to replacing field A_{μ} in the interaction part by superposition of instanton anti-instanton configuration.

$$Z' = \int D\psi D\psi^{+} \langle\langle \exp\int[\psi^{+}(i\hat{\partial}+i\hat{A}+im)\psi+\eta^{+}\psi+\psi^{+}\eta]d^{4}x\rangle\rangle \qquad (2)$$

$$A_{\mu}=\sum_{I} A_{\mu}^{I}+ \sum_{\bar{I}} A_{\mu}^{\bar{I}}$$

Where <<...>> denotes the average over the collective coordinates: positions of all pseudoparticles z, their orientation in color space U_I(following to (b) approximation we will substitute all sizes of pseudoparticles by $\rho=(600MeV)^{-1}$). However, note that Z' is not similar to the former model, since in the definition (2) the propagator has the following form

$$S' = - \frac{\langle\langle Det(i\hat{\nabla}+im)(i\hat{\nabla}+im)^{-1}\rangle\rangle}{\langle\langle Det(i\hat{\nabla}+im)\rangle\rangle} \qquad (3)$$

Isolation of uncertainties from this expressions demands to change the averaging mechanism , i.e., propagator (3) is not equal to the one obtained in [5].

$$S=\langle\langle-(i\hat{\partial}+i\hat{A}+im)\rangle\rangle \qquad (4)$$

Therefore, in our opinion, if we wish to construct a theory similar to (4), it is necessary to start from

$$Z=\int D\psi D\psi^+ D\chi D\chi^+ \langle\langle\exp\int[\psi^+(i\hat{\nabla}+im)\psi+\chi^+(i\hat{\nabla}+im)\chi+\eta^+\psi+\psi^+\eta]d^4x\rangle\rangle \qquad (5)$$

where we introduce bosonic spinor fields χ,χ^+ to compensate for contribution from fermions degree of freedom. Otherwise it is necessary to modify the whole averaging procedure. Remember, that the distribution function was obtained when the fermion fields were rejected.

Following (c), we will replace the exact Green functions in the field of one instanton by a model one[5]

$$S_I(x,y)=S_O(x,y)-\frac{\Phi^I(x)\Phi^{I+}(y)}{im} \qquad (6)$$

where $S_O(x,y)$ is the free propagator, $\Phi^I(x)$ is the zero mode for the I-th pseudoparticle: it is a right (left)-handed Weyl spinor for the (anti)instanton. This model is true at small momenta $\rho p \ll 1$ (the range important to the SBCS) and this propagator becomes a free one at large momenta. Using

$$S_0^{-1} = -(i\hat{\partial}+im) \quad \text{and} \quad S_I^{-1} - S_0^{-1} = -i\hat{A}_I$$

we have

$$-i\hat{A} = \sum_I (S_I^{-1} - S_0^{-1}) + \sum(I \to \bar{I})$$

$$S = \left[S_0^{-1} + \sum_I (S_I^{-1} - S_0^{-1}) + \sum(I \to \bar{I})\right]^{-1} \tag{7}$$

From (6) and (7) we have

$$i\hat{\nabla}+im = i\hat{\partial}+im+\sum_I \left[\left(1+(i\hat{\partial}+im)\frac{\Phi_I \Phi_I^+}{im}\right)^{-1} - 1\right](i\partial+im) + [I \to \bar{I}] \tag{8}$$

Using the chiral properties and normalization condition of zero modes

$$\langle \Phi_I^+ | i\hat{\partial}+im | \Phi_I \rangle = im \qquad \left[1+i\hat{\partial}\frac{\Phi_I \Phi_I^+}{im}\right]^{-1} = 1 - i\hat{\partial}\frac{\Phi_I \Phi_I^+}{im}$$

$$\left[1+(i\hat{\partial}+im)\frac{\Phi_I \Phi_I^+}{im}\right]^{-1} = 1 - \frac{1}{2}(i\hat{\partial}+im)\frac{\Phi_I \Phi_I^+}{im}$$

from (5), in chiral limit we obtain

$$Z(\eta,\eta^+) = \int D\psi D\psi^+ D\chi D\chi^+ \exp\int[\psi^+ i\hat{\partial}\psi + \chi^+ i\hat{\partial}\chi + \eta^+\psi + \psi^+\eta] d^4x \times$$

$$\times \langle\langle \prod_I \left[1 - \frac{1}{im}\int \psi^+ i\hat{\partial}\Phi_I d^4x \int \Phi_I^+ i\hat{\partial}\psi d^4y\right] \times \tag{9}$$

$$\times \exp\left[\frac{i}{m}\int \chi^+ i\hat{\partial}\Phi_I d^4x \int \Phi_I^+ i\hat{\partial}\chi d^4y\right] \prod(I \to \bar{I}) \rangle\rangle$$

and with account of mass corrections we obtain

$$Z(\eta,\eta^+)=\int D\psi D\psi^+ D\chi D\chi^+ \exp\int[\psi^+(i\hat{\partial}+im)\psi+\chi^+(i\hat{\partial}+im)\chi+\eta^+\psi+\psi^+\eta]d^4x\times$$

$$\times\langle\langle\prod_I \exp\left[-\frac{1}{2im}\int\chi^+(i\hat{\partial}+im)\Phi_I d^4x\int\Phi_I^+(i\hat{\partial}+im)\chi d^4y\right]\times \qquad (10)$$

$$\times\left(1-\frac{1}{2im}\int\psi^+(i\hat{\partial}+im)\Phi_I d^4x\int\Phi_I^+(i\hat{\partial}+im)\psi d^4y\right)\prod(I\to\bar{I})\rangle\rangle$$

Mass corrections will be analyzed in section 3. And now turn to the chiral case. Using the Grassmanian properties of ψ,ψ^+ for $Z(\eta,\eta^+)$ we have

$$Z(\eta,\eta^+)=\int D\psi D\psi^+ D\chi D\chi^+ \exp\int[\psi^+ i\hat{\partial}\psi+\chi^+ i\hat{\partial}\chi+\eta^+\psi+\psi^+\eta]d^4x\times$$

$$\langle\langle\prod_I \exp\left[\frac{i}{m}\int\chi^+ i\hat{\partial}\Phi_I d^4x\int\Phi_I^+ i\hat{\partial}\chi d^4y\right]\times \qquad (11)$$

$$\times\left(1-\frac{1}{im}\int\psi^+ i\hat{\partial}\Phi_I d^4x\int\Phi_I^+ i\hat{\partial}\psi d^4y\right)\prod_{\bar{I}}(I\to\bar{I})\rangle\rangle$$

Now the theory with Z' can be obtained from Z, if the variables χ,χ^+ are omitted

$$Z'(\eta,\eta^+)=\int D\psi D\psi^+ \exp\int[\psi^+ i\hat{\partial}\psi+\eta^+\psi+\psi^+\eta]d^4x\times \qquad (12)$$

$$\times\langle\langle\prod_I\left(1-\frac{1}{im}\int\psi^+ i\hat{\partial}\Phi_I d^4x\int\Phi_I^+ i\hat{\partial}\psi d^4y\right)\prod_{\bar{I}}(I\to\bar{I})\rangle\rangle$$

The expression (12) differs from the result of [10] since (12) is normalized not to m as in [10], but to the finite quantities in [11]. Investigate now the problem of how far we may normalize the theory to m, putting m=0 from the very beginning. Let us calculate (12) with nonzero m, and then tend m to zero. Average Z over positions and orientation of pseudoparticles, using the density matrix composed of zero fermion modes [5]:

$$\Phi^{I}_{ia}(x)\Phi^{I+}_{jb}(y)=\int\frac{d^4k\,d^4q}{(2\pi)^8}\exp[ik(x-z_I)-iq(y-z_I)]\,\frac{\phi(k)\phi(q)}{8|k||q|}\times$$

$$\left(\hat{k}\gamma_\mu\gamma_\nu\hat{q}\,\frac{1-\gamma_5}{2}\right)_{ij}\left(U_I\tau^-_\mu\tau^+_\nu U^+_I\right)_{ab} \qquad (13)$$

Here a,b(i,j) are color (spinor) indices, $\tau^{\pm}_{\mu}$ are $N_c\times N_c$ matrices with $(\vec{\tau},\mp i)$ standing in the left upper corner (all other elements are zero)τ are Pauli matrices, $\hat{k}=k_\mu\gamma_\mu$. The function $\phi(k)$ is connected with Fourier-transformed zero modes.

$$\phi(k)=\pi\rho^2\frac{d}{dz}[I_0(z)K_0(z)-I_1(z)K_1(z)]\Big|_{z=|k|\rho/2}=\begin{cases}-2\pi\rho/|k| & |k|\rho\ll 1\\ -12\pi/k^4\rho^2 & |k|\rho\gg 1\end{cases}$$

For anti-instantons $\gamma_5\to-\gamma_5$, $\tau^{\pm}_{\mu}\to\tau^{\mp}_{\mu}$, when averaging over the orientations with the Haar measure normalized to unity, we use the relations

$$\int dU=1,\qquad \int dU\,U_{mi}U^+_{jn}=\frac{1}{N}\,\delta_{ij}\delta_{mn}\ ,$$

$$\int dU\,U_{kp}U_{lq}U^+_{mr}U^+_{ns}=\frac{1}{N_c^2-1}\left[\delta_{kr}\delta_{ls}\left(\delta_{mp}\delta_{nq}-\frac{1}{N_c}\delta_{np}\delta_{mq}\right)+\binom{m\to n}{r\to s}\right] \qquad (14)$$

We can obtain

$$Z'(\eta,\eta^+)=\int D\psi D\psi^+\exp\int[\psi^+ i\hat{\partial}\psi+\eta^+\psi+\psi^+\eta]d^4x\ \left(Y_+\right)^{N_+}\left(Y_-\right)^{N_-}$$

where

$$Y_\pm=1-\frac{1}{imVN_c}\int\psi^+(k)\,\frac{1\pm\gamma_5}{2}\psi(k)a^2(k)\,\frac{d^4k}{(2\pi)^4}\qquad a(k)=|k|\phi(k)$$

$N_\pm$ is the number of (anti)instantons. Then in the thermodynamic limit $N_+=N_-=N/2$, $V\to\infty$, $N\to\infty$, $N/V=$const using the well known equation $\lim_{N\to\infty}(1+x/N)^N=e^x$ we obtain

$$Z'(\eta,\eta^+)=\int D\psi D\psi^+ \exp\int[\psi^+(-\hat{k}+i\frac{N}{2VN_c}\frac{\phi^2 k^2}{m})\psi+\eta^+\psi+\psi^+\eta]\frac{d^4k}{(2\pi)^4}$$

It means that $M(k)\sim m^{-1}$ and $\lim m\to 0$ does not exist. Theory is singular! Thus, the method can be applied only when m tends to zero from the very beginning[10], which is equivalent to the approximate cancellation of Det in exp.(3).

2.Let us now return to Z. First integrate (11) over the orientation matrices U_I, keeping only planar terms(the leading order over $1/N_c$) which means neglecting of the cross terms. Denote the integrals under consideration by I', I_{ij}^{ab}, $\lambda\equiv i/m$

$$I'=\int dU\ \exp[\lambda\int\chi^+ i\hat{\partial}\Phi_I d^4x\int\Phi_I^+ i\hat{\partial}\chi d^4y]$$

$$I_{ij}^{ab}(k,q)=\int dU\ [\hat{k}\Phi_I(k)]_i^a[\Phi_I^+(q)\hat{q}]_j^b\ \exp[\lambda\int\chi^+ i\hat{\partial}\Phi_I d^4x\int\Phi_I^+ i\hat{\partial}\chi d^4y] \tag{15}$$

It follows from the explicit expressions (13) that I' may contain only scalar and tensor productions of left components. However, note that tensor terms are suppressed in $N_c\to\infty$ limit. Thus, I' is a function of the variable t, I'(t), $t=\lambda/N_c(\chi_L^+\chi_L)(z)$ and the brackets mean convolution

$$(A_i^{+a}B_j^b)(z)=\int A_i^{+a}(k)B_j^b(q)\ \exp[iz(k-q)]a(k)a(q)\frac{d^4k\ d^4q}{(2\pi)^8}$$

Note that I' and I_{ij}^{ab} are connected

$$\lambda I_{ij}^{ab}(k,q)+\lambda^2\frac{\partial I_{ij}^{ab}(k,q)}{\partial\lambda}=\frac{\delta^2 I'}{\delta\chi_i^{+a}(k)\delta\chi_j^b(q)}$$

and to resolve this differential equation we find

$$I_{ij}^{ab}(k,q)=\frac{1}{N_c}\,\frac{I'(t)-I'(0)}{t}\,\delta^{ab}\left(\frac{1+\gamma_5}{2}\right)_{ij}a(k)a(q)\exp[z(k-q)]+ \tag{16}$$

$$+\frac{\lambda}{N_c^2}\,\frac{1}{t}\left\{\frac{d}{dt}I'(t)-\frac{I'(t)-I'(0)}{t}\right\}a(k)a(q)\exp[z(k-q)](\chi_j^{+b}\chi_i^{a})$$

From the observation[12], that the group integration is equivalent to the projection of a tensor product of fundamental representation onto the singlets of the group (the eq.(14) is an example, also see ref.[11]) we can obtain

$$I'=\frac{1}{1-\frac{\lambda}{N_c}(\chi_L^{+}\chi_L)} \tag{17}$$

Substituting (16) into (11) and integrating over position of pseudoparticles we obtain

$$Z(\eta,\eta^{+})=\int D\psi D\psi^{+}D\chi D\chi^{+}\exp[-\mathcal{S}_0+(\eta^{+}\psi)+(\psi^{+}\eta)]\times$$

$$\times\left\{\frac{1}{V}\int d^4zK_{+}\right\}^{N_{+}}\left\{\frac{1}{V}\int d^4zK_{-}\right\}^{N_{-}} \tag{18}$$

$$K_{+}=I'(t)+\frac{i}{mN_c}\left\{\frac{I'(t)-I'(0)}{t}\right\}(\psi_L^{+}\psi_L)+$$

$$+\left(\frac{i}{mN_c}\right)^2\frac{1}{t}\left\{\frac{d}{dt}I'(t)-\frac{I'(t)-I'(0)}{t}\right\}(\psi_L^{+}\chi_L)(\chi_L^{+}\psi_L)$$

$$K_{-}=K_{+}(L\to R)\qquad \mathcal{S}_0=-\int[\psi^{+}i\hat{\partial}\psi+\chi^{+}i\hat{\partial}\chi]d^4x$$

This expression is finite over m, and one can use chiral limit here[13].

As will be established later, the action contains the effective mass $M(k)\sim(N_c)^0$, and the interaction potential

$\sim(N_c)^{-1}$, caused by the circumstance, that the number of instantons grows with N_c, hence they must be calculated stage by stage. In propagator calculation the ghost fields χ,χ^+ can be integrated exactly, which was done when the quark mass was included, and approximately, when the dependence of the scalar production $(\chi_L^+\chi_L)(z)$ on z could be ignored. Such an approximation, which is true in the chiral limit, is based on the fact that the function a(k) quickly falls off for the mean values of the ghost fields $\chi^+\chi\sim 1/k^2$ (see eq.19) and the integral accumulates in the region of $k\sim q\sim 0$. In fact, the integrals over ψ and χ are factorized (due to stage by stage calculation) and the determinant which arises as a result of integration over χ,χ^+(see Appendix), is exactly calculated

$$\int_0^\infty dx dy P(x,y)\exp(-x-y)\left[1-\frac{x+y}{4N_c}+\frac{xy}{4N_c^2}\left(\frac{p^2}{m^2}+\frac{1}{4}\right)\right]^{-2N_c}$$

$$p_\mu = i\int\frac{a^2(s)}{s^2}\, s_\mu \exp[is(z-\tilde{z})]\,\frac{d^4s}{(2\pi)^4} \qquad (19)$$

i.e. in the chiral limit $m\to 0$ the dependence on $z,\tilde{z}$ disappears. Remember, that our $Z(\eta,\eta^+)$ is normalized to unity and any suppression is compensated for by analogous contribution from Det of ψ,ψ^+, i.e. dependence on $z,\tilde{z}$ in $I'(t)$ can be ignored.

It should be also emphasized that when mass corrections are taken into account, integration over χ,χ^+ is done exactly and the result in the limit $m\to 0$ coincides with the approximate one. The advantage of this approximation consists in the possibility of calculation of interaction potential which is hard to find off the chiral limit.

In connection with the aforesaid, now write the action keeping $O(1/N_c)$ terms, and substituting $(\chi_L^+\chi_L)(z)\to(\chi_L^+\chi_L)(0)$

$$\mathcal{S}=\mathcal{S}_0-N_+\ln\left[1+\frac{i}{mN_c}(\chi_L^+\chi_L)(0)+\frac{i}{mVN_c}\int\psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}\right]+\begin{bmatrix}L\to R\\ N_+\to N_-\end{bmatrix} \quad (20)$$

In thermodynamic limit

$$\mathcal{S}=\mathcal{S}_0-\frac{N}{2}\ln\left[1+\frac{i}{mN_c}(\chi_L^+\chi_L)(0)\right]-$$

$$-i\,\frac{N}{2mVN_c}\,\frac{\int\psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}}{1+\frac{i}{mN_c}(\chi_L^+\chi_L)(0)}-(L\to R) \quad (21)$$

And $Z(0,0)$ we can write in the form

$$Z(0,0)=\int D\psi D\psi^+ D\chi D\chi^+\frac{d\omega_+}{2\pi}\frac{d\mu_+}{\mu_+^2}\frac{d\omega_-}{2\pi}\frac{d\mu_-}{\mu_-^2}\exp\left[-\mathcal{S}_0-\frac{N}{2}\ln\mu_+ +\right.$$

$$+i\frac{N\mu_+}{2mVN_c}\int\psi_L^+\psi_L(k)a^2(k)\frac{d^4k}{(2\pi)^4}+i\omega_+\left(\frac{1}{\mu_+}-1-\frac{i}{mN_c}(\chi_L^+\chi_L)(0)\right]+ \quad (22)$$

$$\left.+\begin{Bmatrix}L\to R\\ (\omega)\mu_+\to(\omega)\mu_-\end{Bmatrix}\right]$$

After integration over χ,χ^+,ψ,ψ^+ using (19) with $p_\mu=0$ we have

$$Z(0,0)=\int d\mu_+d\mu_-\frac{g(\mu_+,\mu_-)}{\mu_+^2\mu_-^2}\exp\left[-\frac{N}{2}\ln\mu_+-\frac{N}{2}\ln\mu_- +\right.$$

$$\left.+2VN_c\int\ln\left\{1+\frac{\mu_+\mu_-}{m^2}\left(\frac{N}{2VN_c}\right]^2\frac{a^4(k)}{k^2}\right\}\frac{d^4k}{(2\pi)}\right] \quad (23)$$

where

$$g(\mu_+,\mu_-)=\int\frac{d\omega_+}{2\pi}\frac{d\omega_-}{2\pi}\left(1+\frac{\omega_++\omega_-}{4N_c}\right)^{-2N_c}\exp\left[i\omega_+\left(\frac{1}{\mu_+}-1\right)+i\omega_+\left(\frac{1}{\mu_+}-1\right)\right]$$

In thermodynamic limit $\ln[g(\mu_+,\mu_-)/\mu_+^2\mu_-^2]$ must be rejected in comparison with the terms proportional to V or N.

The integration over $\mu_\pm$ can be performed by the steepest descent method, since $\Delta\mu_\pm$ is of the order $1/\sqrt{N}$. We get $\mu_+=\mu_-=m\varepsilon$, where ε is determined from the gap equation derived in [5], which naturally emerges when integrated with $\mu_\pm$.

$$1=\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\,\frac{M^2(p)}{p^2+M^2(p)}\quad M(k)=\frac{N\varepsilon}{?VN_c}\,a^2(k)\quad \varepsilon\sim\left(\frac{VN_c}{N}\right)^{1/2}\tag{24}$$

In accordance with the results of [5,10], in this order over $1/N_c$ we obtain a theory of non-interacting quarks with M(k) dynemical mass

$$Z(0,0)=\mathrm{const}\int D\psi D\psi^+\,\exp\left[\int\psi^+[-\hat{k}+iM(k)]\psi\,\frac{d^4k}{(2\pi)^4}\right]\tag{25}$$

From (21) and (25) we can see that the mean value $\langle(\chi_L^+\chi_L)(0)\rangle$ is defined by

$$\left[1+\frac{i}{mN_c}\langle(\chi_L^+\chi_L)(0)\rangle\right]^{-1}=m\varepsilon\tag{26}$$

Let us now consider the next order over $1/N_c$ in the action

$$\begin{aligned}\mathcal{S}=\mathcal{S}_0-N_+\ln\Bigg[&1+\frac{i}{mN_c}(\chi_L^+\chi_L)(0)+\left(\frac{i}{mN_c}(\chi_L^+\chi_L)(0)\right)^2+\\ &\frac{i}{mVN_c}\int\psi_L^+\psi_L(k)a^2(k)\,\frac{d^4k}{(2\pi)^4}\left[1+\frac{i}{mN_c}(\chi_L^+\chi_L)(0)\right]+\\ &+\left(\frac{i}{mN_c}\right)^2\int\frac{d^4z}{V}(\psi_L^+\chi_L)(z)(\chi_L^+\psi_L)(z)\Bigg]-\begin{pmatrix}L\to R\\ N_+\to N_-\end{pmatrix}\end{aligned}\tag{27}$$

Substituting the mean value(26) instead of colorless production

$$\left(\frac{i}{mN_c}(\chi_L^+\chi_L)(0)\right)^2 \to \left(<\frac{i}{mN_c}(\chi_L^+\chi_L)(0)>\right)^2$$

in chiral limit we have

$$\mathcal{L}=\int\left(\psi^+[\hat{k}-iM(k)]\psi+\chi^+\hat{k}\chi\right)\frac{d^4k}{(2\pi)^4}+2\,\frac{V}{N}\int\sqrt{\overline{M(k)M(q)M(p)M(l)}}\,\delta^4(k+p-q-l)\times$$

$$\times\left(\psi_L^+(k)\chi_L(q)\right)\left(\chi_L^+(p)\psi(l)\right)\frac{d^4k\,d^4p\,d^4q\,d^4l}{(2\pi)^{12}}+(L\to R) \qquad (28)$$

Thus, we had started from a nonlinear theory and had obtained a nonlinear theory with nonlocal interactions ($N_f=1$). This action has two essential advantages:

a)perturbation theory is applicable as the vertex function is small($\sim N/2VN_c\times 1/N_c$) and besides, the model is superconvergent.

b)potential leaves the ghost fields without mass. This is obvious from the calculation of fig.1 type diagrams. Correction to the propagator is

$$\left(\frac{2V}{N}\right)^2 N_c M(p)\,\frac{\hat{p}}{p^2}\,\frac{1-\gamma_5}{2}\int\frac{d^4k\,d^4q}{(2\pi)^8}\,M(p+k)M(q)M(q-k)\,\frac{\hat{q}-\hat{k}+iM(q-k)}{(q-k)^2+M^2(q-k)}\times$$

$$\times Sp\left[\frac{\hat{p}+\hat{k}}{(p+k)^2}\,\frac{1+\gamma_5}{2}\,\frac{\hat{q}+M(q)}{q^2+M^2(q)}\,\frac{1-\gamma_5}{2}\right]\frac{1+\gamma_5}{2}\,\frac{\hat{p}}{p^2}=\alpha(p,k)\,\frac{1+\gamma_5}{2}\,\frac{\hat{p}}{p^2}$$

where $\alpha\sim 1/N_c$, i.e. in the leading order over $1/N_c$ the propagator remains unchanged. Also, in the same order the propagator for quarks remains invariable, this being obvious from calculation of the fig.2. type diagrams.

3.Now consider the massive case (eq. (10)). Then for $\mathcal{L}$ instead of (20) we have

$$\mathcal{S}=-\int[\psi^{+}(i\hat{\partial}+im)\psi+\chi^{+}(i\hat{\partial}+im)\chi]d^{4}x-$$

$$N_{+}\ln\left[1+\frac{i}{2mVN_{c}}(\chi_{L}^{+}\chi_{L})(m)+\frac{i}{2mVN_{c}}(\psi_{L}^{+}\psi)_{L}(m)\right]+\begin{pmatrix}L\to R\\ N_{+}\to N_{-}\end{pmatrix} \qquad (29)$$

where the scalar production

$$(\psi_{L}^{+}\psi)_{L}(m)=\int\psi^{+}(k)\left(1-\frac{im\hat{k}}{k^{2}}\right)\frac{1+\gamma_{5}}{2}\left(1-\frac{im\hat{q}}{q^{2}}\right)\psi(q)\times$$

$$\times\exp[iz(k-q)]a(k)a(q)\frac{d^{4}k\ d^{4}q}{(2\pi)^{8}}$$

and similarly for $(\chi_{L}^{+}\chi_{L})(m)$ and $L\to R$.

Important is the circumstance, that theories with few fields and definitely different effective mass and potential may eventually bring to identical results. Thus, expending the logarithm in action around different values, we get different vertex functions and effective masses, which must be yield the same Green functions. Naturally, we are interested in the case which brings us, as in chiral limit, to a stable propagator in the leading order over $1/N_{c}$. Introduce the arbitrary parameter s in the logarithm, any value of which must not affect the final results. We choose that value of s, which leaves the effective mass of quarks stable.

$$\mathcal{S}=\int[\psi^{+}(i\hat{\partial}+im)\psi+\chi^{+}(i\hat{\partial}+im)\chi]d^{4}x+ \qquad (30)$$

$$+N_{+}\ln\left[1+\frac{i}{2mVN_{c}}(\psi_{L}^{+}\psi)_{L}(m)+\frac{is}{2mVN_{c}}(\chi_{L}^{+}\chi_{L})(m)+\frac{i(1-s)}{2mVN_{c}}(\chi_{L}^{+}\chi_{L})(m)\right]+$$

$$+\begin{pmatrix}L\to R\\ N_{+}\to N_{-}\end{pmatrix}$$

Expanding ln around

$$1+\frac{is}{2mVN_c}(\chi_L^+\chi_L)(m)$$

after steps analogous to the chiral limit, we obtain the following partition function:

$$Z(0,0)=\int D\psi D\psi^+ D\chi D\chi^+ \frac{d\theta_+}{2\pi}\frac{d\mu_+}{\mu_+^2}\frac{d\theta_-}{2\pi}\frac{d\mu_-}{\mu_-^2}\exp\Bigg\{-\frac{N}{2}\ln 2m\mu_+ - N\frac{1-s}{s}m\mu_+ +$$

$$+\int\psi^+\left[-\hat{k}+im+i\frac{N}{2VN_c}a^2(k)\left(1-\frac{im\hat{k}}{k^2}\right)\left(\mu_+\frac{1+\gamma_5}{2}+\mu_-\frac{1-\gamma_5}{2}\right)\left(1-\frac{im\hat{k}}{k^2}\right)\right]\psi\frac{d^4k}{(2\pi)^4}+$$

$$+\int\chi^+\left[-\hat{k}+im+i\frac{N}{2VN_c}a^2(k)\left(1-\frac{im\hat{k}}{k^2}\right)\left(\theta_+\frac{1+\gamma_5}{2}+\theta_-\frac{1-\gamma_5}{2}\right)\left(1-\frac{im\hat{k}}{k^2}\right)\right]\frac{d^4k}{(2\pi)^4}+$$

$$+\frac{N}{s}m\theta_+-\frac{N}{2s}\frac{\theta_+}{\mu_+}+(L\to R,\mu_+\to\mu_-,\theta_+\to\theta_-)\Bigg\} \qquad (31)$$

which brings us to two connected gap equations for quarks and ghosts fields

$$\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\frac{M^2(p)-mM(p)}{p^2-2mM(p)+M^2(p)}=1+2\frac{1-s}{s}m\varepsilon-\frac{1}{s}\frac{\theta}{\varepsilon}$$

$$\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\frac{L^2(p)-mL(p)}{p^2-2mL(p)+L^2(p)}=\frac{2m\theta}{s}-\frac{1}{s}\frac{\theta}{\varepsilon} \qquad (32)$$

$$M(k)=\frac{N\varepsilon}{2VN_c}a^2(k) \qquad L(k)=\frac{N\theta}{2VN_c}a^2(k)$$

From these equations it follows that: i) up to linear approximation over m, the effective ghost mass $L(k)=0$ ($\theta=0$) as in the chiral limit (all other solutions for θ turn to complex mass) ii)in the chiral case for any s we will obtain the same gap equation iii)when $\theta=0$, s is complex ($\omega=iN/2[\theta-\mu(1-s)]/s$, see eq. (21)), i.e. M(k) is real when $s\to\infty$. Thus, from (29) we have for $\mathcal{E}(m)$

$$\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\;\frac{M^2(p)-mM(p)}{p^2-2mM(p)+M^2(p)}=1-2m\mathcal{E} \qquad (33)$$

and the propagator

$$S^{-1}=\left(1-\frac{2mM}{k^2}\right)\hat{k}\;-\;i(M+m) \qquad (34)$$

which coincides with the results of diagrammatic calculations of [6]. Our results differ from those of [11], where in the kinetic term $(i\hat{\partial}+im)$ m was neglected as subleading in N_c. But it is not true. Let us denote the neglected m by μ for difference and after calculations analogous to[11], in the integral part of the gap equation in[11] we obtain $m-2\mu$ instead of m! If one takes μ=m, one can find the gap equation identical to (33).

The chosen value of s brings us to a real interaction potential with coupling constant $\sim 1/N_c$, which means that corrections to color objects are suppressed, i.e. our result for effective propagator (34) is stable.

Thus, the averaging procedure in quenched approximation proposed in this work leads to an action that contains already in case of $N_f=1$ an interaction potential. We restore the quark propagator and gap equation(in chiral limit as well as with account of mass corrections) obtained previously by diagrammatic approach, the results being not affected by the potential. Effective interaction allows to use perturbation theory and find higher point Green functions, which will be presented in forthcoming publications.

Appendix

In this appendix we calculate the determinant which arises as a result of integration over the ghost fields with mass corrections. In eq. (18) these corrections lead to the following replacements: in the kinetic part $i\hat{\partial}\to i\hat{\partial}+im$, and in $K_\pm$ $\psi(k)\to(1-im\hat{k}/k^2)\psi(k)$, $\psi^+(k)\to\psi^+(k)(1-im\hat{k}/k^2)$
and the same for χ,χ^+, and besides, in denominators m is replaced to 2m. Then the integral over χ,χ^+ can be performed onto the integral

$$\int_0^\infty dxdyP(x,y)\exp(-x-y)\int D\chi D\chi^+\exp\left[\int\chi^+(k)(-\hat{k}+im)\chi(k)\frac{d^4k}{(2\pi)^4}+\right.$$

$$+\frac{i}{2mN_c}\int a(k)a(q)\chi^+(k)\left(1-\frac{im\hat{k}}{k^2}\right)\left\{x\exp[iz(k-q)]\frac{1+\gamma_5}{2}+\right. \tag{A1}$$

$$\left.\left.+y\exp[i\tilde{z}(k-q)]\frac{1-\gamma_5}{2}\right\}\left(1-\frac{im\hat{q}}{q^2}\right)\chi(q)\frac{d^4kd^4q}{(2\pi)^8}\right]$$

P(x,y) here is a polynomial. Extraction of $\mathrm{Det}(i\hat{\partial}+im)$ brings us to $\mathrm{Det}(E-T)$ $T=\alpha T_1+\beta T_2$ where

$$\alpha=\frac{ix}{2mN_c}\qquad\beta=\frac{iy}{2mN_c}$$

$$T_1(k,q)^{ab}_{ij}=a(k)a(q)\exp[iz(k-q)]\left[\frac{\hat{k}}{k^2}\frac{1+\gamma_5}{2}\left(1-\frac{im\hat{q}}{q^2}\right)\right]_{ij}\delta_{ab} \tag{A2}$$

$$T_2(k,q)^{ab}_{ij}=a(k)a(q)\exp[i\tilde{z}(k-q)]\left[\frac{\hat{k}}{k^2}\frac{1-\gamma_5}{2}\left(1-\frac{im\hat{q}}{q^2}\right)\right]_{ij}\delta_{ab}$$

They satisfy the following relations

$$T_1^2=-i\frac{m}{2}T_1\quad T_2^2=-i\frac{m}{2}T_2\quad T_1T_2T_1=p^2T_2\quad T_2T_1T_2=p^2T_1 \tag{A3}$$

where we use the normalization condition of zero modes

$$2\int \frac{a^2(s)}{s^2} \frac{d^4s}{(2\pi)^4}=1$$

and by p_μ we denote

$$p_\mu=i\int a^2(s) \frac{s_\mu}{s^2}\exp[-is(z-\tilde{z})]\frac{d^4s}{(2\pi)^4}$$

Using eq.(A3) one can easily derive

$$\left[E-tT\right]^{-1}=E+\frac{1}{D}\left[(\alpha t+i\frac{m}{2}\alpha\beta t^2)T_1+(\beta t+i\frac{m}{2}\alpha\beta t^2)T_2+\alpha\beta t^2(T_1T_2+T_2T_1)\right]$$

$$D=1+i\frac{m}{2}(\alpha+\beta)-(p^2+m^2/4)\alpha\beta t^2 \qquad (A4)$$

From the identity

$$\mathrm{Det}(E-T)=\exp\left[Sp\int_0^1 \frac{-T}{E-tT}dt\right]$$

we finally obtain

$$\mathrm{Det}(E-T)=\left[1-\frac{x+y}{4N_c}+\frac{xy}{4N_c^2}\left(\frac{p^2}{m^2}+\frac{1}{4}\right)\right]^{-2N_c}$$

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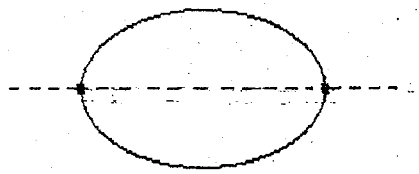

Fig.1

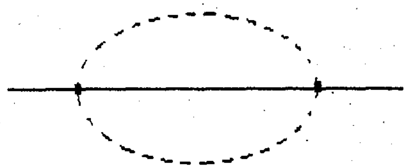

Fig.2

Fig.1,2–Self. energetic diagrams for ghost(dashed curve) and quark (solid curve) fields.

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