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CANONICAL QUANTIZATION OF DIRAC SPINNING PARTICLE IN THE EXTERNAL MAGNETIC FIELD

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ԴԻՐԱԿԻ ՍՊԻՆՕԺՏՎԱԾ ՄԱՍՆԻԿԻ ԿԱՆՈՆԱՎՈՐ ՔՎԱՆՏԱՑՈՒՄԸ ԱՐՏԱՔԻՆ ՄԱԳՆԻՍԱԿԱՆ ԴԱՇՏՈՒՄ.

Դիրակի սպինօժտված մասնիկի կանոնավոր քվանտացումը արտաքին մագնիսական դաշտում կատարված է այնպիսի տրամաչափության ընտրությամբ, որը թույլ է տալիս նկարագրել զանգվածով, ինչպես նաև անզանգված մասնիկներ չորսաչափ տարածությունում: Ստացված են Նյուտոն-Վիգների տիպի կոորդինատներ և իմպուլսներ արտաքին մագնիսական դաշտում և քննարկվում է քվանտացման տվյալ եղանակի կապը Բլոունտի պատկերի հետ:

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ГРИГОРЯН Г.В., ГРИГОРЯН Р.П.

КАНОНИЧЕСКОЕ КВАНТОВАНИЕ ДИРАКОВСКОЙ СПИНОВОЙ ЧАСТИЦЫ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ.

Проведено каноническое квантование дираковской спиновой частицы во внешнем магнитном поле в калибровке, позволяющей описывать как массивные, так и безмассовые частицы, в пространстве размерности $D=4$. Получены координаты и импульсы типа Ньютона-Вигнера для частицы во внешнем магнитном поле и обсуждается связь данной схемы квантования с картиной Блоунта.

Ереванский физический институт.

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CANONICAL QUANTIZATION OF DIRAC SPINNING PARTICLE IN THE EXTERNAL MAGNETIC FIELD

The canonical quantization of Dirac spinning particle in the external magnetic field is carried out in a gauge which allows to describe both massive and massless particles in the spacetime dimension D=4. The Newton-Wigner type coordinates and momenta for a particle in the external magnetic field are found and the connection of this quantization scheme and the Blount picture is discussed.

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In this paper the quantization of relativistic spinning particle in an external magnetic field is carried out starting with a classical pseudomechanical action, where the spin of the particle is described by variables, which are elements of Grassmann algebra. As in [1], where a free relativistic spinning particle was canonically quantized, we will choose the quantization scheme in which all additional gauge fixing constraints are introduced into a theory already at the classical level. This reduces the theory to a system with a second class constraints only, which is then quantized using Dirac's quantization scheme. Again, as in [1], we take one of the additional gauge fixing constraints in the form $x_0 - \tau\varkappa = 0$, thus describing simultaneously both particles and antiparticles already at the classical level [3]. Because of the complexity of the Dirac brackets of the independent dynamical variables of the theory, the operator realization in terms of the initial dynamical variables seems improbable. Thus it's even more important here than in the case of free particle [1] to find new variables, in terms of which the quantum commutation relations would be canonical . Such variables are found.

In analogy with [1] we introduce "classical" spin tensor and Pauli-Lubanski vector in the presence of the external magnetic field (which are gauge invariant generalizations of corresponding quantities of the free particle):their expression in terms of canonical variables is found. Note that contrary to the case of free particle, the spin tensor and Pauli- Lubanski vector are not supergauge invariant and are not conserved in the presence of the external magnetic field

The quantization of the theory is carried out in terms of canonical variables. Even in writing quantum relations, corresponding to classical relations between initial and canonical variables one encounters the problem of operator ordering. It turns out that if one accepts the symmetric (Weyl) quantization scheme,then in a definite gauge ($\xi_5 = 0$) one comes to a Blount picture [4] of the spinning particle in the external magnetic field, just as in the case of free particle we had Dirac theory in the Foldy-Wouthuysen representation [3].

In sect.2 ,following Dirac prescription, the complete set of constraints is found; in sect.3 the Dirac brackets of physical variables are given and a transition from the initial variables to canonical ones is performed; sect.4 is devoted to the quantization of the theory and the relation of the resulting quantized theory to the Blount picture is discussed.

2. Consider the action of the theory of relativistic spinning particle in the external electromagnetic field [6-8]

$$S=\frac{1}{2}\int d\tau\left[\frac{(\dot{x}^{\mu})^2}{e}+em^2\; i(\xi_\mu\dot{\xi}^{\mu}-\xi_5\dot{\xi}_5) - i\chi(\frac{\xi_\mu\dot{x}^{\mu}}{e}-m\xi_5)+ \right. \tag{1}$$
$$\left. +2g\,\dot{x}^{\mu}A_\mu+ig\,e_{\mu\nu}\xi^{\mu}\xi^{\nu}\right].$$

here $\mu = 0,1,2,3$, x^μ - particle coordinate, ξ^μ - Grassmann variables, describing spin degrees of freedom, ξ_5, χ и e are additional fields (e is an even element,ξ_5,χ are odd elements of Grassmann algebra) g is the charge of the particle, A^μ is the four-potential of the electromagnetic field, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; the dote denotes the differentiation over τ along the trajectory; the derivatives over Grassmann variables are left.

The action (1) is invariant under reparametrization transformations with a parameter u

$$\delta x^\mu= 2u p^\mu\ ,\ \delta\xi^\mu= 2guF^{\mu\nu}\xi_\nu\ ,\delta\xi_5= 0$$
$$\delta A_\mu= 2uF_\nu\partial^\nu A_\mu,\ \delta e = 2\dot{u},\ \ \delta\chi= 0. \tag{2}$$

and supergauge transformations with a parameter v

$$\delta x^\mu= iv\xi^\mu,\ \ \delta\xi^\mu= vp^\mu,\ \delta\xi_5= vm,$$
$$\delta A_\mu= iv\xi_\nu\partial^\nu A_\mu,\ \delta e = iv\chi,\ \delta\chi = 2\dot{v} \tag{3}$$

where

$$p_\mu = \frac{\dot{x}_\mu}{e} - \frac{i\chi}{2e}\xi_\mu \tag{4}$$

menta, canonically conjugate to x^μ, e, ξ_μ and χ we

o

$$_\mu = \frac{\partial L}{\partial\dot{x}^\mu} = \frac{\dot{x}_\mu}{e} - \frac{i\chi}{2e}\xi_\mu + g\,A_\mu \equiv p_\mu + gA_\mu,$$
$$\pi_e= \frac{\partial L}{\partial\dot{e}} = 0,\ \pi_\mu = \frac{\partial L}{\partial\dot{\xi}^\mu} = \frac{i}{2}\xi_\mu , \tag{5}$$

$$\pi_5 = \frac{\partial L}{\partial \dot{\xi}_5} = \frac{i}{2}\xi_5 , \quad \pi_\chi = \frac{\partial L}{\partial \dot{\chi}} = 0.$$

The relations (5) (with the exception of the first one) are primary constraints:

$$\Phi_\mu \equiv \pi_\mu - \frac{i}{2}\xi_\mu \approx 0 , \mu = 0,1,2,3; \quad \Phi_4 = \pi_5 + \frac{i}{2}\xi_5 \approx 0, \tag{6}$$

$$\Phi_6 \equiv \pi_e \approx 0, \quad \Phi_{11} \equiv \pi_\chi \approx 0.$$

The numbering of constraints is chosen so as to make the final matrix of Poisson brackets $C_{nn'} = \{ \Phi_n, \Phi_{n'} \}$ of all the constraints of the theory more compact.

The canonical Hamiltonian of the theory is given by

$$H = \dot{x}^\mu P_\mu + \dot{\xi}^\mu \pi_\mu + \dot{\xi}_5 \pi_5 - L = \frac{e}{2}\left[\mathcal{P}^2 - m^2 - i\, g\, F_{\mu\nu}\xi^\mu\xi^\nu \right] + \frac{i\chi}{2}\left[\mathcal{P}_\mu \xi^\mu - m\xi_5\right], \tag{7}$$

$$\mathcal{P}_\mu = P_\mu - gA_\mu .$$

Following Dirac [2] we construct the total Hamiltonian of the theory

$$H^T = H + i\lambda^\mu \Phi_\mu + i\lambda_4 \Phi_4 + \lambda_6 \Phi_6 + i\lambda_{11}\Phi_{11} \tag{8}$$

(λ are Lagrange multipliers) and find the secondary constraints

$$\Phi_5 \equiv \mathcal{P}_\mu \xi^\mu - m\xi_5 \approx 0 , \quad \Phi_7 = \mathcal{P}^2 - m^2 - igF_{\mu\nu}\xi^\mu\xi^\nu \approx 0 \tag{9}$$

As in the free particle case [1] we have four first class constraints: in addition to $\Phi_7, \Phi_6, \Phi_{11}$ there is yet one more first-class constraint, which is a linear combination of the constraints Φ_μ, Φ_4, Φ_5. Thus to remove the degeneration of the theory we have to add four new constraints, which we choose in

the form

$$\Phi_8 \equiv x_0 - \varkappa\tau \approx 0 , \quad \Phi_9 \equiv a\xi_0 + b\xi_5 \approx 0, \tag{10}$$

$$\Phi_{10} \equiv e + \frac{1}{\omega} \approx 0 , \quad \Phi_{12} \equiv \chi \approx 0$$

where

$$\omega = \sqrt{\mathcal{P}_i^2 + m^2 + igF_{\mu\nu}\xi^\mu\xi^\nu} , \quad \mathcal{P}_0 = -\varkappa\omega , \tag{11}$$

a and b are parameters, which do not become zero simultaneously (see [1] for a discussion of restrictions on a,b), $\varkappa = \pm 1$. Notice that contrary to the free particle case $\mathcal{P}_0$ is no longer a real valued quantity and $\varkappa$ denotes here only the sign of the SQR in (11). Nevertheless again $\varkappa = +1$ corresponds to a particle, while $\varkappa = -1$ - to antiparticle

The constraints (6), (9) and (10) represent the total set of constraints of the theory and now they are all second-class.

3. As we have mentioned in the introduction. further considerations will be carried out for the four-potential $A_\mu(x) = (0, A_i(x))$, which corresponds to a constant (in time) magnetic field. Now we will perform a canonical transformation on the variables x^μ, P_μ K to obtain variables x'^μ, P'_μ defined by the relations

$$x'^0 = x^0 - \varkappa\tau , \quad x'^i = x^i, \quad P'_\mu = P_\mu \tag{12}$$

(the corresponding generating function is $W = x^\mu P'_\mu - \tau\varkappa P'_0$). In terms of these new variables the constraint Φ_8 becomes $x'^0 \approx 0$, while all other constraints remain unchanged. Thus the total set of constraints (6), (9), (10) now doesn't depend on time explicitly and we can use the standard Dirac quantization scheme for the theory with a second-class constraints. Under the above described canonical transformation the Hamiltonian of the system on the constraint surface becomes

$$H_{phys} = \sqrt{p_i^2 + m^2 + igF_{ik}\xi_i\xi_k} = \tilde{\omega} \tag{13}$$

Counting Dirac brackets for the complete set of constraints (6), (9), (10) for the independent variables, for which we choose x^i, P_i, ξ^i, we find

$$\{x^i, x^j\}_D = \frac{i\gamma}{\alpha^2}\left[\xi^i\xi^j + \frac{b}{\beta P_0}(p^k\xi^k)(\xi^i p^j - \xi^j p^i)\right],$$

$$\{x^i, P_j\}_D = -\delta^i_j - \frac{i\gamma}{\alpha^2}g(\partial_j A_k)\left[\xi^i\xi_k + \frac{b}{\beta P_0}(p^n\xi^n)(\xi^i P_k - \xi_k p^i)\right],$$

$$\{x^i, \xi^j\}_D = \frac{\gamma}{\alpha^2}\left[\xi^i p^j - \frac{b}{\beta P_0}(p^n\xi^n)p^i p^j + ig\frac{b}{\beta P_0}(p^n\xi^n)F^{jk}\xi^k\xi^i\right],$$

$$\{P_i, P_j\}_D = ig^2\frac{\gamma}{\alpha^2}(\partial_i A_k)(\partial_j A_n)\left[\xi_k\xi_n + \frac{b}{\beta P_0}(p^n\xi^n)(\xi_k P_n - \xi_n P_k)\right],$$

$$\{\xi^i, P_j\}_D = \frac{g\gamma}{\alpha^2}\left[p^i(\partial_j A_k)(\xi_k - \frac{b}{\beta P_0}(p^n\xi^n)P_k) + \frac{ib}{\beta P_0}(p^n\xi^n)\xi^l\xi^m\left(gF^{il}\partial_j A^n + \frac{1}{2}p^i\partial_j F^{ln}\right)\right], \tag{14}$$

$$\{\xi^i, \xi^j\}_D = -i\left(\delta^{ij} - \frac{p^i p^j}{\alpha^2}\gamma\right) + \frac{b\gamma g}{\beta P_0\alpha^2}\xi^l(p^n\xi^n)(F^{il}p^j + F^{jl}p^i),$$

where $\alpha = aP_0 + bm$, $\beta = am + bP_0$, $\gamma = a^2 - b^2$,

$$P_0 = p_0 = -\varkappa\sqrt{p_i^2 + m^2 + igF_{ik}\xi_i\xi_k} = -\varkappa\tilde{\omega}.$$

Comparing these formulae with corresponding relations for the free particle case ,[1], it's easy to see that now the

expressions for the Dirac brackets for independent variables are much more complicated. Hence it is more desirable here than in the free case to pass from the variables x^i, P_i, ξ_i to new variables q^i, Π_i, ψ^i, for which the Dirac brackets would be canonical

$$\{q^i, q^j\}_D = 0, \quad \{\psi^i, \psi^j\}_D = -i\delta^{ij}, \quad \{q^i, \Pi_j\}_D = \delta^i_j,$$
$$\{q^i, \psi^j\}_D = \{\Pi_i, \Pi_j\}_D = \{\Pi_i, \psi^j\}_D = 0 \tag{15}$$

Such a variables can be found. Their relation to the old variables are given by the expressions

$$q^i = x^i - i\xi^i\frac{(a + b\varkappa)(p^l\xi^l)}{\beta(m + \tilde{\omega})},$$

$$\Pi_i = P_i + ig(\partial_i A_m)\xi_m\frac{(a + b\varkappa)(p^l\xi^l)}{\beta(m + \tilde{\omega})}, \tag{16}$$

$$\psi^i = \xi^i + p^i\frac{(a + b\varkappa)(p^l\xi^l)}{\beta(m + \tilde{\omega})}$$

where $\beta = bP_0 + am$.

Note that while the expression for q^i и ψ^i are gauge invariant generalizations of corresponding expressions for the free particle, the expression for the canonical momentum contains a new summand proportional to g , which is the reflection of the fact that the Dirac bracket $\{P_i, P_j\}_D$ is different from zero in the presence of the external field..

From the relations (16) one can obtain inverse relations which connect the variables $x^i, p_i = P_i - gA_i(x), \xi^i$ with the new ones $q^i, \pi_i = \Pi_i - gA_i(q), \psi^i$:

$$x^i = q^i - i\psi^i \frac{(a + b\varkappa)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)},$$

$$p_i = \pi_i + igF_{il}\psi_l \frac{(a + b\varkappa)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)}, \qquad (17)$$

$$\xi^i = \psi^i + \pi^i \frac{(a + b\varkappa)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)},$$

where

$$\Omega = \sqrt{\pi_i^2 + m^2 + igF_{ij}(q)\psi_i\psi_j}\,, \qquad (18)$$

In terms of new variables the Hamiltonian of the theory becomes

$$H_{phys} = \Omega \qquad (19)$$

In describing spin variables of the free particle in [1] the quantity $\tilde{\xi}^\mu = \xi^\mu - (p^\mu/m)\xi_5$ was introduced, in terms of which the spin tensor was expressed $S^{\mu\nu} = i\tilde{\xi}^\mu\tilde{\xi}^\nu$. Being supergauge invariant it was also conserved in time (on equations of motion). . As a consequence of that the spin tensor $S^{\mu\nu}$ was conserved too. Note that the total angular momentum $J^{\mu\nu}$ of the free particle was also conserved. In the presence of the external field however both the total angular momentum and the spin of the particle are not conserved. Nevertheless we will introduce the quantity

$$S^{\mu\nu}_{(A)} = i\tilde{\xi}^\mu_{(A)}\tilde{\xi}^\nu_{(A)}\,, \quad \tilde{\xi}^\mu_{(A)} = \xi^\mu - \frac{p^\mu}{m}\xi_5, \qquad (20)$$

which is a gauge invariant generalization of the tensor $S^{\mu\nu}$ (the superscript "A" denotes the presence of the external field). In terms of the variables q, π, ψ the quantity $\tilde{\xi}^\mu_{(A)}$ is given by

$$\tilde{\xi}^0_{(A)} = -\frac{\varkappa}{m}(\pi^j\psi^j), \quad \tilde{\xi}^i_{(A)} = \psi^i + \frac{\pi^i(\pi^j\psi^j)}{m(m + \Omega)}, \qquad (21)$$

Note that $\tilde{\xi}^\mu_{(A)}$, and hence $S^{\mu\nu}_{(A)}$, do not depend on parameters a and b of the fermion gauge fixing ϕ_σ (though they are not supergauge invariant). Introducing now in analogy with the free particle case the vectors

$$S_i^{\tilde{\xi}(A)} = -\frac{i}{2}\varepsilon_{ijk}\tilde{\xi}^{(A)}_j\tilde{\xi}^{(A)}_k\,, \quad S_i^\psi = -\frac{i}{2}\varepsilon_{ijk}\psi_j\psi_k \qquad (22)$$

(S_i^ψ is the spin vector expressed in terms of the variables ψ, which describe the spin of the particle in the rest frame), we find the relation between them :

$$S_i^{\tilde{\xi}(A)} = \frac{\Omega}{m}S_i^\psi - \frac{\pi_i(\pi_j S_j^\psi)}{m(m + \Omega)} \qquad (23)$$

which generalizes the corresponding relation of the free particle case [1] to the presence of the external field. Note that in deriving of this formula the fact that ψ_i is three dimensional was used (the terms containing multiplication of four or more ψ were equaled to zero). In terms of the variables S_i^ψ the relations (17) are given by

$$x^i = q^i - \frac{(a + b\varkappa)\varepsilon_{ijk}S_j^\psi\pi_k}{(m + \Omega)(bm - a\varkappa\Omega)},$$

$$p_i = \pi_i - g\frac{(a + b\varkappa)(S_i^\psi(B_k\pi_k) - \pi_i(S_k^\psi B_k))}{(m + \Omega)(bm - a\varkappa\Omega)}, \qquad (24)$$

$$\Omega = \sqrt{\pi_i^2 + m^2 - 2g\,S_k^\psi B_k}\,.$$

where $B_i(q) = \frac{1}{2}\varepsilon_{ijk}F_{jk}(q)$. Now let us introduce the analog of the classical Pauli-Lubanski vector in the presence of the external magnetic field:

$$W_\mu^{(A)} = -\frac{i}{2}\epsilon_{\mu\nu\lambda\sigma}P^\nu\tilde\xi^\lambda_{(A)}\tilde\xi^\sigma_{(A)} = -\frac{i}{2}\epsilon_{\mu\nu\lambda\sigma}P^\nu\xi^\lambda_{(A)}\xi^\sigma_{(A)} \qquad (25)$$

From the independence of the $\tilde\xi^{(A)}$ on the fermionic gauge fixing parameters it follows that $W_\mu^{(A)}$ doesn't't depend on them either. From (21) it easy to express $W_\mu^{(A)}$ in terms of q, π, ψ:

$$W_0^{(A)} = \pi_i S_i^\psi, \qquad W_i^{(A)} = -\varkappa\left[m S_i^\psi + \frac{\pi_i(\pi_k S_k^\psi)}{(m+\Omega)}\right], \qquad (26)$$

which is a gauge invariant generalization of the corresponding relations from [1].

It's evident that it's convenient to quantize the theory in terms of the variables q, Π, ψ, for which the Dirac brackets are given by (15). Introducing operators corresponding to this variables $\hat q$, $\hat\Pi$, $\hat\psi$, we write down corresponding commutation relations for them through the rule $[\ ,\ ] = i\hbar\{\ ,\ \}_D$:

$$[\hat q^i, \hat q^j]_- = [\hat\Pi_i, \hat\Pi_j]_- = [\hat q^i, \hat\psi^j]_- = [\hat\Pi_i, \hat\psi^j]_- = 0,$$
$$[\hat q^i, \hat\Pi_j]_- = i\hbar\delta^i_j, \qquad [\hat\psi^i, \hat\psi^j]_+ = \hbar\delta^{ij} \qquad (27)$$

The last of this relations generates a three dimensional Clifford algebra. As is well known, the unique finite dimensional irreducible representation for the operators $\hat\psi^i$, is then given by the Pauli matrices:

$$\hat\psi^i = \pm\left(\frac{\hbar}{2}\right)^{\frac{1}{2}}\sigma^i = \varkappa\left(\frac{\hbar}{2}\right)^{\frac{1}{2}}\sigma^i, \qquad i = 1,2,3. \qquad (28)$$

The transition to the operators $\hat x, \hat P, \hat\xi$, corresponding to the initial variables of the theory x, P, ξ, using the expressions

(17), is complicated (contrary to the free particle case) by the problem of ordering of the operators $\hat q$, $\hat\Pi$, $\hat\psi$ in corresponding quantum relations. Here we will adhere to the symmetric (Weyl) quantization. For the Grassmann variables the symmetric quantization is defined as follows [6,9]: the classical function $f(\psi)$ is expanded in powers of ψ:

$$f(\psi) = \sum_{\nu=0}^{n}\sum_{\{i\}} f_{i_1\ldots i_\nu}\psi^{i_1}\ldots\psi^{i_\nu} \qquad (29)$$

(due to a nilpotency of ψ the series terminate; in our case $n = 3$) The transition to the quantum relations corresponding to (29) is achieved by replacing of ψ by $\hat\psi$ after antisymmetrization over all indeces of the coefficients of the expansion (29).

Taking into account that $\hat S_i^\psi = (\hbar/2)\,\sigma_i$, we obtain for quantum operators $\hat x, \hat P$, $\hat{\tilde S}^{\xi(A)}$ and $\hat H_{phys}$ the expressions

$$\hat x_i \rightleftarrows q_i - \frac{\hbar}{2}\,\frac{(a\varkappa + b)\,\epsilon_{ijk}\sigma_j\pi_k}{(bm - a\varkappa\tilde\Omega)(m + \tilde\Omega)},$$
$$\hat P_i \rightleftarrows \pi_i - g\,\frac{\hbar}{2}\,\frac{(a\varkappa + b)\,[\sigma_i(\pi_k B_k) - \pi_i(\sigma_k B_k)]}{(bm - a\varkappa\tilde\Omega)(m + \tilde\Omega)}, \qquad (30)$$
$$\hat{\tilde S}^{\xi(A)} \rightleftarrows \frac{\hbar\tilde\Omega}{2m}\sigma_i - \frac{\hbar}{2m}\,\frac{\pi_i(\pi_k\sigma_k)}{(m + \tilde\Omega)},$$
$$\hat H_{phys} \rightleftarrows \tilde\Omega\left(1 - g\,\frac{\hbar}{2}\,\frac{(B_k\sigma_k)}{\tilde\Omega^2}\right),$$

where the symbol $\rightleftarrows$ denotes the correspondence between the operator and it's Weyl transformation [10]; $\tilde\Omega = (\pi_i^2 + m^2)^{1/2}$

The presence in (30) of $\tilde{\Omega}$ instead of Ω (compare with (23), (24)) is connected again with the fact that we have dropped out all terms containing powers of ψ higher than three.

5. To compare our results with those obtained in different quantization schemes, we rewrite the expressions for the operators $\hat{x}_i$ и $\hat{P}_i$ in the gauge $\xi_5 \approx 0$ (a = 0):

$$\hat{x}_i \rightleftarrows q_i - \frac{\hbar}{2} \frac{\epsilon_{ijk}\sigma_j\pi_k}{m(m+\tilde{\Omega})}$$

$$\hat{P}_i \rightleftarrows \pi_i - g\frac{\hbar}{2} \frac{[\sigma_i(\pi_k B_k) - \pi_i(\sigma_k B_k)]}{m\,(m+\tilde{\Omega})} . \qquad (31)$$

Apart from that we will find the velocity operator $\hat{v}_i = d\hat{x}_i/dx_o$. Making use of the relation

$$v^i = \frac{dx^i}{dx_o} = \varkappa \frac{dx^i}{d\tau} = \varkappa \{ x^i, H_{phys} \}_D = -\varkappa \frac{P_i}{\Omega} \qquad (32)$$

and using the expression for P_i in the $\xi_5 \approx 0$ gauge, we find for the operator $\hat{v}_i$

$$\hat{v}^i = \frac{d\hat{x}^i}{dx_o} \rightleftarrows -\varkappa\Big[\frac{\pi_i}{\tilde{\Omega}} + g\frac{\hbar}{2} \frac{\pi_i(B_k\sigma_k)(m^2 + m\tilde{\Omega} + \tilde{\Omega}^2)}{m\,(m+\tilde{\Omega})\,\tilde{\Omega}^3} - $$

$$- g\frac{\hbar}{2}\frac{\sigma_i(\pi_k B_k)}{m\tilde{\Omega}\,(m+\tilde{\Omega})} \Big] \qquad (33)$$

The formulae (31), (32), and also the expression $S^{\tilde{\xi}(A)}$ in (30), in the first approximation over g, exactly coincide with analogous formulae given in [10] (for no electric field and for particles without anomalous magnetic moment). They are just the expressions for the position operator, the momentum, the

velocity and the spin in the Blount picture [4,10].

One can also write the equation of spin in this picture (compare with the equation given in [10]).

$$\frac{dS_i^{\tilde{\xi}(A)}}{dx_o} \rightleftarrows \varkappa \frac{g}{\tilde{\Omega}} \epsilon_{ijk} S_j^{\tilde{\xi}(A)} B_k \qquad (34)$$

And finally we write down the quantum analogs of the relations (26) for Pauli-Lubanski vector:

$$\hat{W}_o^{(A)} \rightleftarrows \frac{\hbar}{2}(\pi_k\sigma_k),\ \hat{W}_i^{(A)} \rightleftarrows -\varkappa\frac{\hbar}{2}\Big[m\sigma_i + \frac{\pi_i(\pi_k\sigma_k)}{m+\tilde{\Omega}}\Big] \qquad (35)$$

Here we want to call attention to the following fact. In the free particle case [1] it was found that one quantum operator corresponded (up to a numerical factor) to two classical objects $m\xi^\mu$ and W^μ. To clarify the situation in the present case we write down the quantum analogs of the relations (21):

$$\hat{\tilde{\xi}}^o_{(A)} \rightleftarrows -\frac{1}{m}\Big(\frac{\hbar}{2}\Big)^{\frac{1}{2}} (\pi\psi) ,$$

$$\tilde{\xi}^i_{(A)} \rightleftarrows \varkappa\Big(\frac{\hbar}{2}\Big)^{\frac{1}{2}} \Big[\sigma^i + \frac{\pi_i(\pi_k\sigma_k)}{m(m+\tilde{\Omega})} + g\frac{\hbar}{2}\frac{\pi_i(\pi_k B_k)}{m(m+\tilde{\Omega})^2\tilde{\Omega}}\Big] \qquad (36)$$

The last summand in $\tilde{\xi}^i_{(A)}$ comes from the term ,proportional to ψ^3 in the expansion of the denominator in (21) in powers of ψ. Comparing (36) with (35) we see, that in the presence of the external magnetic field the operators $\tilde{\xi}^\mu_{(A)}$ и $W^\mu_{(A)}$ are differing from each other (the additional summand in (36) is proportional to interaction constant and looks like a quantum correction)

Finally we note, that from the formulae (16) it follows,

References

1. Grigoryan G.V.,Grigoryan R.P. // Yad.Phys.1991. V.53:/.P.1737.

2. Dirac P.A.M., Lectures on Quantum Mechanics, Belfer Graduate School of Science (Yeschiva University Press, New York, 1964).

   Gitman D.M., Tyutin I.V., Quantization of Fields with Constraints, (Springer Verlag, Berlin, 1990).

3. Gitman D.M., Tyutin I.V., JETPh Letters 51 (1990) 188

4. Blount E.I.// Phys.Rev. 1962.V.126. P.1636; 1962.V.128.P.2454.

5. L.L.Foldy,S.A.Wouthuysen// Phys.Rev.1950.V.78.P.29.

6. Berezin F,A.,Marinov M.S., Ann.Phys (N.Y.) 104 (1977) 336

7. Barducci A. et al.// Nuovo Cim. 1976. V.35A.P.377.

8. Galvao C., Teitelboim C.//J.Math. Phys.1980.V.21.P.1863.

9. Henneaux M.,Teitelboim C.// Ann.Phys.(N.Y.).1982 .V.143. P.127.

10. S.R.de.Groot,L.G.Sattorp. Foundations of electodynamics. North-Holland Publishing Company - Amsterdam. 1972

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КАНОНИЧЕСКОЕ КВАНТОВАНИЕ ДИРАКОВСКОЙ СПИНОВОЙ ЧАСТИЦЫ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ.

(на английском языке, перевод авторов)

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