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CANONICAL QUANTIZATION OF DIRAC SPINNING PARTICLE IN THE EXTERNAL ELECTROMAGNETIC FIELD

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ԴԻՐԱԿԻ ՍՊԻՆՕԺՏՎԱԾ ՄԱՍՆԻԿԻ ԿԱՆՈՆԱՎՈՐ ՔՎԱՆՏԱՑՈՒՄԸ ԱՐՏԱՔԻՆ ԷԼԵԿՏՐԱՄԱԳՆԻՍԱԿԱՆ ԴԱՇՏՈՒՄ.

Դիրակի սպինօժտված մասնիկի կանոնավոր քվանտացումը արտաքին էլեկտրամագնիսական դաշտում չորսաչափ տարածությունում կատարված է այնպիսի տրամաչափության ընտրությամբ, որը թույլ է տալիս նկարագրել նաև հակամասնիկներ դասական տեսության շրջանակներում: Ստացված են Նյուտոն–Վիգների տիպի կոորդինատներ և իմպուլսներ արտաքին էլեկտրամագնիսական դաշտում և քննարկվում է քվանտացման տվյալ եղանակի կապը Բլոունտի պատկերի հետ։

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1. Introduction.

In this paper the quantization of relativistic spinning particle in an external electromagnetic field is carried out starting with a classical pseudo mechanical action, where the spin of the particle is described by variables, which are elements of Grassmann algebra. As in [1], where a free relativistic spinning particle was canonically quantized, we will choose the quantization scheme in which all additional gauge fixing constraints are introduced into a theory already at the classical level. This reduces the theory to a system with a second class constraints only, which is then quantized using Dirac's quantization scheme [2]. Again, as in [1], we take one of the additional gauge fixing constraints in the form $x_0 - \tau\varkappa = 0$, thus describing simultaneously both particles and antiparticles already at the classical level [3]. Because of the complexity of the Dirac brackets of the independent dynamical variables of the theory, the operator realization in terms of the initial dynamical variables seems improbable. Thus it's even more important here than in the case of free particle [1] to find new variables, in terms of which the quantum commutation relations would be canonical . Such variables are found. The article goes in the same lines as the quantization of the spinning particle in the external magnetic field constant in time carried out in [4].

In analogy with [1] we introduce "classical" spin tensor and Pauli-Lubanski vector in the presence of the external electromagnetic field (which are gauge invariant generalizations of corresponding quantities of the free particle):their expression in terms of canonical variables is found (note that contrary to the case of free particle, the spin tensor and Pauli- Lubanski vector are not supergauge invariant and are not conserved in the presence of the external electromagnetic field). The quantization of the theory is carried out in terms of canonical variables. Even in writing quantum relations, corresponding to classical relations between initial and canonical variables one encounters the problem of operator ordering. It turns out that if one accepts the symmetric (Weyl) quantization scheme,then in a definite gauge ($\xi_5 \approx 0$) one comes to a Blount picture [5] of the spinning particle in the external electromagnetic field, just as in the case of free particle we had Dirac theory in the Foldy-Wouthuysen representation [1,3].

In sect.2 ,following Dirac prescription, the complete set of constraints is found; in sect.3 the Dirac brackets of physical variables are given and a transition from the initial variables to canonical ones is performed; sect.4 is devoted to the quantization of the theory and the relation of the resulting quantized theory to the Blount picture is discussed.

2.Constraints

Consider the action of the theory of relativistic spinning particle in the external electromagnetic field [6-8]

$$S=\frac{1}{2}\int d\tau\left[\frac{(\dot{x}^\mu)^2}{e}+em^2-i(\xi_\mu\dot{\xi}^\mu-\xi_5\dot{\xi}_5)-i\chi\left(\frac{\xi_\mu\dot{x}^\mu}{e}-m\xi_5\right)+\right.$$

$$\left.+2g\dot{x}^\mu A_\mu+igeF_{\mu\nu}\xi^\mu\xi^\nu\right]. \tag{1}$$

here $\mu=0,1,2,3$, x^μ - particle coordinate, ξ^μ - Grassmann variables, describing spin degrees of freedom, ξ_5, χ и e are additional fields (e is an even element, ξ_5,χ are odd elements of Grassmann algebra) g is the charge of the particle, A^μ is the four-potential of the electromagnetic field, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; the dote denotes the differentiation over τ along the trajectory; the derivatives over Grassmann variables are left.

The action (1) is invariant under supergauge transformations with a parameter ε

$$\begin{aligned}&\delta x^\mu=i\varepsilon\xi^\mu,\quad \delta\xi^\mu=\varepsilon\mathcal{P}^\mu,\quad \delta\xi_5=\varepsilon m,\\&\delta A_\mu=i\varepsilon\xi_\nu\partial^\nu A_\mu,\quad \delta e=i\varepsilon\chi,\quad \delta\chi=2\dot{\varepsilon},\end{aligned} \tag{2}$$

where

$$\mathcal{P}_\mu=\frac{\dot{x}_\mu}{e}-\frac{i\chi}{2e}\xi_\mu. \tag{3}$$

Going through the steps analogous to those of [4] we come to the complete set of constraints

$$\Phi_\mu=\pi_\mu-\frac{i}{2}\xi_\mu,\ \mu=0,1,2,3\ ;\qquad \Phi_4=\pi_5+\frac{i}{2}\xi_5,$$

$$\Phi_5=\mathcal{P}_\mu\xi^\mu-m\,\xi_5,\quad \Phi_6=a\xi_0+b\xi_5,\quad \Phi_7=\mathcal{P}^2-m^2-igF_{\mu\nu}\xi^\mu\xi^\nu,$$

$$\Phi_8=x_0-\varkappa\tau,\quad \Phi_9=\pi_e,\quad \Phi_{10}=e+\frac{1}{\omega}, \tag{4}$$

where π_μ, π_5, π_e are momenta, canonically conjugate to ξ^μ, ξ_5, e

respectively, $\omega = \sqrt{\mathcal{P}_i^2 + m^2 + igF_{\mu\nu}\xi^\mu\xi^\nu}$.

Now we will perform a canonical transformation on the variables x^μ, p_μ to x'^μ, p'_μ defined by the relations

$$x'^0 = x^0 - \varkappa\tau \ , \quad x'^i = x^i, \quad p'_\mu = p_\mu \tag{5}$$

(the corresponding generating function is $W = x^\mu p'_\mu - \tau\varkappa p'_0$).

Under the above described canonical transformation the Hamiltonian of the system on the constraint surface becomes

$$H_{phys} = \sqrt{\mathcal{P}_i^2 + m^2 + igF_{\mu\nu}\xi^\mu\xi^\nu} + g\varkappa A_0 = \omega + g\varkappa A_0 \tag{6}$$

3. Dirac brackets and Newton-Wigner coordinates

Counting Dirac brackets for the complete set of constraints (4), for the independent variables, for which we choose x^i, $\mathcal{P}_i$, ξ^i, we find

$$\{x^i, x^j\}_D = \frac{i\gamma}{\alpha^2}\left[\xi^i\xi^j + \frac{b}{\beta\mathcal{P}_0}(\mathcal{P}^k\xi^k)(\xi^i\mathcal{P}^j - \xi^j\mathcal{P}^i)\right],$$

$$\{x^i, \mathcal{P}_j\}_D = -\delta^i_j - \frac{i\gamma}{\alpha^2}gF_{jk}\left[\xi^i\xi_k + \frac{b}{\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)(\xi^i\mathcal{P}_k - \xi_k\mathcal{P}^i)\right],$$

$$\{x^i, \xi^j\}_D = \frac{\gamma}{\alpha^2}\Big[\xi^i\mathcal{P}^j - \frac{b}{\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)\mathcal{P}^i\mathcal{P}^j + ig\frac{b}{\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)F^{jk}\xi^k\xi^i -$$

$$- \frac{2iagb}{\alpha\beta\mathcal{P}_0}\mathcal{P}_i(\mathcal{P}^n\xi^n)F_{0k}\xi_k\xi_j\Big],$$

$$\{\mathcal{P}_i, \mathcal{P}_j\}_D = gF_{ij} + ig^2\frac{\gamma}{\alpha^2}F_{ik}F_{jn}\xi_k\xi_n +$$

$$+ ig^2\frac{\gamma b}{\alpha^2\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)\xi_k\mathcal{P}_l(F_{ik}F_{jl} - F_{il}F_{jk}),$$

$$\{\xi^i,\mathcal{P}_j\}_D = \frac{g\gamma}{\alpha^2}\Big[\mathcal{P}^i F_{jk}\Big(\xi_k - \frac{b}{\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)\mathcal{P}_k\Big) +$$

$$+ \frac{ib}{\beta\mathcal{P}_0}(\mathcal{P}^n\xi^n)\xi^l\xi^k\Big(gF^{il}F_{jk} + \frac{1}{2}\mathcal{P}^i\partial_j F_{lk} - \frac{2ag}{\alpha}\mathcal{P}_i F_{0l}F_{jk}\Big)\Big],$$

$$\{\xi^i,\xi^j\}_D = -i(\delta^{ij} - \frac{\mathcal{P}^i\mathcal{P}^j}{\alpha^2}\gamma) + \frac{b\gamma g}{\beta\mathcal{P}_0\alpha^2}\xi^k(\mathcal{P}^n\xi^n)(F^{ik}\mathcal{P}^j + F^{jk}\mathcal{P}^i) +$$

$$+ \frac{2agb\gamma}{\alpha^3\beta\mathcal{P}_0}\mathcal{P}_i\mathcal{P}_j(\mathcal{P}^n\xi^n)\xi_k F_{0k}, \qquad (7)$$

where $\mathcal{P}_0 = -\varkappa\sqrt{\mathcal{P}_i^2 + m^2 + igF_{\mu\nu}\xi^\mu\xi^\nu} = -\varkappa\omega$, $\alpha = a\mathcal{P}_0 + bm$, $\beta = am + b\mathcal{P}_0$, $\gamma = a^2 - b^2$,

Again as in [1,4] we pass from the variables x^i, p_i, ξ_i to new (Newton-Wigner) variables q^i, Π_i, ψ^i, for which the Dirac brackets would be canonical

$$\{\psi^i,\psi^j\}_D = -i\delta^{ij}, \qquad \{q^i,\Pi_j\}_D = \delta^i_j,$$

$$\{q^i,q^j\}_D = \{q^i,\psi^j\}_D = \{\Pi_i,\Pi_j\}_D = \{\Pi_i,\psi^j\}_D = 0 . \qquad (8)$$

Their relation to the old variables are given by the expressions

$$q^i = x^i - i\xi^i\frac{(a + b\varkappa)(\mathcal{P}^l\xi^l)}{\beta(m + \omega)},$$

$$\Pi_i = p_i + ig(\partial_i A_m)\xi_m\frac{(a + b\varkappa)(\mathcal{P}^l\xi^l)}{\beta(m + \omega)}, \qquad (9)$$

$$\psi^i = \xi^i + \mathcal{P}^i \frac{(a + b\varkappa)(\mathcal{P}^l \xi^l)}{\beta(m + \omega)}$$

where $\beta = b\mathcal{P}_0 + am$.

Note that while the expressions for q^i и ψ^i are gauge-invariant generalizations of corresponding expressions for the free particle, the expression for the canonical momentum contains a new summand proportional to g, which is the reflection of the fact that the Dirac bracket $\{p_i, p_j\}_D$ is different from zero in the presence of the external field. From the relations (9) one can obtain inverse relations which connect the variables x^i, $\mathcal{P}_i = p_i - gA_i(x)$, ξ^i with the new ones q^i, $\pi_i = \Pi_i - gA_i(q,\tau)$, ψ^i:

$$x^i = q^i - i\psi^i \frac{(a\varkappa + b)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)},$$

$$\mathcal{P}_i = \pi_i + igF_{il}\psi_l \frac{(a\varkappa + b)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)}, \qquad (10)$$

$$\xi^i = \psi^i + \pi^i \frac{(a\varkappa + b)(\pi^k\psi^k)}{(m + \Omega)(bm - a\varkappa\Omega)},$$

where

$$\Omega = \sqrt{\pi_i^2 + m^2 + igF_{ij}(q,\tau)\psi_i\psi_j} \qquad (11)$$

Following [9], we find for the Hamiltonian of the theory in terms of the variables q^i, π_i, ψ^i the expression

$$H_{phys} = \Omega + \varkappa gA_0(q,\tau) + ig\varkappa \frac{F_{0i}(q,\tau)(\pi_k\psi_k)\psi_i}{\Omega(\Omega + m)}. \qquad (12)$$

In describing spin variables of the free particle in [1] the quantity $\tilde{\xi}^\mu = \xi^\mu - (p^\mu/m)\xi_5$ was introduced, in terms of which the spin tensor was expressed $S^{\mu\nu} = i\tilde{\xi}^\mu\tilde{\xi}^\nu$. Being supergauge invariant it was also conserved in time (on equations of motion). As a consequence of that the spin tensor $S^{\mu\nu}$ was conserved too. Note that the total angular momentum $J^{\mu\nu}$ of the free particle was also conserved. In the presence of the external field however both the total angular momentum and the spin of the particle are not conserved. Nevertheless we will introduce the quantity

$$S^{\mu\nu}_{(A)} = i\tilde{\xi}^{\mu}_{(A)}\tilde{\xi}^{\nu}_{(A)} , \qquad \tilde{\xi}^{\mu}_{(A)} = \xi^{\mu} - \frac{\pi^{\mu}}{m}\xi_5 , \tag{13}$$

which is a gauge invariant generalization of the tensor $S^{\mu\nu}$ (the subscript "A" denotes the presence of the external field). In terms of the variables q, π, ψ the quantity $\tilde{\xi}^{\mu}_{(A)}$ is given by

$$\tilde{\xi}^{0}_{(A)} = -\frac{\varkappa}{m}(\pi^j\psi^j) , \qquad \tilde{\xi}^{i}_{(A)} = \psi^i + \frac{\pi^i(\pi^j\psi^j)}{m(m+\Omega)} \tag{14}$$

Note that $\tilde{\xi}^{\mu}_{(A)}$, and hence $S^{\mu\nu}_{(A)}$, do not depend on parameters $\varkappa$, i of the fermion gauge ϕ_6 (though they are not supergauge invariant). Introducing now in analogy with the free particle case the vectors

$$S_i^{\tilde{\xi}(A)} = -\frac{i}{2}\varepsilon_{ijk}\tilde{\xi}^{(A)}_{j}\tilde{\xi}^{(A)}_{K} . \qquad S_i^{\psi} = -\frac{i}{2}\varepsilon_{ijk}\psi_j\psi_k \tag{15}$$

(S_i^{ψ} , the spin vector expressed in terms of the variables ψ, describes the spin of the particle in the rest frame), we find the relation between them :

$$S_i^{\tilde{\xi}(A)} = \frac{\Omega}{m}S_i^{\psi} - \frac{\pi_i(\pi_j S_j^{\psi})}{m(m+\Omega)} , \tag{16}$$

which generalizes the corresponding relation of the free particle case [1] to the presence of the external field. Note that in deriving of this formula the fact that ψ_i is three dimensional was used (the terms containing multiplication of four or more ψ were equaled to zero). In terms of the variables S^{ψ}_i the relations (17) are given by

$$x^i = q^i - \frac{(a\varkappa + b)\varepsilon_{ijk}S^{\psi}_j\pi_k}{(m + \Omega)(bm - a\varkappa\Omega)} ,$$

$$\mathcal{P}_i = \pi_i - g\frac{(a\varkappa + b)(S^{\psi}_i(B_k\pi_k) - \pi_i(S^{\psi}_kB_k)}{(m + \Omega)(bm - a\varkappa\Omega)} , \qquad (17)$$

$$\Omega = \sqrt{\pi_i^2 + m^2 - 2gS^{\psi}_kB_k} ,$$

where $B_i(q) = \frac{1}{2}\varepsilon_{ijk}F_{jk}(q)$. Now let us introduce the analog of the "classical" Pauli-Lubanski vector in the presence of the external electromagnetic field:

$$W^{(A)}_{\mu} = -\frac{i}{2}\varepsilon_{\mu\nu\lambda\sigma}\mathcal{P}^{\nu}\tilde{\xi}^{\lambda}_{(A)}\tilde{\xi}^{\sigma}_{(A)} = -\frac{i}{2}\varepsilon_{\mu\nu\lambda\sigma}\mathcal{P}^{\nu}\xi^{\lambda}_{(A)}\xi^{\sigma}_{(A)} . \qquad (18)$$

From the independence of the $\tilde{\xi}^{(A)}$ on the fermionic gauge fixing parameters it follows that $W^{(A)}_{\mu}$ doesn't depend on them either. From (18) it easy to express $W^{(A)}_{\mu}$ in terms of q, π, ψ :

$$W^{(A)}_0 = \pi_i S^{\psi}_i , \qquad W^{(A)}_i = -\varkappa\left[mS^{\psi}_i + \frac{\pi_i(\pi_kS^{\psi}_k)}{(m + \Omega)}\right], \qquad (19)$$

which is a gauge invariant generalization of the corresponding relations for the free particle case.

4.Quantization

It's evident that it's convenient to quantize the theory in terms of the variables q, Π, ψ, for which the Dirac brackets are given by (8). Introducing operators $\hat{q}$, $\hat{\Pi}$, $\hat{\psi}$, corresponding to this variables, we write down corresponding commutation relations for them through the rule $[\,,\,]=i\hbar\{\,,\,\}_D$:

$$[\hat{q}^i,\hat{q}^j]_-=[\hat{\Pi}_i,\hat{\Pi}_j]_-=[\hat{q}^i,\hat{\psi}^j]_-=[\hat{\Pi}_i,\hat{\psi}^j]_-=0,$$
$$[\hat{q}^i,\hat{\Pi}_j]_-=i\hbar\delta^i_j\,,\quad [\hat{\psi}^i,\hat{\psi}^j]_+=\hbar\delta^{ij} \tag{20}$$

The last of this relations generates a Clifford algebra in three dimensional space. As is well known, the unique finite dimensional irreducible matrix representation for the operators $\hat{\psi}^i$, is then given by the Pauli matrices:

$$\hat{\psi}^i=\pm\left(\frac{\hbar}{2}\right)^{\frac{1}{2}}\sigma^i=\varkappa\left(\frac{\hbar}{2}\right)^{\frac{1}{2}}\sigma^i,\ i=1,2,3. \tag{21}$$

The transition to the operators $\hat{x},\hat{p},\hat{\xi}$, corresponding to the initial variables of the theory - x,p,ξ, using the expressions (10), is complicated (contrary to the free particle case) by the problem of ordering of the operators $\hat{q}$, $\hat{\Pi}$, $\hat{\psi}$ in corresponding quantum relations. Here we will adhere to the symmetric (Weyl) quantization. For the Grassmann variables the symmetric quantization is defined as follows [6,10]: the classical function $f(\psi)$ is expanded in powers of ψ:

$$f(\psi)=\sum_{\nu=0}^{n}\sum_{\{i\}}f_{i_1\ldots i_\nu}\psi^{i_1}\ldots\psi^{i_\nu} \tag{22}$$

(due to a nilpotency of ψ the series terminate; in our case $n=3$) The transition to the quantum relations corresponding to (22) is achieved by replacing of ψ by $\hat{\psi}$ after antisymmetrization over all indexes of the coefficients of the expansion (22).

Taking into account that $\hat{S}^{\psi}_i=(\hbar/2)\sigma_i$, we obtain for quantum operators $\hat{x}$, $\hat{\mathcal{P}}$, $\hat{S}^{\tilde{\xi}(A)}$ and $\hat{H}_{phys}$ the expressions

$$\hat{x}_i \rightleftarrows q_i - \frac{\hbar}{2}\frac{(a\varkappa + b)\,\varepsilon_{ijk}\sigma_j\pi_k}{(bm - a\varkappa\tilde{\Omega})(m + \tilde{\Omega})},$$

$$\hat{\mathcal{P}}_i \rightleftarrows \pi_i - g\frac{\hbar}{2}\frac{(a\varkappa + b)\,[\sigma_i(\pi_k B_k) - \pi_i(\sigma_k B_k)]}{(bm - a\varkappa\tilde{\Omega})(m + \tilde{\Omega})},$$

$$S^{\tilde{\xi}(A)} \rightleftarrows \frac{\hbar}{2}\frac{\tilde{\Omega}}{m}\sigma_i - \frac{\hbar}{2}\frac{\pi_i(\pi_k\sigma_k)}{m\,(m + \tilde{\Omega})}, \tag{23}$$

$$\hat{H}_{\Phi} \rightleftarrows \tilde{\Omega} - g\frac{\hbar}{2}\frac{(B_k\sigma_k)}{\tilde{\Omega}} - g\varkappa\frac{\hbar}{2}\frac{\varepsilon_{ijk}\pi_i\sigma_j E_k}{\tilde{\Omega}(m + \tilde{\Omega})} + g\varkappa A_0$$

where $E_k = F_{0k}$, the symbol $\rightleftarrows$ denotes the correspondence between the operator and it's Weyl transformation [11]; $\tilde{\Omega} = (\pi_i^2 + m^2)^{1/2}$. The presence in (23) of $\tilde{\Omega}$ instead of Ω (compare with (16), (17)) is connected again with the fact that we have dropped out all terms containing powers of ψ higher than three.

To compare our results with those obtained in different quantization schemes, we rewrite the expressions for the operators $\hat{x}_i$ and $\hat{\mathcal{P}}_i$ in the gauge $\zeta_5 \approx 0$ (a= 0):

$$\hat{x}_i \rightleftarrows q_i - \frac{\hbar}{2}\frac{\varepsilon_{ijk}\sigma_j\pi_k}{m(m + \tilde{\Omega})},$$

$$\hat{\mathcal{P}}_i \rightleftarrows \pi_i - g\frac{\hbar}{2}\frac{[\sigma_i(\pi_k B_k) - \pi_i(\sigma_k B_k)]}{m\,(m + \tilde{\Omega})}. \tag{24}$$

Apart from that we will find the velocity operator $\hat{v}_i = d\hat{x}_i/dx_0$ Making use of the relation

$$v^i = \frac{dx^i}{dx_0} = \varkappa \frac{dx^i}{d\tau} = \varkappa \{ x^i, H_{phys} \}_D + \frac{\partial x^i}{\partial x_0} \tag{25}$$

we find for the operator $\hat{v}^i$ (in Φ_6 - gauge)

$$\hat{v}^i = \frac{d\hat{x}^i}{dx_0} \rightleftarrows - \varkappa \frac{\pi_i}{\tilde{\Omega}} - g\varkappa \frac{\pi_i (B_k S_k^\Psi)[b(m^2 + m\tilde{\Omega} + \tilde{\Omega}^2) - a\varkappa m\tilde{\Omega}]}{(m + \tilde{\Omega})\tilde{\Omega}^3 (bm - a\varkappa\tilde{\Omega})}$$

$$+ g\varkappa \frac{(b + a\varkappa)\, S_i^\Psi (\pi_k B_k)}{\tilde{\Omega}(m + \tilde{\Omega})(bm - a\varkappa\tilde{\Omega})} - g \frac{(S^\Psi \times E)_i [b(m - \tilde{\Omega}) - 2a\varkappa\tilde{\Omega}]}{\tilde{\Omega}(m + \tilde{\Omega})(bm - a\varkappa\tilde{\Omega})} +$$

$$+ g \frac{\pi_i (\pi \times S^\Psi)_k E_k (2\tilde{\Omega} + m)}{(\tilde{\Omega} + m)^2 \tilde{\Omega}^3} - g \frac{(b + a\varkappa)(\pi \times E)_i (\pi_k S_k^\Psi)}{\tilde{\Omega}(m + \tilde{\Omega})^2 (bm - a\varkappa\tilde{\Omega})} +$$

$$+ g \frac{(b + a\varkappa)(\pi \times S^\Psi)_i (\pi_k E_k)[bm - a\varkappa(2\tilde{\Omega} + m)]}{\tilde{\Omega}(m + \tilde{\Omega})^2 (bm - a\varkappa\tilde{\Omega})^2} \tag{26}$$

One can also write the equation of spin in this picture

$$\frac{d\hat{S}_i^{\tilde{\xi}(A)}}{dx_0} \rightleftarrows \varkappa \frac{g}{\tilde{\Omega}} \varepsilon_{ijk} \left\{ S_j^{\tilde{\xi}(A)} B_k - \frac{(\pi \times S^{\tilde{\xi}(A)})_j E_k}{\tilde{\Omega}} \right\}. \tag{27}$$

The equation (27) doesn't depend on the fermionic gauge parameters . In the gauge $\xi_5 \approx 0$ ($a = 0$) the expression (26) becomes

$$\hat{v}^i = \frac{d\hat{x}^i}{dx_0} \rightleftarrows - \varkappa \frac{\pi_i}{\tilde{\Omega}} - g\varkappa \frac{\pi_i (B_k S_k^\Psi)\ (m^2 + m\tilde{\Omega} + \tilde{\Omega}^2)}{m(m + \tilde{\Omega})\tilde{\Omega}^3}$$

$$+ g\varkappa \frac{S_i^\Psi (\pi_k B_k)}{m\tilde{\Omega}(m + \tilde{\Omega})} + g \frac{\pi_i (\pi \times S^\Psi)_k E_k}{m\tilde{\Omega}^3}. \tag{28}$$

The formulae (24), (28), the expression $S^{\xi(A)}$ in (23) and also the equation (27), in the first approximation over g, coincide with analogous formulae given in [10] (for particles without anomalous magnetic moment and after the replacement $\pi_i \rightarrow -\pi_i$, $A_0 \rightarrow -\varphi$, which is connected with the different signs in the definition of canonically conjugate momenta p_μ) . They are just the expressions for the position operator, the momentum, the velocity, the spin andthe equation of motion for spin in the Blount picture [5,11]. Note, however, that our relations are correct in all orders of g and were obtained without restrictions on potentials.

And finally we write down the quantum analogs of the relations (26) for Pauli-Lubanski vector:

$$\hat{W}_0^{(A)} \rightleftarrows \frac{\hbar}{2}(\pi_k\sigma_k), \quad \hat{W}_i^{(A)} \rightleftarrows -\frac{\hbar}{2}\varkappa\left(m\sigma_i + \frac{\pi_i(\pi_k\sigma_k)}{m+\tilde{\Omega}}\right). \qquad (29)$$

Finally we note, that from the formulae (16) it follows, that in the presence of the electromagnetic field too there exists a gauge a + b$\varkappa$, in which the canonical variables coincide with the initial variables of the theory and the Dirac quantization in terms of the latter coincides with the canonical quantization.

Thus the canonical quantization of relativistic spinning particle in the external electromagnetic field (in the gauge $\xi_5 \approx 0$) results in the Blount picture [5,11], just as the quantization of the free relativistic spinning particle results in the Dirac picture in the Foldy-Wouthuysen representation [3,1].

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ВО ВНЕШНЕМ ЭЛЕКТРОМАГНИТНОМ ПОЛЕ.

(на английском языке, перевод авторов)

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