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МОДЕЛЬ СО СПИНОМ $-\frac{3}{2}$ НА ШЕСТИУГОЛЬНОЙ РЕШЕТКЕ.
ТОЧНОЕ РЕШЕНИЕ

Найдено точное решение модели со спином $-\frac{3}{2}$ на шестиугольной решетке с Гамильтонианом

$$-\beta H = \sum_{<ij>} [J S_i S_j + K S_i^2 S_j^2 + L S_i^3 S_j^3 + \frac{M}{2}(S_i S_j^3 + S_j S_i^3)] - \Delta \sum_i S_i^2$$

на подпространстве обменных констант взаимодействий. Обнаружена критическая линия Изинговского перехода.

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SPIN $-\frac{3}{2}$ MODEL IN A HONEYCOMB LATTICE:
EXACTLY SOLVABLE CASE

A spin $-\frac{3}{2}$ model on a honeycomb lattice with hamiltonian

$$\beta H = \sum_{<ij>} [J S_i S_j + K S_i^2 S_j^2 + L S_i^3 S_j^3 + \frac{M}{2}(S_i S_j^3 + S_j S_i^3)] - \Delta \sum_i S_i^2$$

is solved on the surface in the space of coupling constants and an Ising-type critical line is found.

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3/2 ՍՊԻՆՈՎ ՄՈԴԵԼԻ ՃՇԳՐԻՏ ԼՈՒԾՄԱՆ ԴԵՊՔ ՎԵՑԱՆԿՅԱՆ ՑԱՆՑԻ ՎՐԱ

Վեցանկյան ցանցի վրա 3/2 սպինով մոդելի համիլտոնիանով

$$-\beta H=\sum_{<ij>}\left[JS_iS_j+KS_i^2S_j^2+LS_i^3S_j^3+\frac{M}{2}(S_iS_j^3+S_jS_i^3)\right]-\Delta\sum_i S_i^2$$

ճշտորեն լուծվել է ենթատարածքում փոխանակային փոխազդեցությունների հաստատունը: Ստացվել է Իզինգի տիպի անցումների ամբողջ գիծ:

Երևանի ֆիզիկայի ինստիտուտ
Երևան 1993

A spin - 1 and spin - $\frac{3}{2}$ models have been carefully investigated. These models have become very attractive because of their simplicity and rich fixed-point structure. A spin - 1 model was for the first time introduced by Blume, Emery and Griffits (BEG) [1]. This model has been studied mostly under the molecular field [1] and renormalization group approximation [2]. Recently, Horiguchi [3], Wu [4], Shankar [5] and Tang [6] have exactly solved the BEG model for honeycomb and square lattices when $\exp(K)\text{ch}J=1$. The same result was obtained for the Bethe lattice [7].

The spin - $\frac{3}{2}$ BEG model with dipolar and quadrupolar interactions was introduced to explain phase transition in $DyVO_4$ and its phase diagram was obtained within the mean field approximation [8]. Another spin - $\frac{3}{2}$ model was later introduced to study tricritical properties in ternary field mixture [9], which was also solved in the mean field approximation. Recently, the complete phase diagram of this model has been fully analyzed with the use of two different approaches: mean field and Monte-Carlo methods [10].

In this paper we derive an exact solution of spin -$\frac{3}{2}$ model on the honeycomb lattice on a surface in the space spanned by the coupling constants J, K, L, M and Δ.

The hamiltonian for spin - $\frac{3}{2}$ model may be written as

$$-\beta H=\sum_{<ij>}\left[JS_iS_j+KS_i^2S_j^2+LS_i^3S_j^3+\frac{M}{2}(S_iS_j^3+S_jS_i^3)\right]-\Delta\sum_i S_i^2 \qquad (1)$$

where $S_i=\pm\frac{1}{2}, \pm\frac{3}{2}$.

Here, we will not be concerned about physical origin of these couplings and will treat them as parameters in the calculations.

The partition function defined by this hamiltonian is

$$Z=\sum_{(S)}\exp(-\beta H) \qquad (2)$$

Let us introduce

$$\sum_{s_i=\pm 1/2,\ \pm 3/2} S_i^n \exp(-\bar{\Delta} S_i^2) = 2\exp\left(-\frac{\bar{\Delta}}{4}\right)\left(1+\exp(-2\bar{\Delta})\right), \qquad n = 0$$

$$= \frac{1}{2}\exp\left(-\frac{\bar{\Delta}}{4}\right)\left(1+9\exp(-2\bar{\Delta})\right), \qquad n = 2 \qquad (8)$$

$$= 0, \qquad n = 1,\ 3$$

where n is the number of lines with site i as an end-point and can take on values 0,1,2,3 only for the honeycomb lattice.

According to Wu [11] this fact together with (8) enable us to rewrite (6) as the partition function of a spin $-\frac{1}{2}$ Ising model. Introducing Ising spins $\sigma_i = \pm 1$ we can rewrite (6) as

$$Z = B^N \sum_{\sigma=\pm 1} \prod_{<ij>} \left(1+4\sigma_i\sigma_j \frac{1+9\exp(-2\bar{\Delta})}{1+\exp(-2\bar{\Delta})} \, \mathrm{th}J_0\right) \qquad (9)$$

where $B = \exp\left(-\frac{\bar{\Delta}}{4}\right)\left(1+\exp(-2\bar{\Delta})\right) \alpha_0^{3/2} \exp(-3R/4)$

Identity which can be verified term - by - term by expanding into a high - temperature expantion the right - hand sides of (6) and (9). That is, we have transformed the evaluation of the partition function for our hamiltonian (1) into the evaluation of the partition function for the Ising model. Thuse we obtain exact equivalence

$$Z = B^N (\mathrm{ch}K_I)^{-3N/2} Z_{Ising}(K_I) \qquad (10)$$

Now the right - hand of (9) gives precisely the high - temperature expantion of a spin $-\frac{1}{2}$ Ising partition function $Z_{Ising}(K_I)$ whose nearest neighbor interaction is K_I with

$$\mathrm{th}(K_I) = \frac{1+9\exp(-2\bar{\Delta})}{1+\exp(-2\bar{\Delta})} \, \mathrm{th}J_0 \qquad (11)$$

Since the Ising model is critical at $\mathrm{ch}(2K_I) = 2$, our model is critical at

$$\mathrm{th}|J_0| = \frac{1+9\exp(-2\bar{\Delta})}{1+\exp(-2\bar{\Delta})} \quad \frac{1}{\sqrt{3}} \qquad (12)$$

in the subspace (7). Using the fact that $\bar{\Delta} = \Delta - 3R$ and R is function of J_0 in subspace (7) we can get the λ - line Ising - type phase transition in spin $-\frac{3}{2}$ model in $(\Delta - \frac{T}{J})$ plane.

$$\exp(2\Delta) = \frac{(1-9\sqrt{3}\,\mathrm{th}|J_0|)\cdot(1-81\mathrm{th}^2 J_0)^{3/16}\cdot(1-\mathrm{th}^2 J_0)^{27/16}}{(\sqrt{3}\,\mathrm{th}|J_0|-1)\cdot(1-9\mathrm{th}^2 J_0)^{15/8}} \qquad (13)$$

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