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CANONICAL QUANTIZATION OF THE D=2N DIMENSIONAL RELATIVISTIC SPINNING PARTICLE WITH ANOMALOUS MAGNETIC MOMENT IN THE EXTERNAL ELECTROMAGNETIC FIELD

Yerevan

Canonical quantization of the D=2n dimensional relativistic spinning particle with anomalous magnetic moment in the external electromagnetic field*

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Abstract

The pseudoclassical hamiltonian and action of the D=2n dimensional Dirac particle with anomalous magnetic moment interacting with the external electromagnetic field is found. The Bargmann- Michel-Telegdi equation of motion for the Pauli-Lubanski vector is deduced. The canonical quantization of D=2n dimensional Dirac spinning particle with anomalous magnetic moment in the external electromagnetic field is carried out in the gauge which allows to describe simultaneously particles and antiparticles (massive and massless) already at the classical level. Pseudoclassical Foldy-Wouthuysen transformation is used to obtain canonical (Newton-Wigner) coordinates and in terms of these variables the theory is quantized. The connection of this quantization with the deGroot and Suttorp's description of Dirac particle with anomalous magnetic moment in the external electromagnetic field is discussed.

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Introduction

It is well known [1-4] , that pseudoclassical description of the spinning particle is a theory with constraints which is supersymmetry and reparametrization invariant. Dirac quantization of this theory, when the first class constraints turn into a equations for wave function, leads to a covariant relativistic wave equation for a D=4 dimensional spin $\frac{1}{2}$ particle without anomalous magnetic moment (AMM). The quantization of the particle with AMM was considered in [4], however the authors of that article came to a conclusion, that the pseudoclassical particle with AMM can not be consistently quantized. The reason for such a conclusion was the wrong supersymmetry generator, which after quantization didn't lead to a wave equation of spin $\frac{1}{2}$ particle with AMM. In this paper we show that if one starts with the correct supersymmetry generator, then one can consistently quantize the theory.

The investigation is carried out in D=2n dimensions. The quantized theory in D=4 dimensions is an extension to an arbitrary electromagnetic field of the deGroot and Suttorp generalization [5] of the Blount picture to describe the particle with AMM.

The paper is organized as follows. In sect.2 we derive the hamiltonian and the action (lagrangian) of D=2n dimensional spin $\frac{1}{2}$ particle with anomalous magnetic moment in the external electromagnetic field. In sect.3 the Bargmann-Michel-Telegdi (BMT) equation for the Pauli-Lubanski vector in D=2n is found from the hamiltonian obtained in sect.2. Note, that for D=4 the

BMT equation was found in [1] from the action which didn't contain some of the terms in the action of section 2 proportional to AMM, which however contribute only to quantum corrections to the BMT equation. In sect.4 the pseudoclassical Foldy -Wouthuysen transformation [6,7] is introduced, which is used in section 5 to find out the canonical variables, in terms of which the theory is quantized.

2.The Action of spin $\frac{1}{2}$ particle with anomalous magnetic moment

To find out the action for D=2n dimensional spinning particle with AMM we will suppose that primary constraints for the Grassmann variables ξ^{μ}, $\mu=0,1,..,D-1$ and ξ_{D+1}, describing the spin degrees of freedom are the same as in thecase of the particle without AMM. Namely

$$\Phi_{\mu}=\pi_{\mu}-\frac{i}{2}\xi_{\mu},\ \mu = 0,1,2,..D-1\ ;\ \Phi_{D}=\pi_{D+1}+\frac{i}{2}\xi_{D+1}, \qquad (1)$$

where π_{μ}, π_{D+1} are the momenta conjugate to ξ^{μ} and ξ_{D+1} correspondingly. The Dirac brackets$\{.,.\}^{*}$ of the variables of the theory for this set of constraints are given by the relations

$$\{x_{\mu},\mathcal{P}_{\nu}\}^{*}=g_{\mu\nu},\ \{\xi_{\mu},\xi_{\nu}\}^{*}=ig_{\mu\nu},\ \{\xi_{D+1},\xi_{D+1}\}^{*}=-i,\ \{\mathcal{P}_{\mu},\mathcal{P}_{\nu}\}^{*}=gF_{\mu\nu} \qquad (2)$$

(all other brackets vanish). Here the variables x_{μ} are the coordinates of the particle, $\mathcal{P}_{\mu}=p_{\mu}-gA_{\mu}$, p_{μ}- are the momenta conjugate to x_{μ}, g is the charge of the particle, A^{μ} is the vector-potential of the electromagnetic field, $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$.

The quantization using Dirac brackets (2) will bring to the Dirac equation for a particle with AMM if we take instead of the constraint $\Phi_{D+1}=\mathcal{P}_{\mu}\xi^{\mu}- m\xi_{D+1}$, the constraint

$$\Phi_{D+1}=\mathcal{P}_{\mu}\xi^{\mu} - m\xi_{D+1}+iGF_{\mu\nu}\xi^{\mu}\xi^{\nu}\xi_{D+1}\approx 0 \qquad (3)$$

(the meaning of the parameter G will be clarified below).

Consider now a theory with Dirac brackets (2) and a hamiltonian

$$H=\frac{i\chi}{2}\Phi_{D+1}, \qquad (4)$$

where χ is the Grassmann odd Lagrange multiplier to Φ_{D+1}. The consistency condition will lead to a new constraint

$$\{\Phi_{D+1},\Phi_{D+1}\}^{*}=i\Phi_{D+3}=$$

$$=i\left[\mathcal{P}_{\mu}\mathcal{P}^{\mu}- ig\frac{M}{2}F_{\mu\nu}\xi^{\mu}\xi^{\nu}+4iGF_{\mu\nu}\mathcal{P}^{\mu}\xi^{\nu}\xi_{D+1}+G^{2}\left(F_{\mu\nu}\xi^{\mu}\xi^{\nu}\right)^{2}-m^{2}\right]\approx 0\ , \qquad (5)$$

where $gM=(g-2Gm)$, M (as it will be shown below) is the total magnetic moment of the particle in Bohr magnetons. Now, with Φ_{D+3} taken into account, the extended hamiltonian is given by the expression

$$H_{ext}=\frac{i\chi}{2}\Phi_{D+1}+\frac{e}{2}\Phi_{D+3}, \qquad (6)$$

where e is Grassmann even Lagrange multiplier to the constraint Φ_{D+3} . The consistency equations with the hamiltonian (6) imply no new constraints since due to Jacoby identity

$$\{\Phi_{D+3},\Phi_{D+1}\}^{*}=\{\{\Phi_{D+1},\Phi_{D+1}\}^{*},\Phi_{D+1}\}^{*}=0, \qquad (7)$$

and hence the dynamics of the system is described by the hamiltonian (6).

The action of the relativistic spinning particle with AMM in the external electromagnetic field can be found by Legendre transformation using (6) and the constraints (1):

$$S=\frac{1}{2}\int d\tau\left[\frac{(\dot{x}^{\mu})^{2}}{e}+em^{2}-i\left(\xi_{\mu}\dot{\xi}^{\mu}-\xi_{D+1}\dot{\xi}_{D+1}\right)-i\chi\frac{\xi_{\mu}\dot{x}^{\mu}}{e}-m\xi_{D+1}-iGF_{\mu\nu}\xi^{\mu}\xi^{\nu}\xi_{D+1}+\right.$$

$$\left.+2g\dot{x}^{\mu}A_{\mu}-4iGF_{\mu\nu}\dot{x}^{\mu}\xi^{\nu}\xi_{D+1}+igMeF_{\mu\nu}\xi^{\mu}\xi^{\nu}-eG^{2}\left[F_{\mu\nu}\xi^{\mu}\xi^{\nu}\right]^{2}\right] \qquad (8)$$

where the overdote denotes the differentiation with respect to tau along the trajectory.*)

3. The Bargmann-Michel-Telegdi equation

Now we will use the hamiltonian (6) to deduce the equation of motion of the spin of the particle with AMM in the external electromagnetic field. To describe the evolution of spin consider the Pauli-Lubanski vector in D=2n dimensions [8,9]

$$W_\mu = \frac{(-i)^{(D-2)/2}}{(D-2)!} \epsilon_{\mu\nu\lambda_2\lambda_3\ldots\lambda_{D-1}} \mathcal{P}^\nu \xi^{\lambda_2} \ldots \xi^{\lambda_{D-1}}, \qquad (9)$$

To obtain the covariant BMT equation we will use (as it is done for particle without AMM) the gauge $\chi=0$, $e=\frac{1}{m}$ [1,4]. Differentiating (9) with respect to τ, we have

$$\dot{W}_\mu = \frac{(-i)^{(D-2)/2}}{(D-2)!} \epsilon_{\mu\nu\lambda_2\lambda_3\ldots\lambda_{D-1}} (\dot{\mathcal{P}}^\nu \xi^{\lambda_2} + (D-2)\mathcal{P}^\nu \dot{\xi}^{\lambda_2}) \xi^{\lambda_3} \ldots \xi^{\lambda_{D-1}}. \qquad (10)$$

Using Eq.(6) in the chosen gauge, we find the expressions for $\dot{\mathcal{P}}^\nu$ and $\dot{\xi}^{\lambda_2}$

$$\dot{\mathcal{P}}^\nu = (\mathcal{P}^\nu, H)^* = \frac{1}{2m}\Big[2g\mathcal{P}^\mu F^\nu{}_\mu + igM\partial^\nu F_{\mu\lambda}\xi^\mu\xi^\lambda + 4iGF^\nu{}_\mu F^\mu{}_\lambda \xi^\lambda \xi_{D+1} -$$

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*) After this paper was completed we received the paper [10], in which this lagrangian was obtained in D=4 from the path integral representation for the causal Green's function of the generalized Dirac equation for the particle with AMM in an external electromagnetic field. I.Tyutin told us about papers [11] and [12], where this lagrangian was obtained in superfield formalism.

$$-4iG\partial^{\nu}F_{\mu\lambda}\mathcal{P}^{\mu}\xi^{\lambda}\xi_{D+1}-2G^{2}F_{\mu\lambda}\xi^{\mu}\xi^{\lambda}\partial^{\nu}F_{\sigma\delta}\xi^{\sigma}\xi^{\delta}\Big], \qquad (11)$$

$$\dot{\xi}^{\lambda_2}=\frac{1}{2m}\Big[2gMF^{\lambda_2}{}_{\mu}\xi^{\mu}+4GF^{\lambda_2}{}_{\mu}\mathcal{P}^{\mu}\xi_{D+1}+4iG^{2}F^{\lambda_2}{}_{\mu}\xi^{\mu}F_{\sigma\delta}\xi^{\sigma}\xi^{\delta}\Big], \qquad (12)$$

where $H=H_{ext}\big|_{\chi=0,\, e=1/m}$.

Also we have tensor relations

$$\epsilon^{\mu\nu\lambda_2\lambda_3\cdots\lambda_{D-1}}W_{\mu}=$$

$$=(-i)^{(D-2)/2}\Big[\mathcal{P}^{\nu}\xi^{\lambda_2}\xi^{\lambda_3}\cdots\xi^{\lambda_{D-1}}+(\text{cyclic permutations})\Big], \qquad (13)$$

$$\epsilon^{\mu\nu\lambda_2\lambda_3\cdots\lambda_{D-1}}W_{\mu}\mathcal{P}_{\nu}=(-1)^{(D-2)/2}\Big[-\mathcal{P}^{2}\xi^{\lambda_2}\xi^{\lambda_3}\cdots\xi^{\lambda_{D-1}}+$$

$$(\mathcal{P}_{\sigma}\xi^{\sigma})\Big(\mathcal{P}^{\lambda_2}\xi^{\lambda_3}\cdots\xi^{\lambda_{D-1}}-\mathcal{P}^{\lambda_3}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-1}}+\mathcal{P}^{\lambda_4}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-1}}-\cdots$$

$$-\mathcal{P}^{\lambda_{D-1}}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-2}}\Big)\Big]=(-1)^{(D-2)/2}\Big[-m^{2}\xi^{\lambda_2}\xi^{\lambda_3}\cdots\xi^{\lambda_{D-1}}+$$

$$m\xi_{D+1}\Big(\mathcal{P}^{\lambda_2}\xi^{\lambda_3}\cdots\xi^{\lambda_{D-1}}-\mathcal{P}^{\lambda_3}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-1}}+\mathcal{P}^{\lambda_4}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-1}}-\cdots$$

$$-\mathcal{P}^{\lambda_{D-1}}\xi^{\lambda_2}\cdots\xi^{\lambda_{D-2}}\Big)\Big]+O(\xi^{(D)}), \qquad (14)$$

where the summand $O(\xi^{(D)})$ contains terms, which after quantization contribute to corrections of the order of $\hbar$ to the BMT equation. The last equation was obtained using Eqs.(3), (5).

Taking into account Eqs.(11)-(14) we find the equation of motion of W_{μ}:

$$\dot{W}_{\mu}=g\,\frac{M}{m}F_{\mu\nu}W^{\nu}+\,2G\,\frac{\mathcal{P}_{\mu}}{m}\,F_{\nu\lambda}W^{\nu}\,\frac{\mathcal{P}^{\lambda}}{m}+O(\xi^{(D)}), \qquad (15)$$

Now since

$$u^{\nu}=\dot{x}^{\nu}=(x^{\nu},H)^{*}=\frac{\mathcal{P}^{\nu}}{m}+\frac{2i}{m}GF^{\nu}{}_{\lambda}\xi^{\lambda}\xi_{D+1}, \qquad (16)$$

we find from (15)

$$\dot{W}_\mu = g\frac{M}{m}F_{\mu\nu}W^\nu + 2Gu_\mu F_{\nu\lambda}W^\nu u^\lambda + O(\xi^{(D)}). \qquad (17)$$

This equation after quantization is a generalization to D=2n dimensions of the BMT equation for the motion of the spin of the particle with AMM in an arbitrary external electromagnetic field. Note that the Eq.(17) is form invariant in all dimensions. From (17) it is clear that M is a total magnetic moment of the particle in Bohr magnetons, while (-G) is the AMM of the particle.

4. Foldy-Wouthuysen Transformation

Now we introduce the pseudoclassical canonical Foldy Wouthuysen transformation with a generator of the infinitesimal canonical transformations [6,7]

$$S_{cl} = -2i(\mathcal{P}_j\xi_j)\xi_{D+1}\theta, \qquad (18)$$

where θ is a function of the variables of the theory which will be specified later. The result of the finite canonical transformation of any dynamical quantity f is given by the expression [13]

$$\tilde{f} \equiv e^{S_{cl}}f = f + \{f, S_{cl}\}^* + (1/2!)\{\{f, S_{cl}\}^*, S_{cl}\}^* + \ldots \qquad (19)$$

Applying Eq.(19) to a function A of the independent variables x_i, $\mathcal{P}_j$, ξ_k and taking into account the relations

$$\{A\xi_{D+1}, S_{cl}\}^* = A(2\theta)(\mathcal{P}_j\xi_j), \qquad (20)$$

$$\{A(\mathcal{P}_j\xi_j), S_{cl}\}^* = -A(2\theta)\gamma\xi_{D+1} - 2i(\mathcal{P}_j\xi_j)\xi_{D+1}A\{(\mathcal{P}_i\xi_i), \theta\}^* +$$

$$+2i\{A, (\mathcal{P}_i\xi_i)\}^*(\mathcal{P}_j\xi_j)\xi_{D+1}\theta, \qquad (21)$$

$$\{A(\mathcal{P}_j\xi_j)\xi_{D+1}, S_{cl}\}^* = 0, \qquad (22)$$

where $\gamma = i\{(\mathcal{P}_j\xi_j), (\mathcal{P}_k\xi_k)\}^* = \mathcal{P}_i^2 + igF_{ij}\xi_i\xi_j$, we find for $\tilde{A}$ the expression

$$\tilde{A}=A-\frac{1}{\sqrt{\gamma}}(A,(\wp_j\zeta_j))^*\zeta_{D+1}\sin(2\theta\sqrt{\gamma})+\frac{1}{\gamma}(A,(\wp_j\zeta_j))^*(\wp_k\zeta_k)(\cos(2\theta\sqrt{\gamma})-1)-$$
$$-\frac{1}{(\sqrt{\gamma})^3}((A,(\wp_j\zeta_j))^*,(\wp_k\zeta_k))^*(\wp_i\zeta_i)\zeta_{D+1}\left[\sin(2\theta\sqrt{\gamma})-2\theta\sqrt{\gamma}\right]-$$
$$-\frac{2}{\gamma}(A,(\wp_j\zeta_j))^*(\theta,(\wp_i\zeta_i))^*(\wp_k\zeta_k)\zeta_{D+1}\left[\cos(2\theta\sqrt{\gamma})-1\right]-$$
$$-2i(A,\theta)^*(\wp_k\zeta_k)\zeta_{D+1}. \qquad (23)$$

If we now specify the function θ taking $\mathrm{tg}(2\theta\sqrt{\gamma})=\frac{\sqrt{\gamma}}{\tilde{m}}$, and hence $\sin(2\theta\sqrt{\gamma})=\frac{\sqrt{\gamma}}{\omega}$, $\cos(2\theta\sqrt{\gamma})=\frac{\tilde{m}}{\omega}$, where $\omega=\sqrt{\wp_i^2+\tilde{m}^2+igF_{ij}\zeta_i\zeta_j}=\sqrt{\gamma+\tilde{m}^2}$, $\tilde{m}=m-iGF_{ij}\zeta_i\zeta_j$, then for $\tilde{A}$ we have

$$\tilde{A}=A-i(A,(\wp_j\zeta_j))^*\frac{\zeta_{D+1}(\omega+\tilde{m})+(\wp_j\zeta_j)}{\omega(\omega+\tilde{m})}+\frac{i(A,\gamma)^*(\wp_j\zeta_j)\zeta_{D+1}}{2\omega^2(\omega+\tilde{m})}+$$
$$+\frac{i(A,(\wp_j\zeta_j))^*((\wp_k\zeta_k),\tilde{m})^*(\wp_i\zeta_i)\zeta_{D+1}}{\omega^3(\omega+\tilde{m})}+\frac{i(A,\tilde{m})^*(\wp_j\zeta_j)\zeta_{D+1}}{\omega^2} \qquad (24)$$

This is the expression of the result of the Foldy-Wouthuysen transformation that will be used below to derive the variables which diagonalize the final Dirac brackets.

5. Final Dirac Brackets and Quantization

Following [8,9] we write down the complete set of constraints which includes the constraints which fix the remaining gauge degrees of freedom :

$$\Phi_\mu=\pi_\mu-\frac{i}{2}\zeta_\mu,\ \mu=0,1,2,..D-1\ ;\ \Phi_D=\pi_{D+1}+\frac{i}{2}\zeta_{D+1}, \qquad (25)$$

$$\Phi_{D+3}=\wp_\mu\wp^\mu-igMF_{\mu\nu}\xi^\mu\xi^\nu+4iGF_{\mu\nu}\wp^\mu\xi^\nu\xi_{D+1}+G^2(F_{\mu\nu}\xi^\mu\xi^\nu)^2-m^2,$$

$$,\Phi_{D+4}=x_0' \ ; \ \Phi_{D+5}=\pi_e, \ \Phi_{D+6}=e-\frac{\varkappa}{\wp_0} \tag{26}$$

$$\Phi_{D+1}=\wp_\mu\xi^\mu-m\xi_{D+2}+iGF_{\mu\nu}\xi^\mu\xi^\nu\xi_{D+1}, \ \Phi_{D+2}=a\xi_0+b\xi_{D+1} \tag{27}$$

here π_e is the canonical momentum, conjugate to the e; $\varkappa=+1$ corresponds to the presence of the particle in the theory, while $\varkappa=-1$ to that of antiparticle; a and b are parameters of the theory, $a^2+b^2\neq0$. When $m\neq0$, the theory has the massless limit ($m\to0$). Note also, that $x_0'=x_0-\varkappa\tau$ and the constraint $\Phi_{D+4}=x_0-\varkappa\tau$ transforms into (19) after a canonical transformation from the variables x^μ, p_μ to variables $x^{\mu\prime}, p_\mu'$, defined by the relations $x_0'=x_0-\varkappa\tau$, $x'^i=x^i$, $p_\mu'=p_\mu$ [8].

We have by definition of Dirac brackets

$$\{\tilde{A},\tilde{B}\}_{D(\Phi)}=\{\tilde{A},\tilde{B}\}^{**}-\{\tilde{A},\varphi_r\}^{**}C^{-1}_{rr'}\{\varphi_{r'},\tilde{B}\}^{**}. \tag{28}$$

Here $\varphi_r=(\Phi_{D+1},\Phi_{D+2})$, $\{.,.\}_{D(\Phi)}$ stands for the final Dirac brackets for the complete set of constraints (25)-(27), $\{.,.\}^{**}$ - for the Dirac brackets for a subset of constraints (25),(26), C^{-1} is the inverse matrix of

$$C_{rr'}=\{\varphi_r,\varphi_{r'}\}^{**}. \tag{29}$$

Now we'll take an advantage of the special structure of the constraints (refConstraintB): one of each pair of constraints is a canonical variable. This allows to prove that for any dynamical variables F and G

$$\{F,G\}^{**}=\{F,G\}^{*}, \tag{30}$$

where in the rhs of Eq.(30)the constraints (26) are taken into account (see e.g.[14]). With account of Eq. (30) the formula (28) takes the form

$$\{\tilde{A},\tilde{B}\}_{D(\Phi)}=\{\tilde{A},\tilde{B}\}^{*}-\{\tilde{A},\varphi_r\}^{*}C^{-1}_{rr'}\{\varphi_{r'},\tilde{B}\}^{*}, \tag{31}$$

where the matrix C is given by

$$C_{rr'}=\{\varphi_r,\varphi_{r'}\}^{*}=\begin{pmatrix} 0 & N \\ N & i(a^2-b^2) \end{pmatrix}, \qquad (32)$$

while N is a certain function, the explicit expression of which, as will be clear below, is immaterial. Also it is easy to prove that

$$\{\tilde{A},\tilde{B}\}^{*}=\widetilde{\{A,B\}}^{*}, \qquad (33)$$

and verify by direct calculation that

$$\{\tilde{A},\Phi_{D+1}\}^{*}\Big|_{\Phi=0}=0, \qquad (34)$$

Then taking into account that $C^{-1}_{(D+2)(D+2)}=0$, we find from (31) and (33) that

$$\{\tilde{A},\tilde{B}\}_{D(\Phi)}=\widetilde{\{A,B\}}^{*}. \qquad (35)$$

Introducing the variables A' : $A'=\tilde{A}\big|_{\Phi=0}$ and taking into account the constraints (27), rewritten in the equivalent form

$$\zeta_{D+1}+\frac{a(\mathcal{P}_j\zeta_j)}{\tilde{\beta}}\approx 0, \qquad \zeta_0-\frac{b(\mathcal{P}_j\zeta_j)}{\tilde{\beta}}\approx 0, \qquad \tilde{\beta}=a\tilde{m}-b\varkappa\omega,$$

, we fined from (24)

$$A'=\tilde{A}\big|_{\Phi=0}=A+i\{A,(\mathcal{P}_j\zeta_j)\}^{*}(\mathcal{P}_k\zeta_k)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})}, \qquad (36)$$

Now making use of the property of the Dirac brackets, which states, that the Dirac brackets of the constraints with any dynamical variable vanish, we derive from Eq.(35), (36) that

$$\{\tilde{A},\tilde{B}\}_{D(\Phi)}=\{A',B'\}_{D(\Phi)}=(\{A,B\}^{*})'=$$

$$=\{A,B\}^{*}+i\{\{A,B\}^{*},(\mathcal{P}_j\zeta_j)\}^{*}(\mathcal{P}_k\zeta_k)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})}. \qquad (37)$$

If now we take for A,B the variables x_i, $\mathcal{P}_j$, ζ_k, then on account of (36) we find the expressions

$$
\begin{aligned}
x_i' &= x_i - i\zeta_i(\mathcal{P}_j\zeta_j)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})} \equiv q_i,\\
\mathcal{P}_i' &= \mathcal{P}_i + igF_{ij}\zeta_j(\mathcal{P}_k\zeta_k)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})} \equiv \pi_i,\\
\zeta_i' &= \zeta_i + \mathcal{P}_i(\mathcal{P}_j\zeta_j)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})} \equiv \psi_i,
\end{aligned}
\tag{38}
$$

If now we apply the formula (37) to variables q_i, π_j, ψ_k we find the relations

$$
\begin{aligned}
&\{\psi_i,\psi_j\}_{D(\Phi)} = \{\zeta_i,\zeta_j\}^{*} = -i\delta_{ij},\ \{q_i,\pi_j\}_D = \{x_i,\mathcal{P}_j\}^{*} = -\delta_{ij},\\
&\{q_i,\psi_j\}_{D(\Phi)} = \{x_i,\zeta_j\}^{*} = 0,\quad \{q_i,q_j\}_D = \{x_i,x_j\}^{*} = 0,\\
&\{\pi_i,\pi_j\}_{D(\Phi)} = gF_{ij}(x) - ig\partial_k F_{ij}\zeta^k(\mathcal{P}_l\zeta_l)\frac{(a+b\varkappa)}{\tilde{\beta}(\omega+\tilde{m})} = gF_{ij}(q),
\end{aligned}
\tag{39}
$$

which prove, that the variables q_i, π_j, ψ_k are canonical. Note, that the expressions (38) coincide for D=4, with the canonical variables, which were found in [9] by diagonalization of final Dirac brackets.

Expressions of initial variables in terms of canonical ones are given by the following

$$
\begin{aligned}
x_i &= q_i - i\psi_i(\pi_j\psi_j)\frac{(b+a\varkappa)}{\tilde{\alpha}(\Omega+\tilde{m}_\psi)},\\
\mathcal{P}_i &= \pi_i + igF_{ij}\psi_j(\pi_k\psi_k)\frac{(b+a\varkappa)}{\tilde{\alpha}(\Omega+\tilde{m}_\psi)},\\
\zeta_i &= \psi_i + \pi_i(\pi_j\psi_j)\frac{(b+a\varkappa)}{\tilde{\alpha}(\Omega+\tilde{m}_\psi)},
\end{aligned}
\tag{40}
$$

where $\tilde{\alpha} = -a\varkappa\Omega + b\tilde{m}_\psi$, $\Omega = \sqrt{\pi_i^2 + \tilde{m}_\psi^2 + igF_{ij}(q)\psi_i\psi_j}$, $\tilde{m}_\psi = m - iGF_{ij}(q)\psi_i\psi_j$.

Using (40) and the relations

$$
\zeta_{D+1} = \varkappa(\pi_j\psi_j)\frac{a}{\tilde{\alpha}},\quad \zeta_0 = -\varkappa(\pi_j\psi_j)\frac{b}{\tilde{\alpha}}
\tag{41}
$$

one can deduce the expression for the physical hamiltonian of the spinning particle with AMM in the external electromagnetic field in D=2n in terms of canonical variables

$$H_{\Phi}=\Omega-g\varkappa A_0-ig\varkappa\frac{F_{0k}\psi_k(\pi_j\psi_j)}{\Omega(\Omega+\tilde{m}_{\psi})}+\frac{2iG\varkappa}{\Omega}\left[F_{0k}\psi_k+\frac{\varkappa F_{ik}\pi_i\psi_k}{(\Omega+\tilde{m}_{\psi})}\right](\pi_j\psi_j). \qquad (42)$$

To quantize the theory by the Berezin, Marinov prescription [1] one must expand Ω, $\tilde{m}_{\psi}$, which enter e.g. (42), in powers of $F_{ij}\psi_i\psi_j$ the expansion terminating in the order $\frac{D-2}{2}$. If however we take into account, that after quantization $\Psi_i\to\sqrt{\frac{\hbar}{2}}\sigma_i$, then to the order of $\hbar$ the quantum Hamiltonian is given by the expression

$$\hat{H}_{\Phi}\rightleftarrows\tilde{\Omega}-g\varkappa A_0-ig\varkappa\hbar\frac{F_{0k}\pi_j(\sigma_k\sigma_j-\sigma_j\sigma_k)}{4\tilde{\Omega}(\tilde{\Omega}+m)}+ig\varkappa\hbar\frac{F_{ij}\sigma_i\sigma_j}{4\tilde{\Omega}}+$$

$$+iG\hbar\frac{(\sigma_k\sigma_j-\sigma_j\sigma_k)}{2\tilde{\Omega}}\left[F_{0k}\hat{\pi}_j-\varkappa F_{ij}\frac{\pi_i\pi_k}{(\tilde{\Omega}+m)}\right] \qquad (43)$$

where $\tilde{\Omega}=\sqrt{\pi_i^2+m^2}$, $\rightleftarrows$ denotes the Weyl correspondence between operators and their symbols.

The expression for the hamiltonian (refPhysHam) in D=4 dimensions to the first order of g coincides with the hamiltonian found in [5].

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КАНОНИЧЕСКОЕ КВАНТОВАНИЕ D=2n-МЕРНОЙ РЕЛЯТИВИСТСКОЙ СПИНОВОЙ ЧАСТИЦЫ С АНОМАЛЬНЫМ МАГНИТНЫМ МОМЕНТОМ ВО ВНЕШНЕМ ЭЛЕКТРОМАГНИТНОМ ПОЛЕ.

Получены псевдоклассические гамильтониан и действие D=2n мерной релятивистской спиновой частицы с аномальным магнитным моментом во внешнем электромагнитном поле. Выведено уравнение Баргмана-Мишеля-Телегди для вектора Паули-Любанского. Проведено каноническое квантование D=2n-мерной релятивистской спиновой частицы с аномальным магнитным моментом во внешнем электромагнитном поле в калибровке, позволяющей описывать одновременно частицу и античастицу (как массивную, так и безмассовую) еще на классическом уровне. Квантование проведено в терминах канонических координат и импульсов типа Ньютона-Вигнера, для получения которых использовано псевдоклассическое преобразование Фолди-Вотхойзена. Обсуждается связь данной схемы квантования с картиной де Гроота и Сатторпа для частицы с аномальным магнитным моментом во внешнем электромагнитном поле.

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CANONICAL QUANTIZATION OF THE D=2N DIMENSIONAL RELATIVISTIC SPINNING PARTICLE WITH ANOMALOUS MAGNETIC MOMENT IN THE EXTERNAL ELECTROMAGNETIC FIELD

The pseudoclassical hamiltonian and action of the D=2n dimensional Dirac particle with anomalous magnetic moment interacting with the external electromagnetic field is found. The Bargmann- Michel-Telegdi equation of motion for the Pauli-Lubanski vector is deduced. The canonical quantization of D=2n dimensional Dirac spinning particle with anomalous magnetic moment in the external electromagnetic field is carried out in the gauge which allows to describe simultaneously particles and antiparticles (massive and massless) already at the classical level. Pseudoclassical Foldy-Wouthuysen transformation is used to obtain canonical (Newton-Wigner) coordinates and in terms of these variables the theory is quantized. The connection of this quantization with the deGroot and Suttorp's description of Dirac particle with anomalous magnetic moment in the external electromagnetic field is discussed.

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(на английском языке, перевод авторов)

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