В. Г. АМБАРЦУМЯН

К РАСЧЕТУ АПЕРИОДИЧЕСКИХ УСКОРИТЕЛЬНЫХ СТРУКТУР

Выведены формулы для расчета ускоряющих полей Е-типа в апериодическом цилиндрическом диафрагмированном волноводе. Учитываются также поля возбуждаемые ускоряющимся пучком, движущимся со скоростью $V \to c$ по оси структуры и сформированным в виде дельтаобразных сгустков.

Формулы получены методом сшивки полей и выражены на языке матриц переходов.

Ереванский Физический Институт
Ереван 1993

Preprint YERPHI 1411(22)-93

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
YEREVAN PHYSICS INSTITUTE

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ON THE CALCULATION OF APERIODIC ACCELERATING STRUCTURES

Նախնատիպ ԵրՖԻ-1411(22)9

Վ.Գ.ՀԱՄԲԱՐՁՈՒՄՅԱՆ

ՈՉ ՊԱՐԲԵՐԱԿԱՆ ՀԱՄԱԿԱՐԳԵՐԻ ՀԱՇՎԱՐԿԻ ՎԵՐԱԲԵՐՅԱԼ

Դուրս են բերված E-տիպի արագացնող դաշտերի հաշվարկի բանաձևեր ոչ պարբերական գլանաձև միջնորմավորված ալիքատարում:

Հաշվի են առնված նաև դելտայաձև խտացումների տեսքով ձևավորված և համակարգի առանցքով V⇒C արագությամբ շարժվող արագացվող փնջով գրգռված դաշտերը:

Բանաձևերը ստացված են դաշտերի կարման եղանակով և արտահայտված են անցումային մատրիցների լեզվով:

Երևանի Ֆիզիկայի Ինստիտուտ
Երևան 1993

The new generation of charged particles' linear accelerators is distinguished by the high acceleration rates and large beam currents, which requires a usage of accelerating fields with higher frequency and the size across of decrease accelerating structures. Hence, there arise problems of field neutralization connected with beam itself, that are supressing the beam structure. One of the methods of such suppression is the use of aperiodic structures.

In this work we derive the formulae for the numerical calculation of the axially symmetric accelerating field (m=0) inside the structures, which consist of the arbitrary number of cavieties and irises. A matrix formalism is applied. The derivations are based on the field-matching technique. We take into acount also the wakefields excited by the train of pointlike bunches with the eN_B charge, where e is the electron charge, N_B is the electrons' number in the bunch, each moving along the axis of the structure at the velocity of C light. The structure is the same as discribed in Ref.[1] (see fig.I,II).

1. The expression of the fields

At this point we follow the Ref.[1]. The K-th Fourier harmonics of the current density and of the fields in the region inside the N-th cavity can be presented in the spatial modes:

$$j(r,\varphi,z,k)=\frac{eN_B}{2\pi}\frac{\delta(r)}{r}\sum_j \exp\{ik(z-S_j)\} \qquad 1.1$$

$$E_z^N(r,z,k)=\sum_{n=1}^{\infty}(\nu_n/b_N)^2 J_0(\nu_n r/b_N)X_n^N(z) \qquad 1.2$$

$$E_r^N(r,z,k)=\frac{2eN_B}{cr}\sum_j \exp\{ik(z-S_j)\}-\sum_{n=1}^{\infty}(\nu_n/b_N)J_1(\nu_n r/b_N)Y_n^N(z) \qquad 1.3$$

$$H_\varphi^N(r,z,k)=\frac{2eN_b}{c\,r}\sum_j \exp\{ik(z-S_j)\}-ik\sum_{n=1}^{\infty}(\nu_n/b^N)J_1(\nu_n r/b^N)X_n^N(z) \qquad 1.4$$

where

$$X_n^N(z)=\vec{C}_n^N\exp\{i\lambda_n^N z\}+\overleftarrow{C}_n^N\exp\{-i\lambda_n^N z\} \qquad 1.5$$

$$Y_n^N(z)=\frac{dX_n^N(z)}{dz};\ \lambda_n^N=\sqrt{k^2-(\nu_n/b^N)^2};\ k=\frac{\omega}{c};\ 0\le z\le g^N \qquad 1.6$$

The longitudinal coordinate Z of the observation point and S_j are calculated from the right end of the first cavity. S_j is the longitudinal coordinate of the J-th bunch at time t = 0.

Similarly for the N-th iris:

$$E_z^N(r,z,k)=\sum_{n=1}^{\infty}(\nu_n/a^N)^2 J_0(\nu_n r/a^N)\xi_n^N(z) \qquad 1.7$$

$$E_r^N(r,z,k)=\frac{2eN_b}{c\,r}\sum_j \exp\{ik(z-S_j)\}-\sum_{n=1}^{\infty}(\nu_n/a^N)J_1(\nu_n r/a^N)\eta_n^N(z) \qquad 1.8$$

$$H_\varphi^N(r,z,k)=\frac{2eN_b}{c\,r}\sum_j \exp\{ik(z-S_j)\}-ik\sum_{n=1}^{\infty}(\nu_n/a^N)J_1(\nu_n r/a^N)\xi_n^N(z) \qquad 1.9$$

where

$$\xi_n^N(z)=\vec{\mathcal{Z}}_n^N\exp\{i\mu_n^N z\}+\overleftarrow{\mathcal{Z}}_n^N\exp\{-i\mu_n^N z\} \qquad 1.10$$

$$\eta_n^N(z)=\frac{d\xi_n^N(z)}{dz};\ \mu_n^N=\sqrt{k^2-(\nu_n/a^N)^2};\ 0\le z\le \ell^N \qquad 1.11$$

In the above expressions ν_n-are the roots of the first order Bessel function $J_0(\nu_n)=0$. For the quantities μ_n^N and λ_n^N we have:

$$\mu_n^N=i\sqrt{(\nu_n/a^N)^2-k^2} \quad \text{if} \quad k^2-(\nu_n/a^N)^2<0 \qquad 1.12$$

$$\lambda_n^N=i\sqrt{(\nu_n/b^N)^2-k^2} \quad \text{if} \quad k^2-(\nu_n/b^N)^2<0 \qquad 1.13$$

2.Fields matching

The continuity conditions on the apertures at $Z = g^N$ and $Z=\ell^N$ are:

$$E_z(r,g^N-0,k)=E_z(r,g^N+0,k) \quad \text{for } 0\le r\le a^{N+1} \qquad 2.1$$

$$E_r(r,\ell^N-0,k)=E_r(r,\ell^N+0,k) \quad \text{for } 0\le r\le a^N \qquad 2.2$$

$$E_r(r,g^N-0,k)=E_r(r,g^N+0,k)\theta(a^{N+1}-r) \quad \text{for } 0\le r\le b^N \qquad 2.3$$

$$E_z(r,\ell^N-0,k)\theta(a^N-r)=E_z(r,\ell^N+0,k) \quad \text{for } 0\le r\le b^N \qquad 2.4$$

From the conditions (2.1÷ 2.4) we obtain:

$$Y_m^N(g^N)=P_m^N(d^N)+\sum_{n=1}^{\infty}T_{mn}(d^N)\eta_n^{N+1}(0) \qquad 2.5$$

$$Y_m^N(0)=P_m^N(p^N)+\sum_{n=1}^{\infty}T_{mn}(p^N)\eta_n^N(\ell^N) \qquad 2.6$$

$$\xi_m^N(\ell^N)=\sum_{n=1}^{\infty}S_{mn}(p^N)X_n^N(0) \qquad 2.7$$

$$\xi_m^{N+1}(0)=\sum_{n=1}^{\infty}S_{mn}(d^N)X_n^N(\ell^N) \qquad 2.8$$

where

$$T_{mn}(\alpha)=\frac{2J_1(\nu_n)J_0(\nu_m\alpha)\nu_n\alpha^2}{J_1^2(\nu_m)(\nu_m^2-\alpha^2\nu_n^2)\nu_m};\ S_{mn}(\alpha)=\frac{2J_0(\nu_n\alpha)\nu_n^2\alpha^2}{J_1(\nu_m)(\nu_m^2-\alpha^2\nu_n^2)\nu_m} \qquad 2.9$$

$$P_m^N(d^N)=T(k)\frac{J_0(\nu_m d^N)}{J_1^2(\nu_m)\nu_m^2}\exp\{ik\sum_{n=1}^{N}(g^n+\ell^n)\} \qquad 2.10$$

$$P_m^N(p^N)=T(k)\frac{J_0(\nu_m p^N)}{J_1^2(\nu_m)\nu_m^2}\exp\{ik\sum_{n=1}^{N}(g^n+\ell^n)-ikg^N\} \qquad 2.11$$

$$J(k)=\frac{4eN_B}{c}\sum_j \exp\{-ikS_j\};\quad \rho^N=\frac{a^N}{b^N};\quad d^N=\frac{a^{N+1}}{b^N} \tag{2.12}$$

3. Derivation of the basic expressions

For the convenience we introduce the following

$$X^N(\alpha)\equiv\begin{pmatrix}X_1^N(\alpha)\\ X_2^N(\alpha)\\ \vdots\end{pmatrix};\qquad Y^N(\alpha)\equiv\begin{pmatrix}Y_1^N(\alpha)\\ Y_2^N(\alpha)\\ \vdots\end{pmatrix} \tag{3.1}$$

$$\xi^N(\alpha)\equiv\begin{pmatrix}\xi_1^N(\alpha)\\ \xi_2^N(\alpha)\\ \vdots\end{pmatrix};\qquad \eta^N(\alpha)\equiv\begin{pmatrix}\eta_1^N(\alpha)\\ \eta_2^N(\alpha)\\ \vdots\end{pmatrix} \tag{3.2}$$

From the (1.5,1.6,1.10,1.11) quantities X_n, ξ_n can be expressed in terms of Y^N, η^N:

$$X^N(0)=\tilde\Lambda_s^N Y^N(g^N)-\tilde\Lambda_t^N Y^N(0) \tag{3.3}$$

$$X^N(g^N)=\tilde\Lambda_t^N Y^N(g^N)-\tilde\Lambda_s^N Y_1^N(0) \tag{3.4}$$

$$\xi^N(0)=\tilde M_s^N \eta^N(\ell^N)-\tilde M_t^N \eta^N(0) \tag{3.5}$$

$$\xi^N(\ell^N)=\tilde M_t^N \eta^N(\ell^N)-\tilde M_s^N \eta^N(0) \tag{3.6}$$

where

$$\left(\tilde\Lambda_s^N\right)_{mk}=\frac{\delta_{mk}}{i\lambda_m^N sh(i\lambda_m^N g^N)};\quad \left(\tilde\Lambda_t^N\right)_{mk}=\frac{\delta_{mk}}{i\lambda_m^N th(i\lambda_m^N g^N)} \tag{3.7}$$

$$\left(\tilde M_s^N\right)_{mk}=\frac{\delta_{mk}}{i\mu_m^N sh(i\mu_m^N \ell^N)};\quad \left(\tilde M_t^N\right)_{mk}=\frac{\delta_{mk}}{i\mu_m^N th(i\mu_m^N \ell^N)} \tag{3.8}$$

Let's introduce the quantities $\tilde\eta^N$ as follows:

$$\tilde\eta^N=\eta^M(0)\quad \text{for the odd } N$$

$$\tilde\eta^N=\eta^M(\ell^M)\quad \text{for the even } N \tag{3.9}$$

$$M=\mathrm{Int}\left(\frac{N+1}{2}\right)$$

Using the formulae (2.5 + 2.8, 3.3 + 3.6) we obtain the basic expressions:

$$E^N\tilde\eta^{N-1}+F^N\tilde\eta^N+G^N\tilde\eta^{N+1}+H^N=0 \tag{3.10}$$

where

for the even N :

$$E^N=\tilde M_s^K \tag{3.11}$$

$$F^N=-S(\rho^K)\tilde\Lambda_t^K T(\rho^K)-\tilde M_t^K \tag{3.12}$$

$$G^N=S(\rho^K)\tilde\Lambda_s^K T(d^K) \tag{3.13}$$

$$H^N=S(\rho^K)\{\tilde\Lambda_s^K P^K(d^K)-\tilde\Lambda_t^K P^K(\rho^K)\} \tag{3.14}$$

and for the odd N:

$$E^N=-S(d^K)\tilde\Lambda_s^K T(\rho^K) \tag{3.15}$$

$$F^N=\tilde M_t^{K+1}+S(d^K)\tilde\Lambda_t^K T(d^K) \tag{3.16}$$

$$G^N=-\tilde M_s^{K+1} \tag{3.17}$$

$$H^N=S(d^K)\{\tilde\Lambda_t^K P^K(d^K)-\tilde\Lambda_s^K P^K(\rho^K)\} \tag{3.18}$$

where

$$K=\mathrm{Int}\left(\frac{N}{2}\right) \tag{3.19}$$

The basic formulae (3.10) are the expression of the theorem of unequivocal determination of the fields.

4. The boundary conditions and determination of the fields

For the left end of the structure $\vec{C}_n^1$ are chosen as known. Similarly for the right end $\overleftarrow{C}^{Nc}$ are chosen as known.

From the formulae (1.5, 1.6, 2.6, 2.7, 3.6) at $N=N_c$ we can express $\eta^{Nc}(\ell^{Nc})$ in terms of $\eta^{Nc}(0)$ and $\overleftarrow{C}_n^{Nc}$:

$$\eta^{Nc}(\ell^{Nc})=\{\tilde{M}_t^{Nc}+iS(p^{Nc})(\Lambda^{Nc})^{-1}T(p^{Nc})\}^{-1}\{\tilde{M}_s^{Nc}\eta^{Nc}(0)-iS(p^{Nc})(\Lambda^{Nc})^{-1}[p^{Nc}(p^{Nc})-2\overleftarrow{C}^{Nc}]\} \qquad 4.1$$

where

$$\Lambda_{mk}^N=\lambda_m^N\delta_{mk};\quad \vec{C}^N\equiv\begin{pmatrix}\vec{C}_1^N\\ \vec{C}_2^N\\ \vdots\end{pmatrix};\quad \overleftarrow{C}^N\equiv\begin{pmatrix}\overleftarrow{C}_1^N\\ \overleftarrow{C}_2^N\\ \vdots\end{pmatrix} \qquad 4.2$$

We can rewrite (4.1) in the terms of $\tilde{\eta}^N$

$$\tilde{\eta}^{2Nc}=A^{2Nc-1}\tilde{\eta}^{2Nc-1}+B^{2Nc-1} \qquad 4.3$$

where

$$A^{2Nc-1}=\{\tilde{M}_t^{Nc}+iS(p^{Nc})(\Lambda^{Nc})^{-1}T(p^{Nc})\}^{-1}\tilde{M}_s^{Nc} \qquad 4.4$$

$$B^{2Nc-1}=-i\{\tilde{M}_t^{Nc}+iS(p^{Nc})(\Lambda^{Nc})^{-1}T(p^{Nc})\}^{-1}S(p^{Nc})(\Lambda^{Nc})^{-1}\{p^{Nc}(p^{Nc})-2\overleftarrow{C}^{Nc}\} \qquad 4.5$$

Let's introduce the "transfer" matrix A^N and B^N according to the expression:

$$\tilde{\eta}^{N+1}=A^N\tilde{\eta}^N+B^N \qquad 4.6$$

With the substitutions in the basic expressions (3.10) we obtain the recurrence equations for the matrix A^N and B^N :

$$A^{N-1}=-(F^N+G^NA^N)^{-1}E^N;\quad B^{N-1}=-(F^N+G^NA^N)^{-1}(G^NB^N+H^N) \qquad 4.7, 4.8$$

Now we use the boudary conditions at the left end of the structure. Let us assume $\varphi^1=0.$, then from the formulae (1.5, 1.6, 2.7) at N=1, (3.6) at N=2 and (4.6) at N=3, we express $\tilde{\eta}^3$ in terms of known quantities $\tilde{C}_n^1$:

$$\tilde{\eta}^3=\{\tilde{M}_s^2A^3-\tilde{M}_t^2-iS(d^1)(\Lambda^1)^{-1}T(d^1)\}^{-1}\times$$

$$\times\{2S(d^1)\vec{C}^1+iS(d^1)(\Lambda^1)^{-1}p^1(d^1)-\tilde{M}_s^2B^3\} \qquad 4.9$$

For the known C^1 and C^{Nc} we determine the "transfer" matrix A^N, B^N and $\tilde{\eta}^3\equiv\eta^2(0)$. Using the formulae (4.6) we obtain the $\tilde{\eta}^N$ ($N=3,4,\ldots 2N_c$) and the fields inside the arbitrary irise. From the matching formulae (2.5, 2.6) we obtain quantities $Y^N(0)$, $Y^N(\varphi^N)$ $N=2,3,\ldots N^c$) and the fields in an arbitrary cavity. This method requires the inversion of the imaginary matrices $(F^N+G^N\tilde{A}^N)$. We propose another way in which the matrices $(F^N+G^NA^N)$ are real.

From the basic equations (3.1) at $N=2N_c-1$ we obtain:

$$\tilde{\eta}^{2Nc-1}=\tilde{A}^{2Nc-2}\tilde{\eta}^{2Nc-2}+\tilde{B}^{2Nc-2}+\bar{B}^{2Nc-2} \qquad 4.10$$

where

$$\tilde{A}^{2Nc-2}=-(F^{2Nc-1})^{-1}E^{2Nc-1} \qquad 4.11$$

$$\tilde{B}^{2Nc-2}=-(F^{2Nc-1})^{-1}H^{2Nc-1} \qquad 4.12$$

$$\bar{B}^{2Nc-2}=\hat{B}^{2Nc-2}\tilde{\eta}^{2Nc} \qquad 4.13$$

$$\hat{B}^{2Nc-2}=-(F^{2Nc-1})^{-1}G^{2Nc-1} \qquad 4.14$$

Using the substitutions in the basic expressions (3.10) we obtain the recurrence formulae for determining $\tilde{A}^N, \tilde{B}^N, \hat{B}^N$:

$$\tilde{\eta}^N = \tilde{A}^{N-1}\tilde{\eta}^{N-1} + \tilde{B}^{N-1} + \hat{B}^{N-1}\tilde{\eta}^{2N_c} \qquad 4.15$$

where

$$\tilde{A}^{N-1} = -(F^N + G^N\tilde{A}^N)^{-1}E^N \qquad 4.16$$

$$\tilde{B}^{N-1} = -(F^N + G^N\tilde{A}^N)^{-1}(G^N\tilde{B}^N + H^N) \qquad 4.17$$

$$\hat{B}^{N-1} = -(F^N + G^N\tilde{A}^N)^{-1}G^N\hat{B}^N \qquad 4.18$$

We define now the "transfer" matrix $\tilde{A}^{N,M}$ and $\tilde{B}^{N,M}$ according to the expression $(M \geq N)$:

$$\tilde{\eta}^M = \tilde{A}^{N,M}\tilde{\eta}^N + \tilde{B}^{N,M} + \hat{B}^{N,M}\tilde{\eta}^{2N_c} \qquad 4.19$$

$$\tilde{A}^{N,M} = \prod_{K=N}^{M-1}\tilde{A}^K;\ \tilde{B}^{N,M} = \sum_{L=N+1}^{M}\tilde{A}^{L,M}\tilde{B}^{L-1};\ \hat{B}^{N,M} = \sum_{L=N+1}^{M}\tilde{A}^{L,M}\hat{B}^{L-1};\ \tilde{A}^{K,K} = I \qquad 4.20$$

We can rewrite (4.9) in another form:

$$\tilde{\eta}^3 = \tilde{\Omega} + \hat{\Omega}\tilde{\eta}^{2N_c} \qquad 4.21$$

where

$$\tilde{\Omega} = \left\{\hat{M}_S^2\hat{A}^3 - \hat{M}_t^2 - iS(d^1)(\Lambda^1)^{-1}T(d^1)\right\}^{-1} \times$$

$$\times\left\{2S(d^1)\vec{C}^1 + iS(d^1)(\Lambda^1)^{-1}p^1(d^1) - \hat{M}_S^2\tilde{B}^3\right\} \qquad 4.22$$

$$\hat{\Omega} = \left\{iS(d^1)(\Lambda^1)^{-1}T(d^1) + \tilde{M}_t^2 - \tilde{M}_S^2\hat{A}^3\right\}^{-1}\tilde{M}_S^2\hat{B}^3 \qquad 4.23$$

Substituting the (4.21) in the (4.19) at N=3, $M = 2N_c - 1$ we obtain:

$$\tilde{\eta}^{2N_c-1} = \tilde{A}^{3,2N_c-1}(\tilde{\Omega} + \hat{\Omega}\tilde{\eta}^{2N_c}) + \tilde{B}^{3,2N_c-1} + \hat{B}^{3,2N_c-1}\tilde{\eta}^{2N_c} \qquad 4.24$$

or

$$\tilde{\eta}^{2N_c-1} = \left\{\tilde{A}^{3,2N_c-1}\hat{\Omega} + \hat{B}^{3,2N_c-1}\right\}\tilde{\eta}^{2N_c} + \tilde{A}^{3,2N_c-1}\tilde{\Omega} + \tilde{B}^{3,2N_c-1} \qquad 4.25$$

(4. 25) and (4.3) give $\tilde{\eta}^{2N_c}$:

$$\tilde{\eta}^{2N_c} = \left\{\tilde{M}_t^{N_c} + iS(p^{N_c})(\Lambda^{N_c})^{-1}T(p^{N_c}) - \tilde{M}_S^{N_c}\left[\tilde{A}^{3,2N_c-1}\hat{\Omega} + \hat{B}^{3,2N_c-1}\right]\right\}^{-1} \times$$

$$\times\left\{\tilde{M}_S^{N_c}\left[\tilde{A}^{3,2N_c-1}\tilde{\Omega} + \tilde{B}^{3,2N_c-1}\right] - iS(p^{N_c})(\Lambda^{N_c})^{-1}\left[P^{N_c}(p^{N_c}) - 2\vec{C}^{N_c}\right]\right\} \qquad 4.26$$

At the known $\tilde{\eta}^{2N_c}$, $\tilde{\eta}^3$ and "transfer" matrix we determine the fields in an arbitrary point inside of structure.

The numerical calculations require the cutting in the matrices and vectors with the infinite dimensions.

The truncation number J^c can be obtained in the calculation process.

The author would like to express his thanks to E.M.Laziev and V.M.Tsakanov for constructive remarks and helpful discussions.

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Pig.1. The general view of structure axis cut and some designations.

Pig.2. The general view of N-cell axis cut and designations.

The manuscript was received in November, 2, 1993

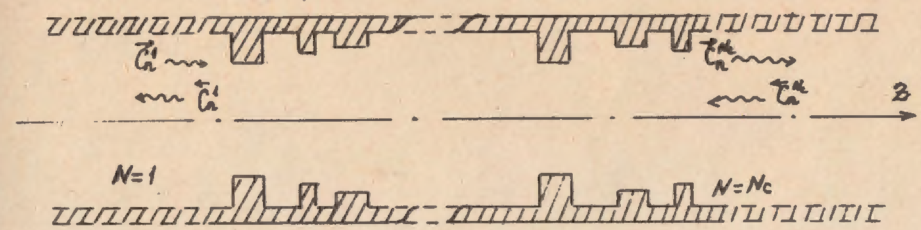

Fig. 1.

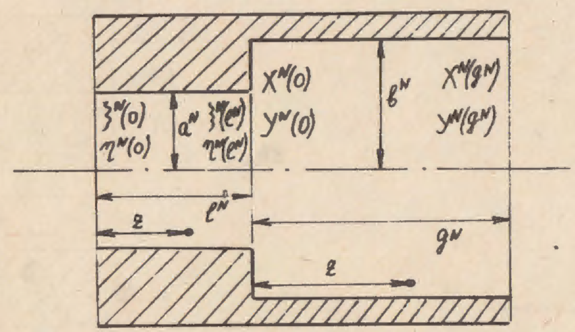

Fig. 2.