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GERASIMOV-DRELL-HEARN SUM RULE AND THE BEHAVIOUR OF NUCLEON  
POLARIZED STRUCTURE FUNCTION AT SMALL  $Q^2$

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**ԳԵՐԱՍԻՄՈՎ-ԴՐԵԼԼ-ՎԵՐՆԻ ԳՈՒՄԱՐԻ ԿԱՆՈՆՆԵՐԸ ԵՎ ՆՈՒԿԼՈՆԻ  
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ՓՈՔՐ Ձ<sup>2</sup> ԴԵՊՋՈՒՄ:**

**Ի Գ Ա Ջ Ն Ա ՈՒ Ի Ր Յ Ա Ն**

Օգտագործելով Գերասիմով-Դրելլ-Վերնի գումարների կանոնը ցույց է տրված, որ նեյտրոնի բնեռացման կառուցվածքային ֆունկցիան փոքր  $\alpha=294t^2$  դեպքում դուրս է գալիս ապիմպտոտիկ արժեքի վրա: Պրոտոնային բնեռացման կառուցվածքային ֆունկցիաների համար ցույց է տրված, որ սկսած  $1.594t^2$  նրա շեղումը ապիմպտոտիկ արժեքից չի գերազանցում 15-20%: Եղած տվյալների հիման վրա,  $\alpha^2 < 394t^2$  դեպքում էներգիաների ռեզոնանսային տիրույթում ստացված են ևուկլոնի բնեռացման կառուցվածքային ֆունկցիայի ինտեգրալների արժեքները  $X=0.45-1$  սահմանում, ուր բացակայում են փորձաքարական տվյալները: Պարզվել է, որ  $\alpha^2=2-394t^2$  դեպքում այդ ինտեգրալները ունեն սիեյնիկյան վաքք, իսկ նրանց մեծությունները համաձայնեցվում են փորձաքարական աշխատանքներում արված կատրապոլյացիաների հետ, մեծ  $X$ -ի տիրույթում:

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# GERASIMOV-DRELL-HEARN SUM RULE AND THE BEHAVIOUR OF NUCLEON POLARIZED STRUCTURE FUNCTION AT SMALL $Q^2$

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## Abstract

Using Gerasimov-Drell-Hearn sum rule it is shown that neutron polarized structure function achieves the asymptotic value at  $Q^2 = 2\text{Gev}^2$ . For proton polarized structure function it is shown that starting from  $1.5\text{ Gev}^2$  its deviation from the asymptotic value does not exceed 15-20%. The integrals over nucleon polarized structure functions are obtained in the interval of  $x = 0.45 \div 1$  on the basis of available data in the resonance energy region at  $Q^2 < 3\text{Gev}^2$ . It appears that at  $Q^2 = 2 + 3\text{Gev}^2$  these integrals have scaling behaviour, and their values agree with the extrapolations to the region of large  $x$  made in experimental works.

Recently the Gerasimov-Drell-Hearn sum rule (GDH) [1, 2] has drawn attention from the point of view to obtain using it an information on the possible scenarios of transition to the asymptotics of the nucleon polarized structure function  $g_1(x, Q^2)$  [3]-[5]. When comparing with the Ellis-Jaffe [6] and Bjorken [7] sum rules this sum rule dictates strong change of the integral over nucleon polarized structure function under variation of  $Q^2$  from 0 to  $\infty$ :

$$\begin{aligned} I_N(Q^2) &= \frac{m^2}{2\pi e^2} \int_{Q^2/2m}^{\infty} \frac{d\nu}{\nu} [\sigma_{1/2}(Q^2) - \sigma_{3/2}(Q^2)] \frac{1}{\nu^2} \Big|_{Q^2=0}^{Q^2=Q^2} \\ &= m \int_{Q^2/2m}^{\infty} g_1^N(\nu, Q^2) \frac{d\nu}{\nu^2} = \end{aligned} \quad (1)$$

$$= \begin{cases} -\frac{k_N^2}{4} & \text{for } Q^2 = 0 \text{ (GDH sum rule [1, 2]),} \\ \frac{2m^2}{Q^2} \Gamma_N(Q^2) \equiv \frac{2m^2}{Q^2} \int_0^1 g_1^N(x) dx & \text{for } Q^2 \rightarrow \infty, \end{cases} \quad (1a)$$

$$(1b)$$

where  $\sigma_{3/2}$  and  $\sigma_{1/2}$  are total absorption cross sections of transverse photons by nucleons with parallel and antiparallel spins,  $m$  and  $k$  are the nucleon mass and anomalous magnetic moments, and in the asymptotics according to the Ellis-Jaffe [6] and Bjorken [7] sum rules we have:

$$\Gamma_p(Q^2 \rightarrow \infty) = \frac{1}{12} \left[ \frac{g_A}{g_V} + \frac{5}{3}(3F - D) \right] = 0.188 \pm 0.014, \quad (2)$$

$$(\Gamma_p - \Gamma_n)(Q^2 \rightarrow \infty) = \frac{1}{6} \frac{g_A}{g_V} = 0.209 \pm 0.001. \quad (3)$$

In this connection an assumption was made [3], that the significant deviation of the measured at  $Q^2 = 10.7 \text{ GeV}^2$  value  $\Gamma_p = 0.126 \pm 0.010 \pm 0.015$  [9]-[11] from the asymptotic one (2) is connected with the large corrections which can be estimated using the GDH sum rule. However, the investigation of the resonance contributions to the  $Q^2$ -evolution of the integral  $I_p(Q^2)$  has shown [4],[11] that the strong change of this integral with increasing  $Q^2$  is connected mainly with the  $\Delta$ -isobar contribution, and after extraction of this contribution it appeared that at  $Q^2 = 10.7 \text{ GeV}^2$  the corrections are not large.

In this work we will use the GDH sum rule in the framework of the approach developed in [4] to investigate the behaviour of the nucleon polarized structure function at small  $Q^2 < 3 \text{ GeV}^2$ . Such consideration is of interest as the recent measurement of the neutron polarized structure function at  $Q^2 = 2 \text{ GeV}^2$  [12] has revealed that  $\Gamma_n(Q^2)$  achieves its asymptotic value at small  $Q^2$ . At the same time according to the results of the Ref [4] at small  $Q^2$   $\Gamma_p(Q^2)$  deviates significantly from its asymptotic value. We will draw special attention to the account of the resonance contributions which are important at small  $Q^2$  and which can not be described by simple extrapolation of the integral  $I_N(Q^2)$  from its value at  $Q^2 = 0$  to the asymptotic value given by relations (1-3).

Let us divide the integrals in (1) into integrals over resonance energy region and over

high energies:

$$I_N(Q^2) = I_N^{\text{Res}} Q^2 + I_N^{\text{HB}}(Q^2). \quad (4)$$

The analyses made in Refs.[12, 14, 15] show that at  $W > 1.8\text{Gev}$  ( $W^2 = 2m\nu - Q^2$ ) the resonance contributions to  $I_N(Q^2)$  are negligible, and in  $I_N^{\text{Res}}(Q^2)$  it is reasonable to be restricted by the energy region with  $W < 1.8\text{Gev}$ . This energy region includes the I-resonance region- the resonance  $P_{33}(1232)$ , the II-resonance region-the resonances  $P_{11}(1440)$ ,  $D_{13}(1520)$ ,  $S_{11}(1535)$  and the III-resonance region - the resonances  $S_{31}(1620)$ ,  $S_{11}(1650)$ ,  $D_{15}(1675)$ ,  $F_{15}(1680)$ ,  $D_{13}(1700)$ ,  $D_{33}(1700)$  and  $P_{13}(1720)$

The integrals  $I_N^{\text{Res}}(Q^2)$  can be quite reliable estimated at  $(Q^2) < 3\text{Gev}^2$  as the main contributions which come from the resonances  $P_{33}(1232)$ ,  $D_{13}(1520)$ ,  $S_{11}(1535)$  and  $F_{15}(1680)$  are well known from the experiments on the proton [15] - [21], and other contributions (and contributions for the neutron) can be obtained using the analysis made in [21]. In this work on the basis of available experimental data on the proton for the members of multiplets  $[70, 1^-] - D_{13}(1520)$ ,  $S_{11}(1535)$  and  $[56, 2^+] - F_{15}(1680)$  and of selection rules which follow from Melosh transformation [22], the amplitudes for other members of these multiplets are obtained. So, amplitudes for remained resonances are also known. These are amplitudes on the neutron for  $D_{13}(1520)$ ,  $S_{11}(1535)$  and  $F_{15}(1680)$  and the amplitudes for  $S_{31}(1620)$ ,  $S_{11}(1650)$ ,  $D_{15}(1675)$ ,  $D_{13}(1700)$ ,  $D_{33}(1700)$  - members of the multiplet  $[70, 1^-]$ , and for  $P_{13}(1720)$  - member of the multiplet  $[56, 2^+]$ .

To calculate resonance contributions to  $I_N^{\text{Res}}(Q^2)$  it is convenient, to explore the narrow resonance approximation which gives:

$$I_N^{\text{Res}}(Q^2) = \sum_{\alpha=I,II,III} I_{\alpha}^N(Q^2), \quad I_{\alpha}(Q^2) = \frac{2m^3}{c^2} \sum_{i \in \text{Res}} \frac{(A_{1/2}^i)^2 - (A_{3/2}^i)^2}{M_i^2 - m^2 + Q^2} \quad (5)$$

where we have represented  $I_N^{\text{Res}}(Q^2)$  as a sum of contributions from the above defined three resonance regions, the amplitudes are normalised according to the condition

$$\sigma_i^i = \frac{4m}{M_i \Gamma_i} (A_i^i)^2,$$

and  $M_i$  and  $\Gamma_i$  are mass and total width of the resonance  $i$ . In the numerical calculations we have used the parametrisations of the amplitudes  $A_i^i$  obtained in Refs.[11],[21] Let

us note that in the I-resonance region we have also taken into account the contribution of the region near threshold. This contribution is determined mainly by the one-pion-exchange in the pion photoproduction on nucleons. It is quite large at  $Q^2 = 0$  due to smallness of the denominator in the integral  $I_n(Q^2)$  (1) near threshold. With increasing  $Q^2$  this contribution falls rapidly.

Summary resonance contributions in three resonance regions are given on fig.1. These contributions are characterized by the large and negative contribution of the resonance  $P_{33}(1232)$  (which is decisive at  $Q^2 = 0$ ), and beginning with  $1 \text{ GeV}^2$  by large and positive contribution of II- and III-resonance regions into integral for proton  $I_n(Q^2)$ . These contributions are determined mainly by the resonances  $D_{13}(1520)$ ,  $S_{11}(1535)$  and  $F_{15}(1680)$  and are characterized by unusually slow decreasing of the amplitudes  $A_{1/2}^p$  with increasing  $Q^2$ . For the resonances  $D_{13}(1520)$  and  $F_{15}(1680)$  this slow decreasing of the amplitudes  $A_{p}^{1/2}$  results in the early (up to  $1 \text{ GeV}^2$ ) change of the sign of the asymmetry  $A = (A_{1/2}^2 - A_{3/2}^2)/(A_{1/2}^2 + A_{3/2}^2)$  for proton, which is observed in many experiments [23]. The neutron amplitudes have not such behaviour, by this reason the contributions of the II- and III- resonance regions into the integral for the neutron  $I_n(Q^2)$  are significantly smaller. As a result the integral over  $g_1(x, Q^2)$  over resonance energy region:

$$\Gamma_N^{\text{Res}}(Q^2) \equiv \Gamma_N(x_{\text{Res}}, Q^2) = \int_{x_{\text{Res}}}^1 g_1^N(x, Q^2) dx, \quad (6)$$

( $x_{\text{Res}} = Q^2 / [(W_{\text{Res}}^{\text{max}})^2 - m^2 + Q^2]$ , in our consideration  $W_{\text{Res}}^{\text{max}} = 1.8 \text{ GeV}$ ) at  $Q^2 = 2 + 3 \text{ GeV}^2$  for the proton is larger than for the neutron, namely:

$$\begin{aligned} \Gamma_p(0.46, 2 \text{ GeV}^2) &= 0.02, & \Gamma_p(0.56, 3 \text{ GeV}^2) &= 0.018, \\ \Gamma_n(0.46, 2 \text{ GeV}^2) &= -0.003, & \Gamma_n(0.56, 3 \text{ GeV}^2) &= 0.003. \end{aligned} \quad (7)$$

These results agree well with the extrapolations made in [8, 9, 12] into the region of large  $x$ , where experimental data are absent:

$$\Gamma_p(0.45, Q^2 = 10.7 \text{ GeV}^2) = 0.017^{[8, 9]}, \quad (8)$$

$$\Gamma_n(0.6, Q^2 = 2 \text{ GeV}^2) = 0.003^{[12]}. \quad (9)$$

So, we obtain that at  $Q^2 = 2 + 3 \text{ Gev}^2$  the contribution of the resonance energy region into  $\Gamma_N(Q^2)$  has scaling behaviour and agrees with estimations made in the experimental works.

Let us consider now the integrals over high energy region with  $W > 1.8 \text{ Gev}$  and suppose, as it is made in Refs.[3, 4] that their  $Q^2$ -evolution is determined by vector dominance:

$$I_N^{HB}(Q^2) = a_N \frac{\mu^2}{\mu^2 + Q^2} + b_N \frac{\mu^4}{(\mu^2 + Q^2)^2}, \quad (10)$$

where  $\mu$  is the mass of  $\rho$ -meson. With this in asymptotics we have:

$$I_N^{Re}(Q^2) \rightarrow 0, \quad I_N^{HB}(Q^2) \rightarrow \frac{2m^2}{Q^2} \Gamma_N(Q^2) \quad \text{at } Q^2 \rightarrow \infty, \quad (11)$$

and from asymptotic sum rules (1b), (2) and (3) it follows:

$$\alpha_p = 0.56, \quad \alpha_n = -0.062. \quad (12)$$

At  $Q^2 = 0$  using GDH sum rule (1a) and the values of  $I_N^{Re}(0)$  (fig.1) we obtain

$$I_p^{HB}(0) = \alpha_p + b_p = 0.200, \quad I_n^{HB}(0) = \alpha_n + b_n = -0.205. \quad (13)$$

So, both parts in the integrals  $I_N(Q^2)$  (4) are determined, and we can turn to the final results for  $\Gamma_N(Q^2)$  which are presented on fig.2.

Our results for the proton at large  $Q^2$  coincide with the results of [4] which are also presented on fig.2. However at  $Q^2 = 1.5 + 3 \text{ Gev}^2$  our results are more close to the asymptotic value of  $\Gamma_p(Q^2)$  than in [4]. This is connected with the fact that in addition to the resonance  $F_{33}(1232)$ , which is taken into account in [4], we have taken into account the contributions of the II- and III-resonance regions which are quite large due to slow decreasing of the amplitudes  $A_{1/2}^p$  for the resonances  $D_{13}(1520)$ ,  $S_{11}(1535)$  and  $F_{15}(1680)$ .

For the neutron the integrals over resonance energy region at  $Q^2 = 2 + 3 \text{ Gev}^2$  turned out to be negligible. Beginning with  $Q^2 = 2 \text{ Gev}^2$  the resonance contributions do not reflect final results, where the integrals over the region with  $W > 1.8 \text{ Gev}$  we have obtained by simple extrapolation from the value at  $Q^2 = 0$  which follows from the GDH sum rule to the asymptotic value given by the Bjorken and Ellis-Jaffe sum rules. With this the

value of the integral over the neutron polarized structure function turned out to be close to the asymptotic value beginning with  $2\text{Gev}^2$ .

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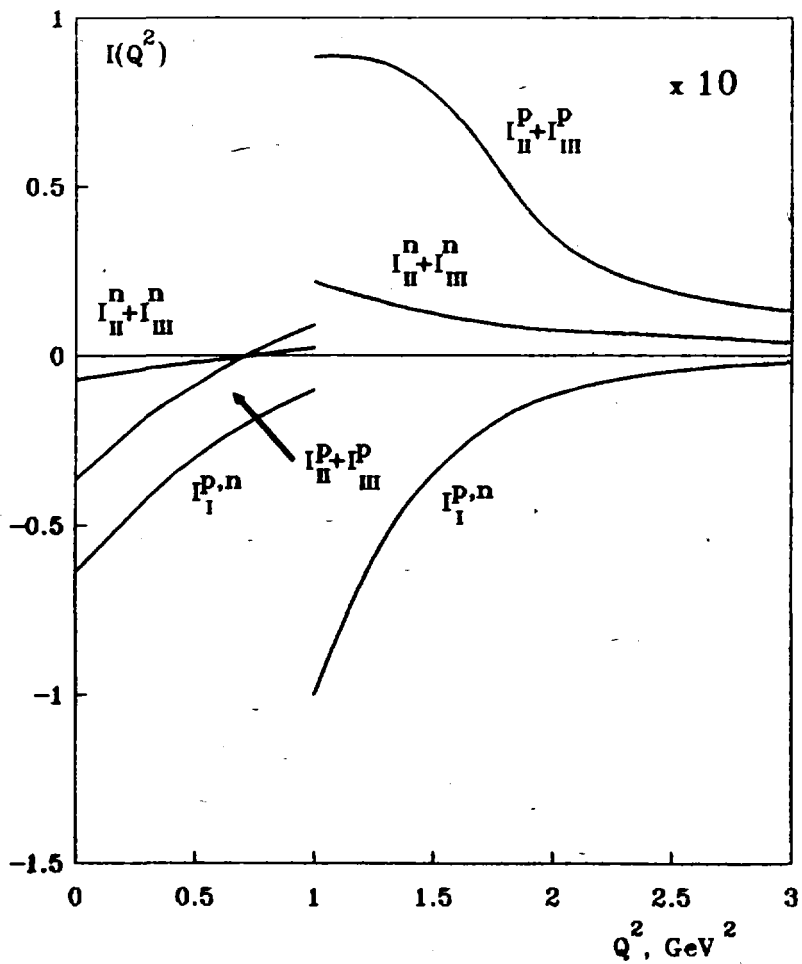


Fig. 1

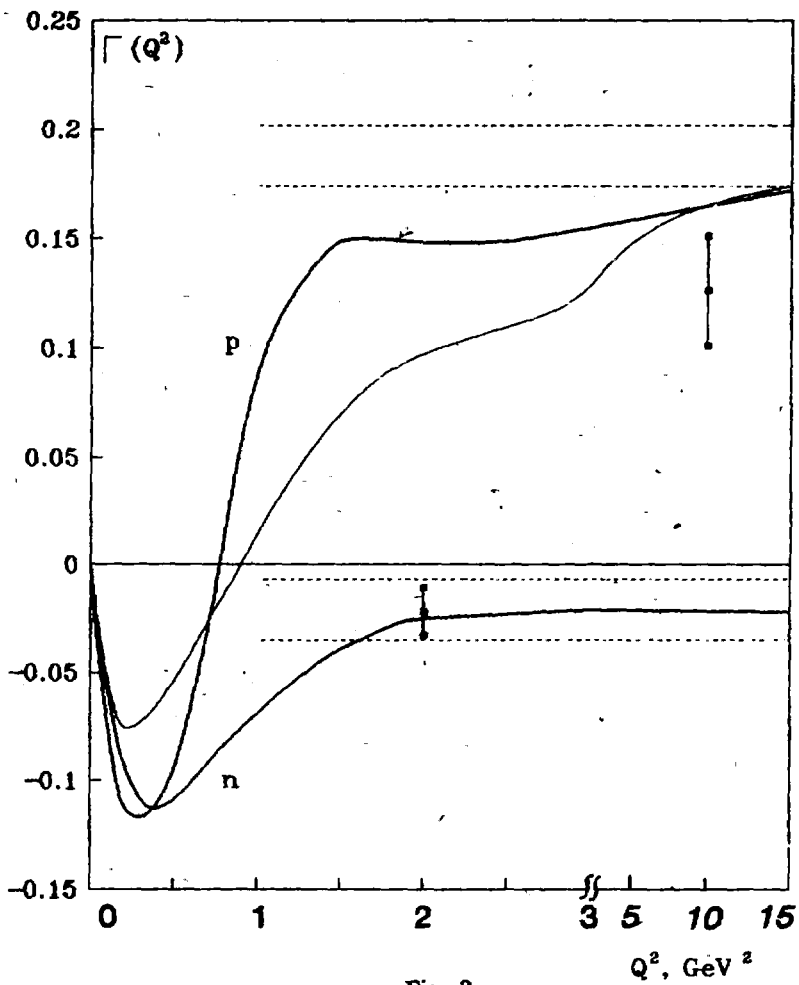


Fig. 2

**Figure caption**

**Fig 1. The first resonance energy region contribution and the summary contribution of the II- and III- resonance energy regions into integral  $I_N(Q^2)$  (1). At  $Q^2 > 1\text{Gev}^2$  the scale is changed tenfold.**

**Fig 2. Theory predictions for  $\Gamma_N(Q^2)$  (1b) and experimental data. Bold curves represent our results, thin curves are results of [4], dashed curves are limits given by the Ellis-Jaffe [6] and Bjorken [7] asymptotic sum rules (2,3). Experimental data are taken from Refs.[8, 9, 12]**

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И. Г. АЗНАУРЯН

ПРАВИЛО СУММ ГЕРАСИМОВА-ДРЕЛЛА-ХЕРНА И ПОВЕДЕНИЕ  
ПОЛЯРИЗАЦИОННОЙ СТРУКТУРНОЙ ФУНКЦИИ НУКЛОНА ПРИ МАЛЫХ  $Q^2$ 

Используя правило сумм Герасимова-Дрелла-Херна, показано, что поляризационная структурная функция нейтрона выходит на асимптотическое значение при  $Q^2 = 2 \text{ Гэв}^2$ . Для протонной поляризационной структурной функции показано, что, начиная с  $1.5 \text{ Гэв}^2$ , ее отклонение от асимптотического значения не превышает 15-20%. На основании имеющихся данных в резонансной области энергий при  $Q^2 < 3 \text{ Гэв}^2$  получены значения интегралов от поляризационной структурной функции нуклона в интервале  $x = 0.45 - 1$ , где отсутствуют экспериментальные данные. Оказалось, что при  $Q^2 = 2 - 3 \text{ Гэв}^2$  эти интегралы имеют скейлинговое поведение, а их величины согласуются со сделанными в экспериментальных работах экстраполяциями в область больших  $x$ .

Ереванский Физический Институт

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**И. Г. Азнаурян**

**Правило сумм Герасимова-Дрелла-Херна и поведение  
поляризационной структурной функции нуклона при малых  $Q^2$**

**Технический редактор А. С. Абрамян**

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