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FOCUSING OF RELATIVISTIC ELECTRON BUNCH, MOVING IN CYLINDRICAL PLASMA WAVEGUIDE

Ереван

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ФОКУСИРОВКА РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО СГУСТКА В ЦИЛИНДРИЧЕСКОМ ВОЛНОВОДЕ

Рассмотрена проблема фокусирования и самофокусирования электронных сгустков движущихся с релятивистской скоростью вдоль оси цилиндрического плаэменного волновода с проводящими боковыми поверхностями.

Показано существование периодических и непериодических составляющих полей, возбуждаемых в плаэме. Получены условия самофокусировки электронного сгустка поперечным электрическим и азимутальным магнитными полями. Обсуждается вопрос о фокусировке и ускорении сгустков электронов или позитронов кильватерным полем ведущего сгустка. Получены также условия, когда сгусток в плаэменном волноводе движется без возбуждения кильватерных волн, что может представить интерес в проблеме транспортировки электронных (и позитронных) сгустков.

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ՌԵԼՅԱՏԻՎԻՍՏԻԿ ԷԼԵԿՏՐՈՆԱՅԻՆ ՓՆՋԻ ՖՈԿՈՒՍԱՑՈՒՄԸ ԳԼԱՆԱՁԵՎ ՊԼԱԶՄԱՅԻՆ ԱԼԻՔԱՏԱՐՈՒՄ

Ա.Ց.ԱՄԱՏՈՒՆԻ, Ս.Ս.ԷԼԲԱԿՅԱՆ, Է.Վ.ՍԵՂԲՈՍՅԱՆ

Դիտարկված է հաղորդող մակերևույթով պլազմային ալիքատարի առանցքով շարժվող ռելյատիվիստիկ էլեկտրոնային փնջերի ֆոկուսացման և ինքնաֆոկուսացման պրոբլեմը։ Ցույց է տրված պլազմայում գրգռվող դաշտերի պարբերական և ոչ պարբերական բաղադրիչների գոյությունը։ Ստացված են պայմաններ լայնակի էլեկտրական և ազիմուտալ մագնիսական դաշտերով էլեկտրոնային փնջի ինքնաֆոկուսացման համար։ Քննարկված է վարող փնջի կիլվատերային ալիքում էլեկտրոնային (կամ պոզիտրոնային) փնջերի ֆոկուսացման կամ արագացման հարցը։ Ստացված են նաև պայմաններ պլազմային ալիքատարում փնջի՝ առանց կիլվատերային ալիքների գրգռման շարժման համար, ինչը կարող է հետաքրքրություն ներկայացնել էլեկտրոնային (և պոզիտրոնային) փնջերի տեղափոխման պրոբլեմում։

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1. FORMULATION OF THE PROBLEM

In our previous work [1] we considered in linear approximation the Coulomb field effect on focusing and self-focusing of electron (positron) bunches, moving in overdense plasma in presence of strong external magnetic field B_o applied along the direction of movement. In present work we consider more transparent, in mathematical sense, case of the driving electron bunch moving in cylindrical plasma wave-guide with the conducting internal surface.

Thus, let us consider the cylindrical electron bunch with the length d and radius a, moving in an overdense plasma filling up the cylindrical waveguide with the same radius a and conducting internal surface. It is assumed that the bunch is moving with the velocity v_o along the waveguide axis. We will consider stationary state of the system (steady state) when all the characteristics of the system are functions of $\bar{z} = z - v_o t$ and r. Plasma ions are in rest, and hydrodynamic velocities of plasma electrons have only one component $v_{ez} \equiv v_e$. It is possible to realize by applying a strong constant magnetic field $\vec{B}(0, 0, B_o)$ along the axis of waveguide. The strength of the magnetic field B_o should satisfy the condition that Larmor radius of the plasma electrons must be smaller than the plasma wave length and/or radius of the waveguide. In this case the transverse motion of the plasma electrons is not essential and it is possible to assume that plasma electrons have only longitudinal component of the velocity different from zero. The distribution of bunch electrons $n_b(r,\bar{z})$ is uniform along the $\bar{z}$ and

$$n_b(r) = \begin{cases} n_{bo}(1-r^2/a^2) & r \le a \\ 0 & r > a \end{cases} \qquad r < \bar{z} \le d \qquad (A)$$

The sought quantities are the components of the electric field E_z and E_r and magnetic field B_θ(E-wave). Introducing potentials $\varphi(\bar{z},r)$ and $\vec{A}(\bar{z},r)$ and using the Lorentz gauge we come to the following system of equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial\varphi(\bar{z},r)}{\partial r}\right] + \frac{1}{\gamma^2}\frac{\partial^2\varphi(\bar{z},r)}{\partial\bar{z}^2} = -4\pi e(n_o - n_e(\bar{z},r) - n_b(r)), \quad (1)$$

$$\frac{\partial}{\partial\bar{z}}\left[\sqrt{1+\rho^2(\bar{z},r)} - \beta\rho(\bar{z},r)\right] = \frac{e}{\gamma^2 mc^2}\frac{\partial\varphi(\bar{z},r)}{\partial\bar{z}}, \quad (2)$$

$$\frac{\partial}{\partial\bar{z}}(v_e(\bar{z},r) - v_o)n_e(\bar{z},r) = 0 \quad (3)$$

where $\rho = \dfrac{v_e/c}{\sqrt{1-v_e^2/c^2}}$ - dimensionless momentum of plasma electrons, $n_e(\bar{z},r)$ - plasma electrons density, n_o - plasma electrons density in equilibrium, which is equal to the ion's density, $\gamma = (1-\beta)^{-1/2}$, $\beta = v_o/c$. Assuming that at the points $\bar{z} = \bar{z}_o$, where φ is equal to zero, the momentum $\rho(\bar{z}_o, r) = \rho_o$ and $n_e(\bar{z}_o,r) = n_o$ the system of the eqs. (1), (2), (3) reduces to nonlinear equation for potential $\Phi = \dfrac{e\varphi}{mc^2\gamma^2}$ inside the bunch

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial\Phi}{\partial r}\right] + \frac{1}{\gamma^2}\frac{\partial^2\Phi}{\partial\bar{z}^2} + \beta^2 k_p^2\left\{1 - \frac{\beta(1+\Phi)}{\sqrt{(1+\Phi)^2 - 1/\gamma^2}}\right\} = \frac{\beta^2 k_p^2}{\gamma^2} n_{bo}\left[1 - \frac{r^2}{a^2}\right], \quad (4)$$

where $k_p = \frac{\omega_p}{v_o}$, $\omega_p = \left(\frac{4\pi n_o e^2}{m}\right)^{1/2}$. The potential Φ outside the bunch is found out from eq. (4), when $n_{bo} = 0$. The boundary conditions on the internal surface of the waveguide are $\Phi(\bar{z}, r = a) = 0$, $\frac{\partial\Phi(\bar{z}, r = a)}{\partial\bar{z}} = 0$ and far enough from the bunch front $\Phi(r, \bar{z} \to +\infty) = 0$, $\frac{\partial\Phi(r, \bar{z} \to +\infty)}{\partial\bar{z}} = 0$. From the eq. of the motion (2) it follows that

$$\rho = \gamma^2 \left\{\beta(1+\Phi) - \sqrt{(1+\Phi)^2 - 1/\gamma^2}\right\}, \tag{5}$$

and for the plasma electron density from the continuity eq. (3) it follows

$$n_o - n_e(\bar{z}, r) = -n_o \frac{\rho}{\beta\sqrt{1+\rho^2} - \rho} = n_o\gamma^2\left\{1 - \frac{\beta(1+\Phi)}{\sqrt{(1+\Phi)^2 - 1/\gamma^2}}\right\}. \tag{6}$$

2.FIELDS AND FORCES IN LINEAR APPROXIMATION

In the following we adopt the linear approximation, when $\phi = \frac{e\varphi}{mc^2\gamma^2} \ll 1$. Then the linearized eq. (4) for potential will take the form

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} + \frac{1}{\gamma^2}\frac{\partial^2\varphi}{\partial\bar{z}^2} + \frac{k_p^2}{\gamma^2}\varphi = \frac{m}{e}\frac{n_{bo}}{n_o}k_p^2 v_o^2\left(1 - \frac{r^2}{a^2}\right) \tag{7}$$

The potential outside the bunch is described by eq. (7) with $n_{bo} = 0$. We represent the solution of the eq. (7), with the above mentioned boundary conditions, as Hankel transformation for the finite segment

$$\varphi = 2\sum_{n=1}^{\infty} \frac{J_0(\mu_n \frac{r}{a})}{J_1^2(\mu_n)} \bar{\varphi}(x), \qquad (8)$$

$$\bar{\varphi} = \frac{1}{a^2} \int_0^a \varphi(r,x) r J_0(\mu_n \frac{r}{a}) dr, \qquad (9)$$

where $x = \bar{z}-d$, μ_n – the roots of the Bessel function $J_0(\mu_n)=0$.

Multiplying (7) by $rJ_0(\mu_n)$ and taking the integral on r from 0 to a we obtain the following eq. for the Hankel transform $\bar{\varphi}^b(x)$ inside the bunch $(-d \leq x \leq 0)$

$$\frac{\partial^2 \bar{\varphi}^b}{\partial x^2} + \left[k_p^2 - \left(\frac{\mu_n \gamma}{a}\right)^2\right] \bar{\varphi}^b = \frac{2m}{e} \frac{n_{bo}}{n_o} k_p^2 v_o^2 \gamma^2 \frac{J_2(\mu_n)}{\mu_n^2} \qquad (10)$$

It is possible to obtain both Hankel transform $\bar{\varphi}_+(x)$ ahead the bunch and Hankel transform $\bar{\varphi}_-(x)$ behind the bunch from eq. (10), putting $n_{bo} = 0$.

When coefficient $\varkappa_n^2 = k_p^2 - \left(\frac{\mu_n \gamma}{a}\right)^2 > 0$ in eq. (10) the solution of the equation is periodic; when $\varkappa_n^2 < 0$ nonperiodic. Peroidic component which exists only behind and inside the bunch represents the wake field; nonperiodic component which exists everywhere not too far from the bunch represents the Coulomb field which, as it is known [2], is not Debay screened in plasma when velocity of the bunch is much greater than the thermal velocity of the plasma electrons (which is the case). The sum in (8) will contain, therefore, periodic (up to some number $n \leq n'$, where n' is defined from the condition $\mu_{n'} = k_p a/\gamma$) and nonperiodic terms $(n > n')$. When $a \to \infty$ $\varkappa_n^2 = k_p^2 > 0$ and

nonperiodic part in (8) disappears. When $\mu_n \leq \frac{k_p a}{\gamma} < \mu_1 = 2{,}405$ in (8), then periodic terms are absent, i.e. in "narrow" ($K_p a < \mu_1 \gamma$) waveguide the periodic waves are not excited.
Assuming

$$\varphi_+(r, x \to \infty) = 0, \quad E_{z+}(r, x \to \infty) = -\frac{1}{\gamma^2} \frac{\partial \varphi_+(r, x \to \infty)}{\partial x} = 0$$

and validity of the potential and fields continuity condition on the bunch front (x=0) and it rear end (x=-d):

$$\varphi_+(r, x=0) = \varphi^b(r, x=0), \; \varphi_-(r, x=-d) = \varphi^b(r, x=-d),$$
$$E_{z+}(r, x=0) = E^b(r, x=0), \; E_{z-}(r, x=-d) = E^b(r, x=-d), \tag{11}$$

we come to the following expressions for longitudinal $E_z = -\frac{1}{\gamma^2}\frac{\partial \varphi}{\partial x}$ and transverse $E_r = -\frac{\partial \varphi}{\partial r}$ fields inside and outside the bunch

$$E_{z+} = \frac{2A}{\gamma^2} \sum_{n=n'+1}^{\infty} \frac{J_0(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa'_n \mu_n^2 J_1^2(\mu_n)} \left[\frac{e^{-\varkappa'_n(d+x)} - e^{-\varkappa'_n x}}{2} \right], \quad (12a)$$

$$E_{r+} = - \frac{2A}{a} \sum_{n=n'+1}^{\infty} \frac{J_1(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa'^2_n \mu_n J_1^2(\mu_n)} \left[\frac{e^{-\varkappa'_n x} - e^{-\varkappa'_n(d+x)}}{2} \right],$$

$$0 \leq x \leq \infty$$

$$E_z^b = - \frac{2A}{\gamma^2} \left\{ \sum_{n=1}^{n'} \frac{J_0(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa_n \mu_n^2 J_1^2(\mu_n)} \sin\varkappa_n x - \right.$$

$$\left. - \sum_{n'+1}^{\infty} \frac{J_0(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa'_n \mu_n^2 J_1^2(\mu_n)} \left[\frac{e^{-\varkappa'_n(d+x)} - e^{\varkappa'_n x}}{2} \right] \right\},$$

$$(12b)$$

$$E_r^b = \frac{2A}{a} \left\{ \sum_{n=1}^{n'} \frac{J_1(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa_n^2 \mu_n J_1^2(\mu_n)} (1-\cos\varkappa_n x) - \right.$$

$$\left. - \sum_{n'+1}^{\infty} \frac{J_1(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa'^2_n \mu_n J_1^2(\mu_n)} \left[1 - \frac{e^{-\varkappa'_n(d+x)} + e^{\varkappa'_n x}}{2} \right] \right\},$$

$$-d \leq x \leq 0$$

$$E_{z-} = - \frac{2A}{\gamma^2} \left\{ \sum_{n=1}^{n'} \frac{J_0(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa_n \mu_n^2 J_1^2(\mu_n)} \left[-\sin\varkappa_n(d+x) + \sin\varkappa_n x \right] - \right.$$

$$-\sum_{n'+1}^{\infty} \frac{J_0(\mu_n \frac{r}{a}) J_2(\mu_n)}{\varkappa'_n \mu_n^2 J_1^2(\mu_n)} \left[\frac{e^{\varkappa'_n(d+x)} - e^{\varkappa'_n x}}{2}\right]\Bigg\}, \qquad (12c)$$

$$E_{r-} = \frac{2A}{a}\left\{\sum_{n=1}^{n'} \frac{J_1(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa_n^2 \mu_n J_1^2(\mu_n)}\left[\cos\varkappa_n(d+x) - \cos\varkappa_n x\right] - \right.$$

$$\left. -\sum_{n'+1}^{\infty}\frac{J_1(\mu_n \frac{r}{a})J_2(\mu_n)}{\varkappa'^2_n \mu_n J_1^2(\mu_n)}\left[\frac{e^{\varkappa'_n(d+x)} - e^{\varkappa'_n x}}{2}\right]\right\},$$

$$-\infty \le x \le d$$

where $\varkappa_n = \gamma\sqrt{k_p^2 - \left(\frac{\mu_n\gamma}{a}\right)^2}$, $\varkappa'_n = \gamma\sqrt{\left(\frac{\mu_n\gamma}{a}\right)^2 - k_p^2}$

$$A = \frac{2m}{e}\frac{n_{bo}}{n_o} k_p^2 v_o^2 \gamma^2 , \quad k_p^2 - (\mu_{n'} \gamma/a)^2 = 0. \qquad (13)$$

From (12c) it is evident that when $\varkappa_n d = 2\pi k (k = 1,2,\dots)$ corresponding periodic component of the solutions behind the bunch is equal to zero, i.e. the component of the wake fields at this condition are not excited. When n_o, γ, a satisfy the condition $k_p^2 - \mu_1^2\gamma^2/a^2 < 0$ periodic component is completely absent in sums (12a)-(12c) and only static (nonperiodic) fields moving in plasma with the velocity v_o of the bunch exist in all regions of the plasma and this feature could be used in the beam transportation problem. The forces acting on plasma electrons inside and behind the bunch consequently have the following form

$$f_z = -eE_z^b \ , \quad f_r = -eE_r^b + \frac{e}{c} v_e^b B_\theta^b = -e(1-\beta_e\beta)E_r^b \qquad (14)$$

$$f_z^- = -eE_{z-}, \; f_r^- = -eE_{r-} + \frac{e}{c} v_e^- B_\theta^- = -e(1-\beta_e^- \beta)E_{r-} \qquad (15)$$

where $\beta_e = v_{e/c}$

The forces acting on the bunch electrons are the following

$$f_z^b = -eE_z^b, \; f_r^b = -eE_r^b + \frac{e}{c} v_0 B_\theta^b = -e(1-\beta^2)E_r^b = -\frac{eE_r^b}{\gamma^2} \qquad (16)$$

It should be noted that potentials φ^b, $\varphi_\pm$ and transverse fields E_r^b, $E_{r\pm}$ are proportional to the square of the Lorentz factor γ (up to $\gamma \sim a\, k_p/\mu n$, $\varkappa n \geq 0$), the transverse force f_r^b is independent of γ due to the compensation, arising from azimuthal magnetic field $B_\theta = \beta E_r^b$.

However, the force acting on electrons in the wake $f_r^- = -e(1-\beta_e^-\beta)E_{r-}^-$ in the considered case, when $\Phi \ll 1$, $|\beta| \simeq \rho = \frac{\Phi}{\beta} \ll 1$, is proportional to γ^2. The force f_r^- will also act on the witness (to be accelerated) bunches in wake fields, if its $\beta_\omega < \beta$. The signs of the f_r^b and $f_r^\pm$ define the focusing (−) and defocusing (+) characters of these forces. In the middle of the bunch ($r=0$) and when $a \to \infty$ E_r^b, $E_{r-} = 0$ due to $J_1(0) = 0$ and the focusing forces f_r^b and f_r^- are absent

3. DRIVING BUNCH SELF-FOCUSING CONDITIONS

The total charge and current densities inside the bunch ($-d \le x \le 0$) are the following:

$$Q_\rho = e(n_0 - n_e - n_b), \quad j = -e(n_b v_0 + n_e v_e) \tag{17}$$

where n_b and $n_e = n_e(r,x)$ are given by (A) and (6). From (6), (17) and $\beta_e = p/\sqrt{1+p^2}$ it follows that $j = v_e Q_\rho$, so that where total charge density $Q_\rho = 0$ the current density j is also equal to zero. If total charge density Q_ρ is positive, then in this region we have focusing forces for electrons. In considered case $\Phi \ll 1$ and total charge density

$$Q_\rho = -en_b\left(1 - \Phi/\beta^2 \frac{n_b}{n_e}\right) \tag{18}$$

The condition of the focusing force existence is therefore

$$\Phi/\beta^2 \frac{n_b}{n_e} > 1, \quad \Phi = \frac{e\varphi^b}{mc^2\gamma^2},$$

$$\varphi^b = 2A\left\{\sum_{n=1}^{n'} \frac{J_0(\mu_n \frac{r}{a}) J_2(\mu_n)}{\kappa_n^2 \mu_n^2 J_1^2(\mu_n)}(1 - \cos\kappa_n x) - \right. \tag{19}$$

$$\left. - \sum_{n=n'+1}^{\infty} \frac{J_0(\mu_n \frac{r}{a}) J_2(\mu_n)}{\kappa_n'^2 \mu_n^2 J_1^2(\mu_n)}\left[1 - \frac{e^{\kappa_n' x} + e^{-\kappa_n'(d+x)}}{2}\right]\right\}.$$

$$-d \leq x \leq 0 \,,$$

where A is determined by (13).

Condition (19) defines the common sign of the fields and forces given by the sums in (12-16). However, the terms in series (12-16) are sign changeable and, in general, it is impossible to obtain analytically the sum of the series.

From physical reasons and from the results of [1] one can expect that the Coloumb (nonperiodic) terms in series will have defocusing property, and periodic terms at certain conditions will determine the presence of the focusing forces. The estimates, done below for some particular cases confirm this point

The condition for existence of the periodic terms in solutions (12-16) $\varkappa_n^2 > 0$ or at least $\frac{k_p a}{\gamma} > \mu_1$ ($\mu_1 = 2{,}405$ the first root of the Bessel function $J_0(\mu_1) = 0$) is restricted enough and its fulfilment needs the use of wide bunches or (and) sufficiently dense plasma for focusing.

Let us consider a few numerical examples using input parameters at ANL experiment [3], where for the first time the self-focusing of the relativistic electron bunch moving in plasma column was demonstrated. At the beginning let us consider the fields and forces inside the bunch. When

$$n_0 = 3\cdot 10^{13} cm^{-3}, \ a = 0{,}14 cm,$$

$\gamma = 42$ (E=21Mev) [3] the value $\frac{k_p a}{\gamma} = 0{,}03 \ll \mu_1$ and therefore $n' = 0$, so that the periodic terms in series for fields and forces are absent. When $\mu_1 < k_p a/\gamma < \mu_2$ in (12b) only one of the periodical terms with $n = 1$ remains.

Let us take $n_0 = 3\cdot 10^{17} cm^{-3}$, leaving the other parameters the same. Then the nonperiodical terms in (12b) are small and the leading contribution in series for E_r^b and f_r^b in the expression (12b) and (16) is made by the first periodic term with the n=1 and we have the following approximate formulae for the field:

$$E_r^b \approx \frac{4m}{ea}\frac{n_{bo}}{n_o} k_p^2 v_o^2 \gamma^2 \frac{J_1\left(\mu_1 \frac{r}{a}\right) J_2(\mu_1)}{\varkappa_1^2 \mu_1 J_1^2(\mu_1)} (1-\cos\varkappa_1 x) \qquad (20)$$

Field's gradient for $\mu_1 \frac{r}{a} \ll 1$ when field is lineary dependent on r is

$$G = \frac{f_r^b}{er} = - \frac{E_r^b}{r\gamma^2} \qquad (20a)$$

and when $n_{bo}=2,5\cdot 10^{12}cm^{-3}$ (which coinsides with the value used in ANL experiment), $\mu_1 \frac{r}{a} \approx 0,1$ the amplitude of field $E_r^b =$ $= 4,9\cdot 10^4$CGSE $= 1,5\cdot 10^5 \frac{v}{cm}$ and $G = 4,96\frac{Gs}{cm}$. However, if $n_{bo}=$ $2,5\cdot 10^{16}cm^{-3}$ (and $n_o = 3\cdot 10^{17}cm^{-3}$) the field amplitude and gradient values increase: $E_r^b = 1,47\cdot 10^8 \frac{v}{cm}$, $G = 4,96\cdot 10^4 \frac{Gs}{cm}$. It is possible to obtain even larger values of transverse field and its gradient by decreasing the a value and increasing n_b and γ because $E_r^b \sim \gamma^2/a$ and $G \sim \frac{1}{a^2}$, $(r \sim a)$.
But it is necesary to preserve the $k_p a/\gamma > \mu_1$ condition, to keep in the sums the small number of the periodic terms. For example, if $\gamma=10^2$, $a=10^{-2}$ cm, $n_o = 1,7\cdot 10^{20}cm^{-3}$, $\mu_n \frac{r}{a} \ll 1$ $G =$ $65\frac{MGs}{cm}$, when $\frac{n_{bo}}{n_o}=10^{-3}$.

Generally, the x-dependence of E_r^b is complicated. However in the above considered case when it is possible to approximate the sum by only one term (20), the x-dependence is

simple and given by the factor $(1-\cos \varkappa_1 x)$.
Thus, around the points where $1-\cos \varkappa_1 x \le 2$ focusing takes place: when $1-\cos\varkappa_1 x \approx 0$ the repulsive forces will prevail, due to the nonperiodic terms which are neglected in (20), when focusing fields are large. Therefore, the long beam $\varkappa_1 d \gg 1$ will be bunched with the period $\lambda_b=2\pi/\varkappa_1$. When $\varkappa_1 d < 1$ the focusing of the bunch will take place. In considered case (20) the n_o dependence of the field E_r^b is simple: $E_r^b \sim 1/n_o$. In general case, when terms with the different $\varkappa_n$ are essential in the sum, the $E_r^b(n_o)$ function is more complicated. The longitudinal field E_z^b inside the bunch, when in sum (12b) only the first periodic term is essential, is

$$E_z^b \approx - \frac{4m}{e} \frac{n_{bo}}{n_o} k_p^2 v_o^2 \frac{J_o(\mu_1 \frac{r}{a})J_2(\mu_1)}{\varkappa_1 \mu_1^2 J_1^2(\mu_1)} \sin \varkappa_1 x \qquad (21)$$

The x-dependence of the E_z^b in (21) is simply given by a factor $\sin\varkappa_1 x$, $-d \le x \le 0$,so that for the long bunch, when $\varkappa_1 d \gg 1$, we will have the regions, where bunch electrons are deaccelerated ($\sin\varkappa_1 x < 0$), accelerated ($\sin\varkappa_1 x > 0$) and pushes aside by Coulomb forces, when $\sin\varkappa_1 x \approx 0$. For a short bunch, when $\varkappa_1 d \ll 1$ and (21) is valid we have only deacceleration.

The r -dependence of the longitudinal wake field (21) suggests the idea proposed in phenomenological approach to plasma description [4], to use for acceleration the narrow driven beams ($r_o \ll a$) moving along the waveguide axis in order to increase trasformation ratio.

4. WAKE FIELDS - FOCUSING AND ACCELERATION

Wake fields and corresponding forces are given by eqs. (12c) and (15). Nonperiodic terms in these eqs., proportional to exp $(\varkappa_n(d+x))$ and exp $(\varkappa_n x)$ are small for $x \ll -d$ and even for the distances not too far from the rear of the bunch they are negligible, compared to the periodic terms. The periodic terms are sign changable; the common sign, which determine the character of the forces, acting on the plasma or witness bunch electrons in general, as it was mentioned above, is difficult to obtain.

In what follows, we discuss a particular case, when plasma and bunch parameters are chosen in such a way (see preceding section), that the main contribution comes from the first term of the sum with n=1. Then the expressions for transverse and longitudinal fields take the following simple form

$$E_{r-} = \frac{4m}{ea}\frac{n_{bo}}{n_o}k_p^2 v_o^2 \gamma^2 \frac{J_1(\mu_1 \frac{r}{a})J_2(\mu_1)}{\varkappa_1^2 \mu_1 J_1^2(\mu_1)} \left[\cos\varkappa_1(d+x)-\cos\varkappa_1 x\right],$$

$$E_{z-} \approx \frac{4m}{e}\frac{n_{bo}}{n_o}k_p^2 \quad v_o^2 \frac{J_o(\mu_1 \frac{r}{a})J_2(\mu_1)}{\varkappa_1 \mu_1^2 J_1^2(\mu_1)}\left[\sin\varkappa_1(d+x)-\sin\varkappa_1 x\right] \qquad (22)$$

It is evident from (22) that the fields can be focusing or de-focusing, accelerating or deaccelerating depends on the phases $\varkappa_1 x$, $\varkappa_1(d+x)$. When $n_o = 3 \cdot 10^{17} cm^{-3}$, $a = 0{,}14 cm$, $\gamma = 42$, $n_b = 2{,}5 \quad 10^{16} cm^{-3}$ and $r = a/2$ $E_{r-} = 1{,}5 \cdot 10^9 v/cm$, $E_{z-} = 4{,}5 \cdot 10^7 v/cm$. For $\mu_1 \frac{r}{a} = 0{,}1 \ll 1$ $E_{r-} \approx 1{,}47.10^8 v/cm$ and transverse field gradient is

$$G=\frac{f_r^-}{er} = - \frac{E_{r-}}{r}(1-\beta_e^-\beta) = 0{,}7\cdot 10^2 \frac{MGs}{cm} \qquad (23)$$

As it was noticed above the field gradient is large due to the square of the Lorentz factor γ^2 presence in expression for E_{r-} which in case, when $|\beta_e| \simeq \frac{\phi}{\beta} \ll 1$ is not canceled because of the absence of the compensation, originated by magnetic force.

Let us consider the case, when the witness bunch with the energy smaller than the energy of driving bunch ($\beta_v < \beta_0$) is injected in the wake field. If the length of the witness bunch is smaller than the length of the wake wave $\lambda_v = \frac{2\pi}{\varkappa}$, it is possible to inject it in such a phase of the wake field where the witness bunch will be accelerated and focused at the same time (see also [5]).

The focusing could be strong enough due to the factor γ^2, present in the expression for transverse field and its gradient.

The similar situation will arise in the case of the positron witness bunch, injected in the corresponding accelerating and focusing phase of the fields (22) with $\beta_v < \beta_0$ velocity. In addition, when positron bunch is injected in the vicinity of the driving electron bunch rear the Coulomb fields of the driving bunch will also give an additional accelerating contribution. However, this last case needs more careful consideration.

In conclusion it should be noted that cited quantitative estimates are very approximate and serve only for the qualitative understanding of the origin of the fields of different characters, present in question.

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ABSTRACT

The problem on the focusing of electron bunches moving with the relativistic velocity along the axis of cylindrical overdense plasma waveguide with the conducting internal surface is considered. The existence of periodic and nonperiodic components of the fields, generated in the plasma is shown. The conditions of electron bunch self-focusing by transverse electrical field and azimuthal magnetic field are derived. The possibility of the acceleration and focusing of electron or positron bunches by driving electron bunch wake field is discussed. The conditions, when the bunch in plasma waveguide moves without wake fields generating are obtained, which could be of the interest for the transport of relativistic electron (positron) bunches.

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