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ABSOLUTE CALIBRATION OF MAGNETIC SPECTROMETERS BY THE SIEGBAHN METHOD USING INVERSE COMPTON SCATTERING

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Absolute Calibration of Magnetic Spectrometers by the Siegbahn Method using Inverse Compton Scattering

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Abstract

In this paper we show,that an absolute and accurate calibration of magnetic spectrometers can be performed with the help of the inverse Compton scattering using the absolute or relative energies of recoil electrons. The Monte Carlo simulations show that the absolute calibration of the CEBAF magnets can be performed with a precision below the 10^{-4} level.

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1 Introduction.

High-precision absolute calibration of magnetic spectrometers is an important requirement to the CEBAF physics program, and various methods have been considered [1]. We suggest to use inverse laser scattering [2,3].It has been shown [4,5] that the absolute value of the CEBAF beam enargy can be determined with an accuracy of up to 10^{-5} by the direct measurement of the energy of photons scattered at small angles, and different techniques for the measurement of the energy of scattered photons were considered [5]. In this paper we suggest to evaluate the absolute energy of incident electrons by the measurement of absolute energy of recoil electrons. It is shown that the characteristics of the inverse Compton scattering allow one to carry out absolute and accurate calibration of magnetic spectrometers using only relative values of the momenta of the primary and recoil electrons. This is the extension of the well known Siegbahn idea [6] of the absolute calibration of β -ray magnetic spectrometers in the high energy domain. The results of Monte Carlo simulations are presented. The parameters of the CEBAF electron beam and a realistic laser system have been used.

2 The Principles of the Method.

The energy of the scattered photon ϵ_2, as is well known, is related to the energy of the primary photon ϵ_1 by the following formula

$$\epsilon_2 = \frac{(1-\beta\cos\theta_1)\epsilon_1}{1-\beta\cos\theta_2+(1-\cos\theta)(\epsilon_1/E_1)} \tag{1}$$

where E_1 is the energy of incident electron,$\beta = v/c$, v is the velocity of electron, c is the speed of light, θ is the angle between the momentum of incident and scattered photon, θ_1, θ_2 are the angles between the initial momentum of the electron and incident and scattered photons respectively. Rewrite this expression in the following form

$$\epsilon_2 = \epsilon_1 A(\epsilon_1, \gamma, \theta, \theta_1, \theta_2)\gamma^2 \tag{2}$$

where

$$\gamma^2 = (1-\beta^2)^{-1} \tag{3}$$

and

$$A(\epsilon_1, \gamma, \theta, \theta_1, \theta_2) = \frac{(1 - \beta\cos\theta_1)(1 + \beta\cos\theta_2)}{1 + \beta^2\gamma^2\sin^2\theta_2 + \gamma^2(1 - \cos\theta)(1 + \beta\cos\theta_2)(\epsilon_1/E_1)} \quad (4)$$

The behaviour of the factor A in the case of a monochromatic electron and photon beams is determined mainly by the geometry and angular distributions of both the electron beam and the laser photons, also by the accuracy of determination of the direction of the scattered photons. By choosing a proper experimental geometry and a laser system, the last two terms can be made negligible, then

$$dA/A \simeq (\gamma d\theta_2)^2 \quad (5)$$

where $d\theta_2$ is conditioned by the incident beam divergence.

Let us emphasize the following characteristics of the inverse Compton scattering.

1. Inverse Compton scattering on monochromatic beams with small angular divergence gives monochromatic photons at certain angles of observation. The energy of scattered photons is uniquely related to the angle of emergence. In each concrete case the accurate mean value of the factor A can be determined analytically or with the help of Monte Carlo simulations.

2. For the Lorentz-factor of $\gamma \leq 10^5$ the energy of the photons scattered at small angles with respect to the incident electron momentum is proportional to the square of the beam energy (the mean value of the factor A has a weak dependence on γ).

3. The recoil electrons emerge at infinitesimally small angles with respect to their initial momentum. The multiple scattering and radiation are negligibly small.

The above-mentioned characteristics of the inverse Compton scattering allow one to evaluate the absolute mean energy of incident electrons by the measurement of the absolute mean energy of recoil electrons and vice-versa, and to exploit the Siegbahn idea for a precise and absolute calibration of magnetic spectrometers without any absolute measurements [6] in the high energy domain. Indeed, one has the energy conservation:

$$\epsilon_1 A(\epsilon_1, \gamma, \theta, \theta_1, \theta_2)\gamma^2 - \bar{\gamma} m_e + E_2 - \epsilon_1 = 0 \quad (6)$$

or

$$\bar{\gamma} m_e(1 - E_2/E_1) = \gamma^2 A(\epsilon_1, \gamma, \theta, \theta_1 \theta_2)\epsilon_1(1 - 1/(\gamma^2 A(\epsilon_1, \gamma, \theta, \theta_1, \theta_2))) \quad (7)$$

where

$$\bar{\gamma} = E_1/m_e \quad (8)$$

and E_2 is the energy of the recoil electron, m_e is the rest mass of the electron. The Lorentz factor $\bar{\gamma} = \gamma$ can be determined from the eq. (6) or eq. (7) by the measurement of the absolute value of E_2 or the ratio E_2/E_1.

With the use of a multimode laser system, the Lorentz factor of the incident beam can be expressed by the ratio $a = E_3/E_2$ of the energies of recoil electrons from the two different laser lines, ϵ_{11} and ϵ_{12}.

$$\epsilon_{11}\gamma^2 A(\epsilon_{11}, \gamma, \theta, \theta_1, \theta_2)(1 - 1/(\gamma^2 A(\epsilon_{11}, \gamma, \theta, \theta_1, \theta_2))(1 - a\alpha) = m_e\bar{\gamma}(1-a)\alpha \quad (9)$$

where

$$\alpha = \frac{\epsilon_{11} A(\epsilon_{11}, \gamma, \theta, \theta_1, \theta_2)(1 - 1/(\gamma^2 A(\epsilon_{11}, \gamma, \theta, \theta_1, \theta_2)))}{\epsilon_{12} A(\epsilon_{12}, \gamma, \theta, \theta_1, \theta_2)(1 - 1/(\gamma^2 A(\epsilon_{12}, \gamma, \theta, \theta_1, \theta_2)))} \quad (10)$$

In the case of a parallel electron beam, when $dA/A \leq 10^{-2}$ and the distribution of A is Gaussian like, in the equations (2),(6),(7),(9) the values of $\epsilon_2, E_1, E_2, E_3, \gamma, \bar{\gamma}$ and α can be replaced by their mean values. Then the absolute mean value of the Lorentz factor $\bar{\gamma} = \gamma$ of the incident beam can be determined by the measurement of the absolute mean energy of forward scattered photons (direct way) or by the indirect way by the measurement of the absolute mean energy of recoil electrons as well as by means of the determination of the ratios E_2/E_1 or E_3/E_2 of energies of recoil and incident electrons or of recoil electrons from the two different laser lines, i.e. without any absolute measurements.

3 Results of the Monte Carlo Calculations

Let us consider, for example, the absolute energy determination of the electron beam by the help of eq. (6) and eq. (9) taking into account the real parameters of the electron beam [1] and laser system [7].

The absolute (relative) values of recoil electron energies can be measured by emploing a magnetic spectrometer. The energies of recoil electrons in this case are expressed via bending angles and field integrals. We use the following characteristics of the electron beam: energy interval - 0.5-5.0 GeV; energy spread $(\sigma_{E_1})/E_1 = 2.5 \times 10^{-5}$; angular spread $(\sigma) = 1.5 \times 10^{-5}$; and laser system [7]: energy of laser photons - 1.07, 2.4 and 3.4 eV; angular spread $(\sigma) = 5 \times 10^{-4}$; Energy resolution of magnetic spectrometer $dE_2/E_2 = dE_3/E_3 = 10^{-4}$.

We consider the case when recoil electrons are detected in coincidence with the photons scattered at small ($\leq 1.5 \times 10^{-5}$) angles with respect to the momentum of the incident electrons in head-on collisions. The γ-detector required, must have a good position resolution, but need not measure the γ-energy. However, the determination of the directions of scattered photons with a precision higher than the angular divergence of primary beam is useless.

As follows from our Monte Carlo calculations, the angular divergence of recoil electrons in coinsidence with forward scattered photons for $\gamma = 1000$ is $\simeq 10^{-6}$rad, more than an order of magnitude smaller than the incident beam divergence and decrease with increasing γ. Therefore, the recoil electrons practically move in the direction of incident beam.

The Monte Carlo simulations were carried out taking into account the angular spread of laser photons and the angular and energy spread of incident electrons as well as the resolution of magnetic spectrometer. As a result of this simulation, the mean energy of recoil electrons is determined. The absolute value of the mean Lorentz factor of incident beam is determined using only the mean energies of recoil electrons, absolute- with the help of eq. (6), relative- using eq. (9). The procedure in both cases is similar, and we will describe it in detail only for the last case. In the first step we determine the mean energies of recoil electrons $\bar{E}_2$ and $\bar{E}_3$ from two different laser lines ϵ_{11} and ϵ_{12}. To the first order of magnitude we assume A=4, $\alpha = \epsilon_{11}/\epsilon_{12}$, and the Lorentz factor is determined from the equation:

$$\gamma_i = \frac{m_e \alpha (1-a)}{\epsilon_{11} 4 (1 - a\alpha)} \tag{11}$$

where

$$a = \bar{E}_3/\bar{E}_2. \tag{12}$$

Using this value of γ_i and taking into account the angular characteristics of the incident electron beam and laser photons, we calculate the new mean values of the factor A and α:

$$A_{11} = \int\int\int A(\epsilon_{11}, \gamma_i, \theta, \theta_1, \theta_2) d\theta, d\theta_1, d\theta_2 \tag{13}$$

$$A_{12} = \int\int\int A(\epsilon_{12}, \gamma_i, \theta, \theta_1, \theta_2) d\theta, d\theta_1, d\theta_2 \tag{14}$$

$$\alpha = \frac{\epsilon_{11} A_{11}(1 - 1/(\gamma_i^2 A_{11}))}{\epsilon_{12} A_{12}(1 - 1/(\gamma_i^2 A_{12}))} \tag{15}$$

Next we define a new value of γ_{i+1} by

$$\gamma_{i+1} = \frac{m_e \alpha(1 - a)}{\epsilon_{11} A_{11}(1 - 1/\gamma_i^2 A_{11})(1 - a\alpha)} \tag{16}$$

and we repeat this iterative procedure while $abs(\gamma_i - \gamma_{i+1})/\gamma_i \geq 10^{-5}$. The mean value of the obtained Lorentz factors $\bar{\gamma}_{MC}$ and their statistic errors σ_γ/γ (one standard deviation) are given in Table 1 for different incident values of $\gamma, \epsilon_{11}, \epsilon_{12}$ and for a total number of 10^4 events in each case, respectively. In this case the attainable accuracy is set by the error coming from the determination of the ratio of mean values of momentum of recoil electrons from different laser lines. The mean bending angles of recoil electrons can be determined with high accuracy by selection of a suitable geometry and detector. Then, the attainable accuracy is set by the error coming from the determination of the field integrals ratio $(\int Bdl)_3/(\int Bdl)_2$. The relative accuracy of the order of 10^{-5} can be obtained, but the accuracy of the absolute measurements is about 10^{-4} for CEBAF [8] and SLC [9] magnets. Therefore, the attainable accuracy in each case can be estimated using the maximum accuracy of relative measurements equal to 10^{-5}. The obtained results(σ_∞/γ) are presented in the last column of Table 1 and can be represented with the help of the following formula:

$$\frac{\sigma_\infty}{\bar{\gamma}} = \frac{da}{1 - a} \tag{17}$$

where da is the accuracy of a.

The experimental conditions are particularly favourable when an iron-free magnet is used with the average external field along the path completely compensated for.In that case the field ratio is simplified to be a ratio between two currents through the magnet coils corresponding equivalent points on the E_2 and E_3 of recoil electrons energies from the two different laser lines, which can be measured with an accuracy of better than 10^{-5}.

The absolute mean energy of incident electron beam can be determined by the measurement of the absolute mean energy of recoil electrons,with the help of eq. (6). The obtained results are presented in Table 2.The mean value ($\bar{E}_2$) and the dispersion ($\sigma_{E_2}/\bar{E}_2$) of the recoil electrons energies are presented in adition. Where

$$\sigma_{E_2} = [< (E_2 - \bar{E}_2)^2 >]^{1/2} \tag{18}$$

In this case the attainable accuracy is set by the error coming from the determination of the absolute mean energy of recoil electrons $d\bar{E}_r/\bar{E}_r$, which is mainly conditioned by the accuracy of the field integrals (about 10^{-4}):

$$\frac{\sigma_\infty}{\bar{\gamma}} = \frac{d\bar{E}_r}{\bar{E}_r} \tag{19}$$

Using the laser system proposed in [7] and the 1μA electron beam, one may have $\approx 10^4$ events per second in an angular acceptance of $d\Omega = 10^{-10}$sr and measure the Lorentz factor with an accuracy of the order of 10^{-4} .

This method allows one to obtain monochromatic photon beams with standard and tuned energies in the high energy domain, which can find a wide application.

It is interesting to note, that the accuracy in the measurements of the Lorentz factor by direct and indirect ways allow one to evaluate the absolute value of the atomic constants $h/(m_e c)$ as well as the relation $\bar{\gamma} = \gamma$ with $10^{-4} - 10^{-5}$precision at high energies.

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Table 1. The mean value of extracted Lorentz-factors $\bar{\gamma}_{MC}$ and their errors σ_γ/γ for different values of true Lorentz-factors γ and $\epsilon_{11}, \epsilon_{12}$, obtained with the help of eq. (9). The total number of events in each case is 10^4 .

γ	$\epsilon_{11}(eV)$	$\epsilon_{12}(eV)$	$\bar{\gamma}_{MC}$	σ_γ/γ	σ_{∞}/γ
1000.000	1.07	2.4	999.970	2.1×10^{-5}	1.0×10^{-3}
1000.000	1.07	3.4	999.998	1.2×10^{-5}	5.8×10^{-4}
3000.000	1.07	2.4	3000.10	6.1×10^{-5}	3.5×10^{-4}
3000.000	1.07	3.4	3000.06	4.4×10^{-5}	2.2×10^{-4}
10000.00	1.07	2.4	10001.6	7.0×10^{-4}	8.8×10^{-5}
10000.00	1.07	3.4	10001.8	5.1×10^{-4}	5.2×10^{-5}

Table 2 The same as in Table 1, obtained with the help of eq. (6).The mean value and the dispersion of the recoil electrons energies are given in addition.

γ	$\epsilon_1(eV)$	$\bar{\gamma}_{MC}$	σ_γ/γ	$E_2(MeV)$	σ_{E_2}/E_2
1000.000	1.07	1000.001	1.0×10^{-6}	506.75	2.5×10^{-5}
1000.000	2.4	1000.001	1.0×10^{-6}	501.58	2.5×10^{-5}
1000.000	3.4	1000.001	1.0×10^{-6}	497.75	2.5×10^{-5}
3000.000	1.07	3000.003	1.3×10^{-6}	1495.49	7.2×10^{-5}
3000.000	2.4	3000.004	1.9×10^{-6}	1451.36	1.5×10^{-4}
3000.000	3.4	2999.997	2.5×10^{-6}	1419.80	2.1×10^{-4}
10000.00	1.07	9999.8	2.5×10^{-5}	4722.78	2.2×10^{-3}
10000.00	2.4	9999.6	5.0×10^{-5}	4316.30	4.5×10^{-3}
10000.00	3.4	9999.5	7.5×10^{-5}	4053.97	6.0×10^{-3}

References

[1] Conceptual Design Report, CEBAF Basic Experiment Equipment, April 13, 1990.

[2] F.R.Arutynian, V.A.Tumanian, Phys. Lett. 4 (1963) 176

[3] R.H.Milburn, Phys.Rev.Lett. 10(1963) 75

[4] A.T.Margarian, Laser and a New Nonmagnetic Method for the Tagging of Monochromatic Electrons and Photons, Yerevan Physics Institute, Preprint YERPHI 1403(14)-93(1993). Lepton and Photon Interactions 14th International Symposium, Ithaca, NY 1993, Contributed paper No.39.

[5] A.T.Margarian, Proposal of Electron Beam Absolute Energy Measurement by Laser Compton Scattering, Yerevan Physics Institute, Preprint YERPHI 1414(1)-94 (1994);

[6] K.Siegbahn,β-Ray Spectrometer Theory and Design, High Resolution Spectroscopy,in:β and γ-Ray Spectroscopy, ed. by Kai Siegbahn,North-Holand Pub.Co.,Amsterdam,1955,p.96

[7] B.E.Norum, T.P.Welch, An Intense Polarized Photon Source at CEBAF Hall B, CEBAF-LOI 93-012.

[8] C.Yan, R.Carlini, D.Neuffer, Beam Energy Measurement Using the ARC Beam Line as a Spectrometer, CEBAF-PR-93-004.

[9] M.Levi,J.Nash,S.Watson,Precision Measurements of the SLC Spectrometer Magnets,SLAC-PUB-4654(1989)

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Ա.Տ. ՄԱՐԳԱՐՅԱՆ

ՄԱԳՆԻՍԱԿԱՆ ՍՊԵԿՏՐՈՉԱՓԵՐԻ ԲԱՑԱՐՁԱԿ ՍԱՆԴՂԱԿԱՎՈՐՈՒՄԸ ԶԻԳԲԱՆԻ ՄԵԹՈԴՈՎ ԿՈՄՊՏՈՆՅԱՆ ՀԵՏՑՐՄԱՆ ՕԳՆՈՒԹՅԱՄԲ

Այս աշխատանքում ցույց է տրված մագնիսական սպեկտրոչափերի բացարձակ սանդղակավորման հնարավորությունը Կոմպտոնյան հետցրման օգնությամբ, օգտագործելով ցրված էլեկտրոնների բացարձակ կամ հարաբերական էներգիաները: Մոնտե-Կառլո մոդելավորմամբ ցույց է տրված *CEBAF*-ի մագնիսների բացարձակ սանդղակավորումը 10^{-14} ճշտությամբ:

Երևանի ֆիզիկայի ինստիտուտ

Препринт ЕрФИ-1424(11)-94

А. Т. МАРГАРЯН

АБСОЛЮТНАЯ КАЛИБРОВКА МАГНИТНЫХ СПЕКТРОМЕТРОВ МЕТОДОМ ЗИГБАНА С ИСПОЛЬЗОВАНИЕМ ОБРАТНОГО КОМПТОНОВСКОГО РАССЕЯНИЯ

В этой работе показана возможность абсолютной калибровки магнитных спектрометров с помощью обратного Комптоновского рассеяния, с использованием абсолютных или относительных энергий рассеянных электронов. Монте-Карло расчетами показано возможность абсолютной калибровки магнитов CEBAF-а с точностью 10^{-14}.

Ереванский физический институт

Абсолютная калибровка магнитных спектрометров методом Зигбана с использованием обратного комптоновского рассеяния

Маргарян А. Т.

(на англ. языке)

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