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THE FORMULAE FOR THE SEMI-ANALYTICAL CALCULATIONS OF THE FIELDS IN CYLINDRICAL APERIODIC STRUCTURES

AMBARTSUMIAN V.G.

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ABSTRACT

Formulae are derived to calculate the fields in a cylindrical aperiodic structure. Account is taken also of the fields excited by the accelerated beam moving at a velocity v ≈ c parallel to the structure's axis. The formulae are obtained by the field-matching method and expressed using the "transfer" matrices.

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The formulae for the semi-analytical calculations of the fields in cylindrical aperiodic structures

It is known , that the next generations of the linear accelerators of charged particles are distinguished by the high acceleration rates(~ 100MeV/m) and large currents of accelerated particles(~100mA). This circumstance requires the use of high frequency accelerating fields (~10-20GHz) and accelerating srtuctures with the compact cells (a few cm). But the large currents of charge and small sizes of the cells lead to the effects of beam breakup. At present it is known two methods for suppression of these effects: the damping method and detuned structure [1].

In [2] the formulae had been derived and carried out the semi-analytical calculations of fields in detuned (aperiodic) cylindrical structures for the symmetrical spatial electric modes (m=0). A slightly different way is chosen in [3] to derive formulae for the field calculations at the same case(m=0).

In this work we carry out the derivation of formulae for the semi-analytical calculations of fields in structures which consist the cylindrical cells with the arbitrary sizes (Fig.1) in the case of arbitrary spatial mode(arbitrary m), the special case for which is the structure described in [2].

1. The expansion of fields

The general solution of Maxwell equations in the arbitrary

N_{th} cell we represent as a sum of two solutions: particular solution of inhomogeneous equations and general solution of homogeneous equations. We will not consider here the inhomogeneous solution in detail, because it depends on the concrete distribution of current of the accelerated charges, but we assume that it satisfies the boundary conditions $E/_{\iota}^{a} = 0$ on the cylindrical walls of cell. In appendix we derive the inhomogeneous solution for the important practical case at which point-like charged bunch moves parallel to the structures axis with the small offset $\delta r^{a} \ll a^{N}$ (a^{N} is the radius of Nth cell).

Now we proceed to the homogeneous solution. The Fourier harmonics of homogeneous solution on the frequency ω can be found using electric and magnetic Hertz vectors which have only two non-zero components $\vec{\Phi}_z^{E,N}$ and $\vec{\Phi}_z^{H,N}$ for our case.

Thus $\vec{E}^{N}$ and $\vec{H}^{N}$ can be found according to expressions:

$$\vec{E}^{N} = \nabla(\nabla \vec{\Phi}_z^{E,N}) + \tilde{k}^2 \vec{\Phi}_z^{E,N} + i\tilde{k}[\nabla \vec{\Phi}_z^{H,N}] \qquad 1.1$$

$$\vec{H}^{N} = -i\tilde{k}[\nabla \vec{\Phi}_z^{E,N}] + \nabla(\nabla \vec{\Phi}_z^{H,N}) + \tilde{k}^2 \vec{\Phi}_z^{H,N} \qquad 1.2$$

where $\Phi_z^{E,N}$ and $\Phi_z^{H,N}$ are solutions of wave equation:

$$\Delta \Phi_z^{E(H),N} + \tilde{k}^2 \Phi_z^{E(H),N} = 0; \quad \Phi_z^{E(H),N} = \Phi_\varphi^{E(H),N} = 0; \quad \tilde{k} = \frac{\omega}{c} \qquad 1.3$$

$$\Phi_z^{E,N} = \sum_{m,k} \overset{c}{\Phi}{}_{mk}^{E,N} \left(\overset{c}{\overrightarrow{C}}{}_{mk}^{E,N} e^{ih_{mk}^{E,N} z} + \overset{c}{\overleftarrow{C}}{}_{mk}^{E,N} e^{-ih_{mk}^{E,N} z} \right) + \sum_{m,k} \overset{s}{\Phi}{}_{mk}^{E,N} \left(\overset{s}{\overrightarrow{C}}{}_{mk}^{E,N} e^{ih_{mk}^{E,N} z} + \overset{s}{\overleftarrow{C}}{}_{mk}^{E,N} e^{-ih_{mk}^{E,N} z} \right) \qquad 1.4$$

$$\Phi_z^{H,N} = \sum_{m,k} \overset{c}{\Phi}{}_{mk}^{H,N} \left(\overset{s}{\overrightarrow{C}}{}_{mk}^{H,N} e^{ih_{mk}^{H,N} z} - \overset{s}{\overleftarrow{C}}{}_{mk}^{H,N} e^{-ih_{mk}^{H,N} z} \right) + \sum_{m,k} \overset{s}{\Phi}{}_{mk}^{H,N} \left(\overset{c}{\overrightarrow{C}}{}_{mk}^{H,N} e^{ih_{mk}^{H,N} z} - \overset{c}{\overleftarrow{C}}{}_{mk}^{H,N} e^{-ih_{mk}^{H,N} z} \right) \qquad 1.5$$

$$\left.\begin{aligned} \overset{c}{\Phi}{}_{mk}^{E(H),N} &= J_m(g_{mk}^{E(H),N} r)\cos(m\varphi) \\ \overset{s}{\Phi}{}_{mk}^{E(H),N} &= J_m(g_{mk}^{E(H),N} r)\sin(m\varphi) \end{aligned}\right\} \qquad 1.6$$

$$\left.\begin{aligned} h_{mk}^{E(H),N} &= \sqrt{\tilde{k}^2 - g^{2\,E(H),N}_{mk}} \quad \text{при} \quad \tilde{k}^2 - g^{2\,E(H),N}_{mk} \geq 0 \\ h_{mk}^{E(H),N} &= i\sqrt{g^{2\,E(H),N}_{mk} - \tilde{k}^2} \quad \text{при} \quad \tilde{k}^2 - g^{2\,E(H),N}_{mk} < 0 \\ g_{mk}^{E,N} &= \frac{\nu_{mk}}{a^N}; \quad g_{mk}^{H,N} = \frac{\mu_{mk}}{a^N}; \quad J_m(\nu_{mk}) = J'_m(\mu_{mk}) = 0 \end{aligned}\right\} \qquad 1.7$$

Here Jm are Bessel functions, $\overset{c(s)}{\overrightarrow{\overleftarrow{C}}}{}_{mk}^{E(H),N}$ are constants. The sign (−) before the second terms of sum (1.5) and also the correspondences c⇒s, s⇒c are chosen for convenience. The functions $\overset{c(s)}{\Phi}{}^{E(H),N}$ already satisfy the continuity conditions on the cylindrical walls of cells. For convinience we introduce the usual notations:

$$\overset{c(s)}{X}{}_{mk}^{E(H),N}(z) = \overset{c(s)}{\overrightarrow{C}}{}_{mk}^{E(H),N} e^{ih_{mk}^{E(H),N} z} + \overset{c(s)}{\overleftarrow{C}}{}_{mk}^{E(H),N} e^{-ih_{mk}^{E(H),N} z} \qquad 1.8$$

$$\overset{c(s)}{Y}{}_{mk}^{E(H),N}(z) = \frac{d\overset{c(s)}{X}{}_{mk}^{E(H),N}(z)}{dz} \qquad 1.9$$

According to the (1.1,1.2) for the fields inside the Nth cell we have:

$$E_z^{E,N}(\varphi, r, z, \tilde{k}) = \sum_{m,k} g^{2\,E,N}_{mk} \left[\overset{c}{\Phi}{}_{mk}^{E,N} \overset{c}{X}{}_{mk}^{E,N}(z) + \overset{s}{\Phi}{}_{mk}^{E,N} \overset{s}{X}{}_{mk}^{E,N}(z) \right] \qquad 1.10$$

$$E_r^{E,N}(\varphi, r, z, \tilde{k}) = \sum_{m,k} \left[\overset{c}{\Phi}{}'^{E,N}_{mk} \overset{c}{Y}{}_{mk}^{E,N}(z) + \overset{s}{\Phi}{}'^{E,N}_{mk} \overset{s}{Y}{}_{mk}^{E,N}(z) \right] \qquad 1.11$$

$$E_\varphi^{E,N}(\varphi, r, z, \tilde{k}) = \frac{1}{r} \sum_{m,k} m \left[\overset{c}{\Phi}{}_{mk}^{E,N} \overset{s}{Y}{}_{mk}^{E,N}(z) - \overset{s}{\Phi}{}_{mk}^{E,N} \overset{c}{Y}{}_{mk}^{E,N}(z) \right] \qquad 1.12$$

$$H_z^{E,N}(\varphi,r,z,\tilde{k})=0 \qquad 1.13$$

$$H_r^{E,N}(\varphi,r,z,\tilde{k})=\frac{i\tilde{k}}{r}\sum_{m,k} m\left[\overset{c}{\Phi}{}^{E,N}_{mk}\,\overset{s}{X}{}^{E,N}_{mk}(z)+\overset{s}{\Phi}{}^{E,N}_{mk}\,\overset{c}{X}{}^{E,N}_{mk}(z)\right] \qquad 1.14$$

$$H_\varphi^{E,N}(\varphi,r,z,\tilde{k})=-i\tilde{k}\sum_{m,k}\left[\overset{c}{\Phi}{}'^{E,N}_{mk}\,\overset{c}{X}{}^{E,N}_{mk}(z)+\overset{s}{\Phi}{}'^{E,N}_{mk}\,\overset{s}{X}{}^{E,N}_{mk}(z)\right] \qquad 1.15$$

$$E_z^{H,N}(\varphi,r,z,\tilde{k})=0 \qquad 1.16$$

$$E_r^{H,N}(\varphi,r,z,\tilde{k})=\frac{\tilde{k}}{r}\sum_{m,k}\frac{m}{h^{H,N}_{mk}}\left[\overset{s}{\Phi}{}^{H,N}_{mk}\,\overset{s}{Y}{}^{H,N}_{mk}(z)-\overset{c}{\Phi}{}^{H,N}_{mk}\,\overset{c}{Y}{}^{H,N}_{mk}(z)\right] \qquad 1.17$$

$$E_\varphi^{H,N}(\varphi,r,z,\tilde{k})=\tilde{k}\sum_{m,k}\frac{1}{h^{H,N}_{mk}}\left[\overset{c}{\Phi}{}'^{H,N}_{mk}\,\overset{s}{Y}{}^{H,N}_{mk}(z)+\overset{s}{\Phi}{}'^{H,N}_{mk}\,\overset{c}{Y}{}^{H,N}_{mk}(z)\right] \qquad 1.18$$

$$H_z^{H,N}(\varphi,r,z,\tilde{k})=-i\sum_{m,k}\frac{g^{2\,H,N}_{mk}}{h^{H,N}_{mk}}\left[\overset{c}{\Phi}{}^{H,N}_{mk}\,\overset{c}{Y}{}^{H,N}_{mk}(z)+\overset{s}{\Phi}{}^{H,N}_{mk}\,\overset{s}{Y}{}^{H,N}_{mk}(z)\right] \qquad 1.19$$

$$H_r^{H,N}(\varphi,r,z,\tilde{k})=i\sum_{m,k}h^{H,N}_{mk}\left[\overset{c}{\Phi}{}'^{H,N}_{mk}\,\overset{s}{X}{}^{H,N}_{mk}(z)+\overset{s}{\Phi}{}'^{H,N}_{mk}\,\overset{c}{Y}{}^{H,N}_{mk}(z)\right] \qquad 1.20$$

$$H_\varphi^{H,N}(\varphi,r,z,\tilde{k})=\frac{i}{r}\sum_{mk} m\,h^{H,N}_{mk}\left[\overset{c}{\Phi}{}^{H,N}_{mk}\,\overset{c}{X}{}^{H,N}_{mk}(z)+\overset{s}{\Phi}{}^{H,N}_{mk}\,\overset{s}{X}{}^{H,N}_{mk}(z)\right] \qquad 1.21$$

Here by the prime we mark the derivative with respect to r.

2.The continuity conditions

The continuity conditions for the fields on the transverse walls of cells at z=0 and $z=l^N$ are:

$$\tilde{E}_z^N(\varphi,r,l^N,\tilde{k})=\tilde{E}_z^{N+1}(\varphi,r,0,\tilde{k}) \qquad 0\le r\le a_2^N \qquad 2.1$$

$$\tilde{E}_r^N(\varphi,r,l^N,\tilde{k})\,\theta(a^N-r)=\tilde{E}_r^{N+1}(\varphi,r,0,\tilde{k})\,\theta(a^{N+1}-r) \qquad 2.2$$

$$\tilde{E}_\varphi^N(\varphi,r,l^N,\tilde{k})\,\theta(a^N-r)=\tilde{E}_\varphi^{N+1}(\varphi,r,0,\tilde{k})\,\theta(a^{N+1}-r) \qquad 2.3$$

$$\vec{H}^N(\varphi,r,l^N,\tilde{k})=\vec{H}^{N+1}(\varphi,r,0,\tilde{k}) \qquad 0\le r\le a_2^N \qquad 2.4$$

where for the two arbitrary quantities ρ^N and ρ^{N+1} we introduce the quantities ρ_1^N and ρ_2^N :

$$\left.\begin{aligned}\rho_1^N&=\rho^N\theta_1^N+\rho^{N+1}\theta_2^N\\ \rho_2^N&=\rho^N\theta_2^N+\rho^{N+1}\theta_1^N\\ \theta_1^N&\equiv\theta(a^N-a^{N+1})\,;\quad \theta_2^N\equiv\theta(a^{N+1}-a^N)\end{aligned}\right\} \qquad 2.5$$

In (2.1÷2.4) $\vec{E}$ and $\vec{H}$ are the total fields, l^N is the length of the Nth cell. From (2.4), $\operatorname{div}\vec{H}=0$ follows the condition:

$$\left.\frac{\partial\tilde{H}_z^N}{\partial z}\right|_{(\varphi,r,l^N,\tilde{k})}=\left.\frac{\partial\tilde{H}_z^{N+1}}{\partial z}\right|_{(\varphi,r,0,\tilde{k})} \qquad 2.6$$

As it can be seen below,the conditions (2.1÷2.3),(2.6) and boundary conditions at the structures both ends are sufficient for the unequal determination of fields inside the structure.

3.The fields matching

Multiplying (2.2) by $[r^{2}\,\overset{c(s)}{\Phi}{}^{H,N}_{mk}]_1$, (2.3) by $[r^{2}\,\overset{c(s)}{\Phi}{}^{E,N}_{mk}]_1$ and integrating from "0" to a_1^N. also multiplying (2.1) by $[r\,\overset{c(s)}{\Phi}{}^{E,N}_{mk}]_2$, ( ) by $[r\,\overset{c(s)}{\Phi}{}^{H,N}_{mk}]_2$ and integrating from "0" to a_2^N we determine:

$$\overset{c(s)}{R}{}^{E,N}_{m,r(\varphi),1}\,\overset{c(s)}{\eta}{}^{E,N}_{m,1}+\overset{c(s)}{R}{}^{H,N}_{m,r(\varphi),1}\,\overset{c(s)}{\eta}{}^{H,N}_{m,1}=\overset{c(s)}{R}{}^{E,N}_{m,r(\varphi),2}\,\overset{c(s)}{\eta}{}^{E,N}_{m,2}+$$

$$+\overset{c(s)}{R}{}^{H,N}_{m,r(\varphi),2}\,\overset{c(s)}{\eta}{}^{H,N}_{m,2}+\overset{c(s)}{P}{}^{E,N}_{m,r(\varphi)} \qquad 3.1$$

$$ {}^{c(s)}R^{E(H),N}_{m,z,1}\,{}^{c(s)}\xi^{E(H),N}_{m,1} = {}^{c(s)}R^{E(H),N}_{m,z,2}\,{}^{c(s)}\xi^{E(H),N}_{m,2} + {}^{c(s)}P^{E(H),N}_{m,2} \qquad 3.2 $$

where

$$ \left.\begin{aligned} {}^{c(s)}\eta^{E(H),N}_{m,1(2)} &= {}^{c(s)}Y^{E(H),N}_{m}(\ell^N)\,\Theta^N_{1(2)} + {}^{c(s)}Y^{E(H),N+1}_{m}(0)\,\Theta^N_{2(1)} \\ {}^{c(s)}\xi^{E(H),N}_{m,1(2)} &= {}^{c(s)}X^{E(H),N}_{m}(\ell^N)\,\Theta^N_{1(2)} + {}^{c(s)}X^{E(H),N+1}_{m}(0)\,\Theta^N_{2(1)} \end{aligned}\right\} \qquad 3.3 $$

$$ Y_m \equiv \begin{pmatrix}(Y_m)_1\\(Y_m)_2\\ \vdots\end{pmatrix}; \qquad X_m \equiv \begin{pmatrix}(X_m)_1\\(X_m)_2\\ \vdots\end{pmatrix} \qquad 3.4 $$

The matrices R and vectors P are:

$$ \left({}^{c(s)}R^{E,N}_{m,z,1}\right)_{kn} = \frac{2J_m(\nu_{mn}\alpha_N)\alpha_N^2\nu_{mn}^2}{(\nu_{mk}^2-\alpha_N^2\nu_{mn}^2)J_{m+1}(\nu_{mk})\nu_{mk}} \qquad 3.5 $$

$$ \left({}^{c(s)}R^{E,N}_{m,z,2}\right)_{kn} = \delta_{kn} \qquad 3.6 $$

$$ \left({}^{c(s)}R^{H,N}_{m,z,1}\right)_{kn} = \frac{2J_m'(\mu_{mn}\alpha_N)\alpha_N^3\mu_{mn}^3 h^{H,N}_{mn,1}}{(\mu_{mk}^2-\alpha_N^2\mu_{mn}^2)(\mu_{mk}^2-m^2)J_m(\mu_{mk})h^{H,N}_{mk,2}} \qquad 3.7 $$

$$ \left({}^{c(s)}R^{H,N}_{m,z,2}\right)_{kn} = \delta_{kn} \qquad 3.8 $$

$$ \left({}^{c(s)}R^{E,N}_{m,z,1}\right)_{kn} = -\frac{2J_{m+1}(\nu_{mn})J_m(\mu_{mk})\nu_{mn}^3}{(\mu_{mk}^2-\nu_{mn}^2)^2} \qquad 3.9 $$

$$ \left({}^{c(s)}R^{E,N}_{m,z,2}\right)_{kn} = \frac{2J_m'(\nu_{mn})J_m(\mu_{mk}\alpha_N)\alpha_N^2\nu_{mn}^3}{(\nu_{mn}^2-\mu_{mk}^2\alpha_N^2)^2} + $$

$$ + \frac{J_m'(\mu_{mk}\alpha_N)J_m'(\nu_{mn})\alpha_N^3\nu_{mn}\mu_{mk}}{(\nu_{mn}^2-\mu_{mk}^2\alpha_N^2)} \qquad 3.10 $$

$$ \left({}^{c}R^{H,N}_{m,z,1}\right)_{kn} = -\left({}^{s}R^{H,N}_{m,z,1}\right)_{kn} = -\frac{m\tilde K}{2}\left(1-\frac{m^2}{\mu_{mk}^2}\right)\frac{J_m^2(\mu_{mk})}{h^{H,N}_{mk,1}}\delta_{kn} \qquad 3.11 $$

$$ \left({}^{c}R^{H,N}_{m,z,2}\right)_{kn} = -\left({}^{s}R^{H,N}_{m,z,2}\right)_{kn} = -\frac{m\tilde K}{h^{H,N}_{mn,2}}\,\frac{J_m'(\mu_{mk}\alpha_N)J_m(\mu_{mn})\alpha_N^3\mu_{mk}}{\mu_{mn}^2-\mu_{mk}^2\alpha_N^2} \qquad 3.12 $$

$$ \left({}^{c}R^{E,N}_{m,\varphi,1}\right)_{kn} = -\left({}^{s}R^{E,N}_{m,\varphi,1}\right)_{kn} = -\frac{m}{2}J_{m+1}^2(\nu_{mk})\delta_{kn} \qquad 3.13 $$

$$ \left({}^{c}R^{E,N}_{m,\varphi,2}\right)_{kn} = -\left({}^{s}R^{E,N}_{m,\varphi,2}\right)_{kn} = m\frac{J_m'(\nu_{mn})J_m(\nu_{mk}\alpha_N)\nu_{mn}}{\nu_{mn}^2-\nu_{mk}^2\alpha_N^2} \qquad 3.14 $$

$$ \left({}^{c(s)}R^{H,N}_{m,\varphi,1}\right)_{kn} = \frac{2\tilde K}{h^{H,N}_{mn,1}}\,\frac{J_m'(\nu_{mk})J_m(\mu_{mn})\mu_{mn}^2\nu_{mk}}{(\nu_{mk}^2-\mu_{mn}^2)^2} \qquad 3.15 $$

$$ \left({}^{c(s)}R^{H,N}_{m,\varphi,2}\right)_{kn} = \frac{2\tilde K J_m'(\nu_{mk}\alpha_N)J_m(\mu_{mn})\mu_{mn}^2\nu_{mk}\alpha_N^3}{h^{H,N}_{mn,2}(\mu_{mn}^2-\nu_{mk}^2\alpha_N^2)^2} + $$

$$ + \frac{\tilde K J_m(\nu_{mk}\alpha_N)J_m(\mu_{mn})(\mu_{mn}^2-m^2)\alpha_N^2}{h^{H,N}_{mn,2}(\mu_{mn}^2-\nu_{mk}^2\alpha_N^2)} \qquad 3.16 $$

$$ {}^{c(s)}P^{E,N}_{m,r} = \frac{1}{\pi(a_1^N)^2}\int_0^{2\pi}\int_0^{a_1^N}\left\{[E^Q_r]_2\Theta(a_2^N-r)-[E^Q_r]_1\right\}\left[{}^{c(s)}\Phi^{H,N}_{mk}\right]_1 r^2\,dr\,d\varphi \qquad 3.17 $$

$$ {}^{c(s)}P^{E,N}_{m,\varphi} = \frac{1}{\pi(a_1^N)^2}\int_0^{2\pi}\int_0^{a_1^N}\left\{[E^Q_\varphi]_2\Theta(a_2^N-r)-[E^Q_\varphi]_1\right\}\left[{}^{s(c)}\Phi^{E,N}_{mk}\right]_1 r^2\,dr\,d\varphi \qquad 3.18 $$

$$ {}^{c(s)}P^{E,N}_{m,z} = \frac{2}{\pi J_{m+1}^2(\nu_{mk})(a_2^N)^2}\int_0^{2\pi}\int_0^{a_2^N}\left\{[E^Q_z]_2-[E^Q_z]_1\right\}\left[{}^{c(s)}\Phi^{E,N}_{mk}\right]_2 r\,dr\,d\varphi \qquad 3.19 $$

$$ {}^{c(s)}P^{H,N}_{m,z} = \frac{2}{\pi(a_2^N)^2\left(1-\frac{m^2}{\mu_{mk}^2}\right)J_m^2(\mu_{mk})h^{H,N}_{mk,2}} \times $$

$$ \times\int_0^{2\pi}\int_0^{a_2^N}\left\{\left[\frac{\partial H^Q_z}{\partial z}\right]_2-\left[\frac{\partial H^Q_z}{\partial z}\right]_1\right\}\left[{}^{c(s)}\Phi^{H,N}_{mk}\right]_2 r\,dr\,d\varphi \qquad 3.20 $$

The quantities $h^{H,N}_{mn,1}$ and $h^{H,N}_{mn,2}$ are defined according to (2.5) and $\alpha_N = a^N_2/a^N_1$.

Here $\vec{E}^Q$ and $\vec{H}^Q$ are the Fourier harmonics of inhomogeneous solution. Indexes "1" and "2" are defined according to (2.5).

For the quantities with the different indexes "E" and "H" we introduce the new vector:

$$\beta \equiv \begin{pmatrix} \beta^E \\ \beta^H \end{pmatrix} \qquad 3.21$$

For example, we can write for two vectors $\overset{c(s)}{\eta}{}^{E(H),N}_{m,1(2)}$:

$$\overset{c(s)}{\eta}{}^{N}_{m,1(2)} \equiv \begin{pmatrix} \overset{c(s)}{\eta}{}^{E,N}_{m,1(2)} \\ \overset{c(s)}{\eta}{}^{H,N}_{m,1(2)} \end{pmatrix} \qquad 3.22$$

Let's introduce the following matrices:

$$\overset{c(s)}{T}{}^{N}_{m,1(2)} = \begin{pmatrix} \overset{c(s)}{R}{}^{E,N}_{m,z,1(2)} ; & \overset{c(s)}{R}{}^{H,N}_{m,z,1(2)} \\ \overset{c(s)}{R}{}^{E,N}_{m,\varphi,1(2)} ; & \overset{c(s)}{R}{}^{H,N}_{m,\varphi,1(2)} \end{pmatrix}; \quad \overset{c(s)}{S}{}^{N}_{m,1(2)} = \begin{pmatrix} \overset{c(s)}{R}{}^{E,N}_{m,z,1(2)} ; & 0 \\ 0 ; & \overset{c(s)}{R}{}^{H,N}_{m,z,1(2)} \end{pmatrix} \qquad 3.23$$

Now we can rewrite (3.1) and (3.2):

$$\overset{c(s)}{T}{}^{N}_{m,1} \overset{c(s)}{\eta}{}^{N}_{m,1} = \overset{c(s)}{T}{}^{N}_{m,2} \overset{c(s)}{\eta}{}^{N}_{m,2} + \overset{c(s)}{P}{}^{N}_{m} \qquad 3.24$$

$$\overset{c(s)}{S}{}^{N}_{m,1} \overset{c(s)}{\xi}{}^{N}_{m,1} = \overset{c(s)}{\xi}{}^{N}_{m,2} + \overset{c(s)}{P}{}^{N}_{m,z} \qquad 3.25$$

where

$$\overset{c(s)}{P}{}^{N}_{m} = \begin{pmatrix} \overset{c(s)}{P}{}^{E,N}_{m,z} \\ \overset{c(s)}{P}{}^{E,N}_{m,\varphi} \end{pmatrix}; \quad \overset{c(s)}{P}{}^{N}_{m,z} = \begin{pmatrix} \overset{c(s)}{P}{}^{E,N}_{m,z} \\ \overset{c(s)}{P}{}^{H,N}_{m,z} \end{pmatrix} \qquad 3.26$$

In the expression (3.25) we omit the matrices $\overset{c(s)}{S}{}^{N}_{m,2}$ because they are equal to unit matrices.

As follows from (3,25), at $m \neq 0$ in the stucture cannot exist the fields purely of type "E" or purely of type "H".

Also, because Det($\overset{c(s)}{S}{}^{N}_{m,1}$) = 0(for the structure cannot exist the local transfer matrices), hereafter we shall proceed from the consequences of the theorem of the unequal determination of fields inside any volume at known $\vec{E}_t$ or $\vec{H}_t$ on the boundaries.

4.The basic relations

Before driving the basic relations we introduce a few necessary notations and expressions.

From (3.24) we express $\overset{c(s)}{\eta}{}^{N}_{m,1}$ through $\overset{c(s)}{\eta}{}^{N}_{m,2}$ and $\overset{c(s)}{P}{}^{N}_{m}$:

$$\overset{c(s)}{\eta}{}^{N}_{m,1} = \overset{c(s)}{W}{}^{N}_{m} \overset{c(s)}{\eta}{}^{N}_{m,2} + \overset{c(s)}{V}{}^{N}_{m} \qquad 4.1$$

where

$$\overset{c(s)}{W}{}^{N}_{m} = \left(\overset{c(s)}{T}{}^{N}_{m,1}\right)^{-1} \overset{c(s)}{T}{}^{N}_{m,2}; \quad \overset{c(s)}{V}{}^{N}_{m} = \left(\overset{c(s)}{T}{}^{N}_{m,1}\right)^{-1} \overset{c(s)}{P}{}^{N}_{m} \qquad 4.2$$

Introduce the following matrices and vectors:

$$\overset{c(s)}{\overrightarrow{W}}{}^{N}_{m} = \theta^N_1 + \theta^N_2 \overset{c(s)}{W}{}^{N}_{m}; \quad \overset{c(s)}{\overleftarrow{W}}{}^{N}_{m} = \theta^N_2 + \theta^N_1 \overset{c(s)}{W}{}^{N}_{m} \qquad 4.3$$

$$\overset{c(s)}{\overrightarrow{V}}{}^N_m = \Theta_2^N \overset{c(s)}{V}{}^N_m \; ; \qquad \overset{c(s)}{\overleftarrow{V}}{}^N_m = \Theta_1^N \overset{c(s)}{V}{}^N_m \qquad 4.4$$

It is not difficult to verify that the following expressions hold:

$$\left.\begin{aligned} \overset{c(s)}{Y}{}^N_m(\ell^N) &= \overset{c(s)}{\overleftarrow{W}}{}^N_m \overset{c(s)}{\eta}{}^N_{m,2} + \overset{c(s)}{\overleftarrow{V}}{}^N_m \\ \overset{c(s)}{Y}{}^{N+1}_m(0) &= \overset{c(s)}{\overrightarrow{W}}{}^N_m \overset{c(s)}{\eta}{}^N_{m,2} + \overset{c(s)}{\overrightarrow{V}}{}^N_m \end{aligned}\right\} \qquad 4.5$$

Introduce also the matrices $\tilde{\lambda}^N_{m,s}$, $\tilde{\lambda}^N_{m,t}$:

$$\tilde{\Lambda}^N_{m,s(t)} = \begin{pmatrix} \tilde{\Lambda}^{E,N}_{m,s(t)} & 0 \\ 0 & \tilde{\Lambda}^{H,N}_{m,s(t)} \end{pmatrix} \qquad 4.6$$

$$\left(\tilde{\Lambda}^{E(H),N}_{m,s}\right)_{kn} = \frac{\delta_{kn}}{i h^{E(H),N}_{mk}\, sh\left(i h^{E(H),N}_{mk} \ell^N\right)} \qquad 4.7$$

$$\left(\tilde{\Lambda}^{E(H),N}_{m,t}\right)_{kn} = \frac{\delta_{kn}}{i h^{E(H),N}_{mk}\, th\left(i h^{E(H),N}_{mk} \ell^N\right)} \qquad 4.8$$

Using these matrices we can write:

$$\overset{c(s)}{X}{}^N_m(0) = \tilde{\Lambda}^N_{m,s} \overset{c(s)}{Y}{}^N_m(\ell^N) - \tilde{\Lambda}^N_{m,t} \overset{c(s)}{Y}{}^N_m(0) \qquad 4.9$$

$$\overset{c(s)}{X}{}^N_m(\ell^N) = \tilde{\Lambda}^N_{m,t} \overset{c(s)}{Y}{}^N_m(\ell^N) - \tilde{\Lambda}^N_{m,s} \overset{c(s)}{Y}{}^N_m(0) \qquad 4.10$$

Now we with the use of (3.3,4.1,4.5,4.9,4.10) all quantities in

(3.25) express in terms of $\overset{c(s)}{\eta}{}^{N-1}_{m,2}$, $\overset{c(s)}{\eta}{}^{N}_{m,2}$, $\overset{c(s)}{\eta}{}^{N+1}_{m,2}$ and through free terms:

$$\overset{c(s)}{E}{}^N_m \overset{c(s)}{\eta}{}^{N-1}_{m,2} + \overset{c(s)}{F}{}^N_m \overset{c(s)}{\eta}{}^{N}_{m,2} + \overset{c(s)}{G}{}^N_m \overset{c(s)}{\eta}{}^{N+1}_{m,2} + \overset{c(s)}{H}{}^N_m = 0 \qquad 4.11$$

where

$$\overset{c(s)}{E}{}^N_m = \left(\Theta_2^N - \Theta_1^N \overset{c(s)}{S}{}^N_{m,1}\right) \tilde{\Lambda}^N_{m,s} \overset{c(s)}{\overrightarrow{W}}{}^{N-1}_m \qquad 4.12$$

$$\overset{c(s)}{F}{}^N_m = \overset{c(s)}{S}{}^N_{m,1}\left(\Theta_1^N \tilde{\Lambda}^N_{m,t} - \Theta_2^N \tilde{\Lambda}^{N+1}_{m,t}\right) \overset{c(s)}{W}{}^N_m - \left(\Theta_2^N \tilde{\Lambda}^N_{m,t} - \Theta_1^N \tilde{\Lambda}^{N+1}_{m,t}\right) \qquad 4.13$$

$$\overset{c(s)}{G}{}^N_m = \left(\Theta_2^N \overset{c(s)}{S}{}^N_{m,1} - \Theta_1^N\right) \tilde{\Lambda}^{N+1}_{m,s} \overset{c(s)}{\overleftarrow{W}}{}^{N+1}_m \qquad 4.14$$

$$\begin{aligned} \overset{c(s)}{H}{}^N_m = \Big\{ & \overset{c(s)}{S}{}^N_{m,1} \Big[\Theta_1^N\Big(\tilde{\Lambda}^N_{m,t} \overset{c(s)}{V}{}^N_m - \tilde{\Lambda}^N_{m,s} \overset{c(s)}{\overrightarrow{V}}{}^{N-1}_m\Big) + \Theta_2^N\Big(\tilde{\Lambda}^{N+1}_{m,s} \overset{c(s)}{\overleftarrow{V}}{}^{N+1}_m - \tilde{\Lambda}^{N+1}_{m,t} \overset{c(s)}{V}{}^N_m\Big)\Big] - \\ & - \Big(\Theta_1^N \tilde{\Lambda}^{N+1}_{m,s} \overset{c(s)}{\overleftarrow{V}}{}^{N+1}_m - \Theta_2^N \tilde{\Lambda}^N_{m,s} \overset{c(s)}{\overrightarrow{V}}{}^{N-1}_m\Big)\Big\} - \overset{c(s)}{P}{}^N_{m,t} \end{aligned} \qquad 4.15$$

The expressions (4.11) we call the basic relations. They are the consequences of the theorem of unequal determination of fields.

5. The boundary conditions and determination of the fields

We assume that at the ends of structure the quantities $\overset{c(s)}{\overrightarrow{C}}{}^1_m$ and $\overset{c(s)}{\overleftarrow{C}}{}^{Nc}_m$ are known (Nc is the number of the last cell).

From (4.11) we express the $\overset{c(s)}{\eta}{}^{Nc-2}_{m,2}$ through $\overset{c(s)}{\eta}{}^{Nc-3}_{m,2}$ and $\overset{c(s)}{\eta}{}^{Nc-1}_{m,2}$

$$\overset{c(s)}{\eta}{}^{Nc-2}_{m,2} = \overset{c(s)}{A}{}^{Nc-3}_m \overset{c(s)}{\eta}{}^{Nc-3}_{m,2} + \overset{c(s)}{\tilde{B}}{}^{Nc-3}_m + \overset{c(s)}{\hat{B}}{}^{Nc-3}_m \overset{c(s)}{\eta}{}^{Nc-1}_{m,2} \qquad 5.1$$

where

$$\overset{c(s)}{A}{}^{N_c-3}_{m} = -\left(\overset{c(s)}{F}{}^{N_c-2}_{m}\right)^{-1}\overset{c(s)}{E}{}^{N_c-2}_{m} \qquad 5.2$$

$$\overset{c(s)}{\tilde{B}}{}^{N_c-3}_{m} = -\left(\overset{c(s)}{F}{}^{N_c-2}_{m}\right)^{-1}\overset{c(s)}{H}{}^{N_c-2}_{m} \qquad 5.3$$

$$\overset{c(s)}{\hat{B}}{}^{N_c-3}_{m} = -\left(\overset{c(s)}{F}{}^{N_c-2}_{m}\right)^{-1}\overset{c(s)}{G}{}^{N_c-2}_{m} \qquad 5.4$$

Now with the substitutions in (4.11) we obtain the recurence formulae for $\overset{c(s)}{\eta}{}^{N}_{m,2}$:

$$\overset{c(s)}{\eta}{}^{M}_{m,2} = \overset{c(s)}{A}{}^{M-1}_{m}\overset{c(s)}{\eta}{}^{M-1}_{m,2} + \overset{c(s)}{\tilde{B}}{}^{M-1}_{m} + \overset{c(s)}{\hat{B}}{}^{M-1}_{m}\overset{c(s)}{\eta}{}^{N_c-1}_{m,2} \qquad 5.5$$

where

$$\overset{c(s)}{A}{}^{M-1}_{m} = -\left(\overset{c(s)}{F}{}^{M}_{m} + \overset{c(s)}{G}{}^{M}_{m}\overset{c(s)}{A}{}^{M}_{m}\right)^{-1}\overset{c(s)}{E}{}^{M}_{m} \qquad 5.6$$

$$\overset{c(s)}{\tilde{B}}{}^{M-1}_{m} = -\left(\overset{c(s)}{F}{}^{M}_{m} + \overset{c(s)}{G}{}^{M}_{m}\overset{c(s)}{A}{}^{M}_{m}\right)^{-1}\left(\overset{c(s)}{G}{}^{M}_{m}\overset{c(s)}{\tilde{B}}{}^{M}_{m} + \overset{c(s)}{H}{}^{M}_{m}\right) \qquad 5.7$$

$$\overset{c(s)}{\hat{B}}{}^{M-1}_{m} = -\left(\overset{c(s)}{F}{}^{M}_{m} + \overset{c(s)}{G}{}^{M}_{m}\overset{c(s)}{A}{}^{M}_{m}\right)^{-1}\overset{c(s)}{G}{}^{M}_{m}\overset{c(s)}{\hat{B}}{}^{M}_{m} \qquad 5.8$$

Introduce the matrices $\overset{c(s)}{A}{}^{N,M}_{m}$, $\overset{c(s)}{\tilde{B}}{}^{N,M}_{m}$, $\overset{c(s)}{\hat{B}}{}^{N,M}_{m}$:

$$\overset{c(s)}{\eta}{}^{M}_{m,2} = \overset{c(s)}{A}{}^{N,M}_{m}\overset{c(s)}{\eta}{}^{N}_{m,2} + \overset{c(s)}{\tilde{B}}{}^{N,M}_{m} + \overset{c(s)}{\hat{B}}{}^{N,M}_{m}\overset{c(s)}{\eta}{}^{N_c-1}_{m,2} \qquad 5.9$$

$$\overset{c(s)}{A}{}^{N,M}_{m} = \prod_{K=N}^{M-1}\overset{c(s)}{A}{}^{K}_{m}\,; \quad \overset{c(s)}{\tilde{B}}{}^{N,M}_{m} = \sum_{L=N+1}^{M}\overset{c(s)}{A}{}^{L,M}_{m}\overset{c(s)}{\tilde{B}}{}^{L-1}_{m}\,;$$

$$\overset{c(s)}{\hat{B}}{}^{N,M}_{m} = \sum_{L=N+1}^{M}\overset{c(s)}{A}{}^{L,M}_{m}\overset{c(s)}{\hat{B}}{}^{L-1}_{m}\,; \quad \overset{c(s)}{A}{}^{K,K}_{m} = I \qquad 5.10$$

Hereafter for convinence in the quantities with the indexes c(s) we admit these indexes. We will write these indexes only in quantities which are introduced for the first time.

Using the expressions (1.8, 3.3, 3.25, 4.1, 4.3÷4.5, 4.9,4.10) we express $\eta^{1}_{m,2}$ in terms of $\eta^{2}_{m,2}$ and $\vec{C}^{1}_{m}$:

$$\eta^{1}_{m,2} = \Theta^{1}_{1}\left\{\tilde{\Lambda}^{2}_{m,t} + iS^{1}_{m,1}(h^{1}_{m})^{-1}W^{1}_{m}\right\}^{-1} \times \left\{\tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}\eta^{2}_{m,2} - 2S^{1}_{m,1}\vec{C}^{1}_{m} + \tilde{\Lambda}^{2}_{m,s}\overleftarrow{V}^{2}_{m} - iS^{1}_{m,1}(h^{1}_{m})^{-1}V^{1}_{m} + P^{1}_{m,z}\right)\Big\} +$$

$$+\Theta^{1}_{2}\left\{S^{1}_{m,1}\tilde{\Lambda}^{2}_{m,t}W^{1}_{m} + i(h^{1}_{m})^{-1}\right\}^{-1} \times \left\{S^{1}_{m,1}\tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}\eta^{2}_{m,2} - 2\vec{C}^{1}_{m} + S^{1}_{m,1}\left(\tilde{\Lambda}^{2}_{m,s}\overleftarrow{V}^{2}_{m} - \tilde{\Lambda}^{2}_{m,t}V^{1}_{m}\right) - P^{1}_{m,z}\right\} \qquad 5.11$$

Here h^{1}_{m} is determined according to the rule (3.21). From (5.5) and (5.11) express $\eta^{1}_{m,2}$ through $\eta^{N_c-1}_{m,2}$ and $\vec{C}^{1}_{m}$.

$$\eta^{1}_{m,2} = \overset{c(s)}{\hat{\Omega}}_{m}\eta^{N_c-1}_{m} + \overset{c(s)}{\Omega}_{m}\vec{C}^{1}_{m} + \overset{c(s)}{\tilde{\Omega}}_{m} \qquad 5.12$$

where

$$\overset{c(s)}{\hat{\Omega}}_{m} = \overset{c(s)}{\gamma}_{1}\tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}\hat{B}^{1}_{m} + \overset{c(s)}{\gamma}_{2}S^{1}_{m,1}\tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}\hat{B}^{1}_{m} \qquad 5.13$$

$$\overset{c(s)}{\Omega}_{m} = -2\left(\overset{c(s)}{\gamma}_{1}S^{1}_{m,1} - \overset{c(s)}{\gamma}_{2}\right) \qquad 5.14$$

$$\overset{c(s)}{\tilde{\Omega}}_{m} = \overset{c(s)}{\gamma}_{1}\left\{\tilde{\Lambda}^{2}_{m,s}\left(\overleftarrow{W}^{2}_{m}\tilde{B}^{1}_{m} + \overleftarrow{V}^{2}_{m}\right) - iS^{1}_{m,1}(h^{1}_{m})^{-1}V^{1}_{m} + P^{1}_{m,z}\right\} +$$

$$+\overset{c(s)}{\gamma}_{2}\left\{S^{1}_{m,1}\tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}\tilde{B}^{1}_{m} + S^{1}_{m,1}\left(\tilde{\Lambda}^{2}_{m,s}\overleftarrow{V}^{2}_{m} - \tilde{\Lambda}^{2}_{m,t}V^{1}_{m}\right) - P^{1}_{m,z}\right\} \qquad 5.15$$

$$\overset{c(s)}{\gamma}_{1} = \Theta^{1}_{1}\left\{\tilde{\Lambda}^{2}_{m,t} + iS^{1}_{m,1}(h^{1}_{m})^{-1}W^{1}_{m} - \tilde{\Lambda}^{2}_{m,s}\overleftarrow{W}^{2}_{m}A^{1}_{m}\right\}^{-1} \qquad 5.16$$

$$\overset{c(s)}{\gamma}_2 = \Theta_2^1 \left\{ S^1_{m,1} \left(\tilde\Lambda^2_{m,t} W^1_m - \tilde\Lambda^2_{m,s} A^1_m\right) + i \left(h^1_m\right)^{-1} \right\}^{-1} \qquad 5.17$$

Substituting (5.12) in (5.9) at M=Nc−2 and N=1 we get:

$$\eta^{Nc-2}_{m,2} = \left(A^{1,Nc-2}_m \hat\Omega_m + \hat B^{1,Nc-2}_m\right)\eta^{Nc-1}_{m,2} + A^{1,Nc-2}_m \Omega_m \vec C^1_m + A^{1,Nc-2}_m \tilde\Omega_m + \tilde B^{1,Nc-2}_m$$

Using (1.8, 3.3,3.25,4.1,4.3÷4.5, 4.9, 4.10) we can write for $\eta^{Nc-1}_{m,2}$, $\eta^{Nc-2}_{m,2}$ and $\overleftarrow{C}^{Nc}_m$ the expression like (5.11):

$$\left(\overset{c(s)}{\beta_1}\Theta_1^{Nc-1} + \overset{c(s)}{\beta_2}\Theta_2^{Nc-1}\right)\eta^{Nc-1}_{m,2} = \Theta_1^{Nc-1}\left\{\overset{c(s)}{K_1}\eta^{Nc-2}_{m,2} + \overset{c(s)}{\tilde K_1}\overleftarrow{C}^{Nc}_m + \overset{c(s)}{\hat K_1}\right\} + \Theta_2^{Nc-1}\left\{\overset{c(s)}{K_2}\eta^{Nc-2}_{m,2} + \overset{c(s)}{\tilde K_2}\overleftarrow{C}^{Nc}_m + \overset{c(s)}{\hat K_2}\right\} \qquad 5.19$$

where

$$\overset{c(s)}{\beta_1} = S^{Nc-1}_{m,1}\tilde\Lambda^{Nc-1}_{m,t} W^{Nc-1}_m + i\left(h^{Nc}_m\right)^{-1}; \quad \overset{c(s)}{\beta_2} = \hat\Lambda^{Nc-1}_{m,t} + i S^{Nc-1}_{m,1}\left(h^{Nc}_m\right)^{-1} W^{Nc-1}_m \qquad 5.20$$

$$\overset{c(s)}{K_1} = S^{Nc-1}_{m,1}\tilde\Lambda^{Nc-1}_{m,s}\overrightarrow{W}^{Nc-2}_m; \quad \overset{c(s)}{\tilde K_1} = -2 \qquad 5.21$$

$$\overset{c(s)}{\hat K_1} = S^{Nc-1}_{m,1}\left(\tilde\Lambda^{Nc-1}_{m,s}\vec V^{Nc-2}_m - \tilde\Lambda^{Nc-1}_{m,t} V^{Nc-1}_m\right) + P^{Nc-1}_{m,z} \qquad 5.22$$

$$\overset{c(s)}{K_2} = \hat\Lambda^{Nc-1}_{m,s}\overrightarrow{W}^{Nc-2}_m; \quad \overset{c(s)}{\tilde K_2} = -2S^{Nc-1}_{m,1} \qquad 5.23$$

$$\overset{c(s)}{\hat K_2} = \hat\Lambda^{Nc-1}_{m,s}\vec V^{Nc-2}_m - iS^{Nc-1}_{m,1}\left(h^{Nc}_m\right)^{-1}V^{Nc-1}_m - P^{Nc-1}_{m,z} \qquad 5.24$$

From (5.18) and (5.19) we obtain the dependence of $\eta^{Nc-1}_{m,2}$ on $\vec C^1_m$ and $\overleftarrow{C}^{Nc}_m$.

$$\eta^{Nc-1}_{m,2} = \Theta_1^{Nc-1}\overset{c(s)}{X_1}\left\{K_1 A^{1,Nc-2}_m\Omega_m\vec C^1_m + \tilde K_1\overleftarrow{C}^{Nc}_m + K_1\left(A^{1,Nc-2}_m\tilde\Omega_m + \tilde B^{1,Nc-2}_m\right) + \hat K_1\right\} +$$

$$+\Theta_2^{Nc-1}\overset{c(s)}{X_2}\left\{K_2 A^{1,Nc-2}_m\Omega_m\vec C^1_m + \tilde K_2\overleftarrow{C}^{Nc}_m + K_2\left(A^{1,Nc-2}_m\tilde\Omega_m + \tilde B^{1,Nc-2}_m\right) + \hat K_2\right\} \qquad 5.25$$

where

$$\overset{c(s)}{X_1} = \left\{\beta_1 - K_1\left(A^{1,Nc-2}_m\hat\Omega_m + \hat B^{1,Nc-2}_m\right)\right\}^{-1} \qquad 5.26$$

$$\overset{c(s)}{X_2} = \left\{\beta_2 - K_2\left(A^{1,Nc-2}_m\hat\Omega_m + \hat B^{1,Nc-2}_m\right)\right\}^{-1} \qquad 5.27$$

To determine the fields we can first determine $\eta^1_{m,2}$ from (5.25) and (5.12), then from (5.9) obtain $\eta^M_{m,2}$ for arbitrary M. Now using the formulae (4.5) we determine $Y^N_m(\ell^N)$ and $Y^N_m(0)$ for the arbitrary N, which is equivalent to the determination of the quantities $X^N_m(0)$ and $X^N_m(\ell^N)$. At known X^N and Y^N from formulae for field expression we obtain the fields in an arbitrary point of structure.

Appendix

We consider the practical case in which the charged bunches move parallel to the axix of structure. We assume that the motion of bunch is determined by the rule:

$$\rho(r,\varphi,z,t) = \rho_0(r,\varphi,z-Vt); \quad \vec J = \rho_0\vec V \qquad 1A$$

where

$$\vec V = (0,0,V)$$

The solution of Maxwell equations in the absence of structure which corresponds to distribution (1A) has the form:

$$\vec{E}^{Q}=\vec{E}^{Q}(r,\varphi,z-Vt);\quad \vec{H}^{Q}=\vec{H}^{Q}(r,\varphi,z-t) \qquad 2A$$

Expanding E^{Q}_{z} and E^{Q}_{φ} in Fourier series in φ on the cylindrical wall and making the Fourier transformation of the time t we get:

$$E^{Q}_{z(\varphi)}(a^{N},\varphi,z,\tilde{k})=\sum_{m}\Big[\overset{c}{\xi}{}^{Q}_{z(\varphi),m}(a^{N},\tilde{k})\cos(m\varphi)+ + \overset{s}{\xi}{}^{Q}_{z(\varphi),m}(a^{N},\tilde{k})\sin(m\varphi)\Big]e^{\frac{i\tilde{k}}{\beta}z} \qquad 3A$$

where

$$\tilde{K}=\frac{\omega}{c};\qquad \beta=\frac{V}{c}$$

we find the solution of homogeneous equations by introducing the Hertz vectors:

$$\Phi^{E(H),N}=\sum_{m}I_{m}(gr)e^{\frac{i\tilde{k}}{\beta}z}\Big[\overset{c}{\vec{C}}{}^{E(H),N}_{m}\cos(m\varphi)+\overset{s}{\vec{C}}{}^{E(H),N}_{m}\sin(m\varphi)\Big] \qquad 4A$$

where $g=\tilde{k}/\gamma$; $\gamma=(1-\beta^{2})^{-1/2}$, Im – are the Bessel function of imaginary argument.

According to the formulae in the main text (1.1) and (1.2):

$$\left.\begin{aligned}
E^{E}_{z}&=g^{2}\sum_{m}I_{m}(gr)e^{\frac{i\tilde{k}}{\beta}z}\Big[\overset{c}{\vec{C}}{}^{E,N}_{m}\cos(m\varphi)+\overset{s}{\vec{C}}{}^{E,N}_{m}\sin(m\varphi)\Big]\\
E^{E}_{\varphi}&=\frac{i\tilde{k}}{r\beta}\sum_{m}I_{m}(gr)e^{\frac{i\tilde{k}}{\beta}z}\Big[\overset{s}{\vec{C}}{}^{E,N}_{m}\cos(m\varphi)-\overset{c}{\vec{C}}{}^{E,N}_{m}\sin(m\varphi)\Big]\\
E^{H}_{z}&=0\\
E^{H}_{\varphi}&=i\tilde{k}g\sum_{m}I'_{m}(gr)e^{\frac{i\tilde{k}}{\beta}z}\Big[\overset{c}{\vec{C}}{}^{H,N}_{m}\cos(m\varphi)+\overset{s}{\vec{C}}{}^{H,N}_{m}\sin(m\varphi)\Big]
\end{aligned}\right\} \quad 5A$$

Equating the total E_{φ} and E_{z} with nil on the cylindrical wall we obtain:

$$\overset{c(s)}{\vec{C}}{}^{E,N}_{m}=-\frac{\overset{c(s)}{\xi}{}^{Q}_{z,m}(a^{N},\tilde{k})}{g^{2}I_{m}(ga^{N})} \qquad 6A$$

$$\overset{c}{\vec{C}}{}^{H,N}_{m}=\frac{1}{gI'_{m}(ga^{N})}\left[\frac{\overset{s}{\xi}{}^{Q}_{z,m}(a^{N},\tilde{k})}{\beta a^{N}g^{2}}-\frac{\overset{c}{\xi}{}^{Q}_{\varphi,m}(a^{N},\tilde{k})}{iK}\right] \qquad 7A$$

$$\overset{s}{\vec{C}}{}^{H,N}_{m}=-\frac{1}{gI'_{m}(ga^{N})}\left[\frac{\overset{c}{\xi}{}^{Q}_{z,m}(a^{N},\tilde{k})}{\beta a^{N}g^{2}}+\frac{\overset{s}{\xi}{}^{Q}_{\varphi,m}(a^{N},\tilde{k})}{iK}\right] \qquad 8A$$

At $V\Rightarrow C$ the radial functions Rm(r) are the solution of equation:

$$R''+\frac{R'}{r}-\frac{m^{2}R}{r^{2}}=0 \qquad 9A$$

The finite solutions at finite r are:

$$R_{m}(r)=C_{m}r^{m} \qquad 10A$$

These functions we obtain also in the way:

$$\lim_{g\to 0}\frac{I_{m}(gr)}{g^{m}}=C_{m}r^{m} \qquad 11A$$

Here Cm are constants. Now we can write:

$$\left(\overset{c(s)}{\vec{C}}{}^{H,N}_{m}\right)_{v=c} = -\frac{\overset{c(s)}{\mathcal{E}}_{\varphi,m}(a^N,\tilde{k})}{m\, r^{m-1}\, ik} \qquad 12A$$

In conclusion we give an example $\overset{c(s)}{\mathcal{E}}_{\varphi,m}(a^N,\tilde{k})$ for the more practical case at which point-like bunches move at $v \Rightarrow c$ parallel to the axis at the small distance δr^Q in the plane $\varphi = \varphi^Q$ (in linear approximation to $\delta r^Q/a^N$):

$$\overset{c}{\mathcal{E}}{}^{Q}_{\varphi,1} = \frac{2eN_B}{c(a^N)^2}\,\delta r^Q \cos\varphi^Q ; \quad \overset{s}{\mathcal{E}}{}^{Q}_{\varphi,1} = \frac{2eN_B}{c(a^N)^2}\,\delta r^Q \sin\varphi^Q \qquad 13A$$

In this approximation $\overset{c(s)}{\mathcal{E}}{}^{Q}_{\varphi,m} = 0$ for all $m \neq 1$.

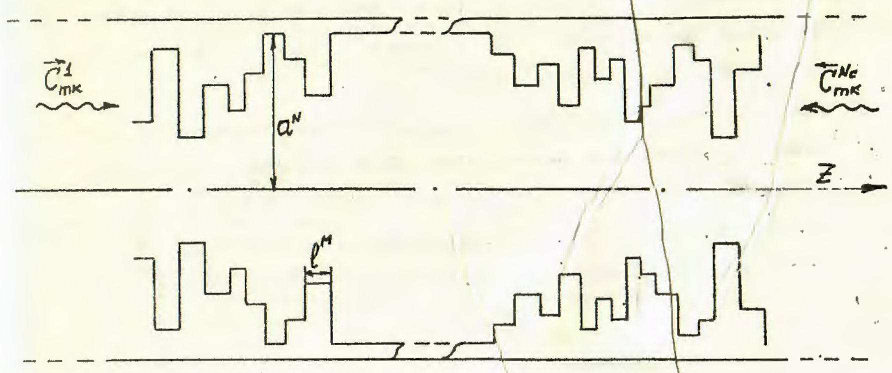

Рис.I Общий вид осевого разреза структуры и некоторые обозначения (размеры произвольны).

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Նախատիպ ԵրՖԻ-1426(13)-94

ԳԼԱՆԱՁԵՎ ԲԱՆԱՁԵՎԵՐ ՈՉ ՊԱՐԲԵՐԱԿԱՆ ԿԱՌՈՒՑՎԱԾՔՆԵՐՈՒՄ ԴԱՇՏԵՐԻ ԿԻՍԱԱՆԱԼԻՏԻԿ ՀԱՇՎԱՐԿՄԱՆ ՀԱՄԱՐ

Վ.Գ. ՀԱՄԲԱՐՁՈՒՄՅԱՆ

Դուրս են բերված դաշտերի հաշվարկի բանաձևեր ոչ պարբերական գլանաձև ալիքատարում։

Հաշվի են առնված նաև համակարգի առանցքին զուգահեռ $v \to c$ արագությամբ շարժվող արագացվող փնջով գրգռված դաշտերը։

Բանաձևերը ստացված են դաշտերի կարման եղանակով և արտահայտված են 'անցումային' մատրիցաների լեզվով։

Երևանի Ֆիզիկայի Ինստիտուտ
Երևան - 1994

Препринт ЕрФИ-1426(13)-94

ФОРМУЛЫ ДЛЯ ПОЛУАНАЛИТИЧЕСКИХ РАСЧЕТОВ ПОЛЕЙ В ЦИЛИНДРИЧЕСКИХ АПЕРИОДИЧЕСКИХ СТРУКТУРАХ

В.Г. Амбарцумян

Выведены формулы для расчета полей в апериодическом цилиндрическом волноводе.

Учтены также поля возбуждаемые пучком частиц движущихся паралельно оси структуры со скоростью $v \to c$.

Формулы получены методом сшивки полей и выражены на языке матриц "переходов".

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