Корректная постановка задачи Коши в ковариантных теориях

Амбарцумян В. Г.

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Технический редактор А. С. Абрамян

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V. G. AMBARTSUMIAN

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V.G. AMBARTSUMIAN

ON THE CORRECT FORMULATION OF THE CAUCHY PROBLEM IN COVARIANT THEORIES. THE COORDINATE SYSTEM

It was demonstrated that the usual indefinity of the Einstein equations'solutions was due to the incorrect formulation of the Cauchy problem. We could formulate the correct problem by means of the obvious introduction of the coordinate system as the physical system.

Yerevan Physics Institute

В. Г. АМБАРЦУМЯН

КОРРЕКТНАЯ ПОСТАНОВКА ЗАДАЧИ КОШИ В КОВАРИАНТНЫХ ТЕОРИЯХ

Показано, что обычная неоднозначность в решениях уравнений Эйнштейна возникает из-за некорректной формулировки задачи Коши. Мы показали также, что возможно корректно сформулировать задачу Коши, если прямо ввести координатную систему как физическую систему.

Ереванский физический институт

Earlier the authors of the "Relativistic Theory of Gravity" [1,2] contended that the comparisons of Einstein's theory of general relativity (TGR) with the experimental data are illegitimate, because Einstein's equations have not unequivocal solution, and it is impossible to formulate the correct Cauchy problem. In our work we demonstrate that this indefinity isn't the specific characteristic of Einstein's theory but is the feature of any covariant theory. We also demonstrate that it relates with the incorrect introduction of coordinate system and also with the incorrect formulation of the Cauchy problem.

1. The standard formulation of the Cauchy problem. Why is it incorrect ?

We shall begin from the standard formulation of the Cauchy problem in Einstein's TGR. The equations for the gravitational field and other substances in TGR are:

$$G_{ik}=\frac{8\pi K}{c^4}T_{ik} \tag{1}$$

$$\frac{\delta L_s}{\delta Q_i}=0 \tag{2}$$

L_s is the Lagrangian of the substance here, Q_i are variables which describe the physical state of the substance. the standard formulation of the Cauchy problem is: to determine the solution of [1,2] system equations, which satisfies the following conditions in any coordinate system at the moment of the time $t=0$:

$$g_{ik}(0,\vec{r})=\bar{g}_{ik}(\vec{r});\quad \dot{g}_{ik}(0,\vec{r})=\bar{\dot{g}}_{ik}(\vec{r}) \tag{3}$$

$$Q_i(0,\vec{r})=\bar{Q}_i(\vec{r});\quad \dot{Q}_i(0,\vec{r})=\bar{\dot{Q}}_i(\vec{r}) \tag{4}$$

The points over the quantities mark the derivation with respect to the time.

We assume that the system (1÷4) has a solution and we shall discuss only the problem of ambiguity. Here we follow the work [3]. If we have any arbitrary solution of the (1÷4) we can make the coordinate transformation (GL(4)), which doesn't change the conditions (3,4), but it isn't the unity transformation in general. Because the system (1,2) is invariant with the respect to that kind of transformation we shall find the new solution of the problem (1÷4). Thus, the problem (1÷4) has not the unequivocal solution. In the case of gravitational field absence the system (1÷4) will turn into the system of equations:

$$R_{iklm}=0 \qquad (5)$$

$$\frac{\delta L_g}{\delta Q_i}=0 \qquad (6)$$

$$\eta_{ik}(0,\vec{r})=\bar{\eta}_{ik}(\vec{r});\quad \dot{\eta}_{ik}(0,\vec{r})=\bar{\dot{\eta}}_{ik}(\vec{r}) \qquad (7)$$

$$Q_i(0,\vec{r})=\bar{Q}_i(\vec{r});\quad \dot{Q}_i(0,\vec{r})=\bar{\dot{Q}}_i(\vec{r}) \qquad (8)$$

Here η_{ik} is the metric tensor of the flat space-time. Repeating the reasonings which we adduced above we come to the conclusion that our assertion is true for the arbitrary covariant theory. We assume that the reason of this kind of uncertainty of the solution is the incorrect formulation of the Cauchy problem, because that formulation contains nothing about the means of coordinate system introduction. The coordinate system must be related to the certain physical process by means of which each point of the space-time must be related to the certain physical event. We must describe any physical system by the system of equations (1,2) or (5,6) and for it we must give the initial data like (4) or (8).

We can introduce this physical system by the large number of bodies (coordinate bodies which fill the 3-space) with any set of related coordinates- $\vec{R}^0$ and with the standard clocks marking their own time - τ. If these coordinate bodies don't collide, the variables $(\vec{R}^0,\tau)$ will describe any coordinate system $\mathcal{K}$. We can introduce an arbitrary coordinate system $K(\vec{r},t)$ in which the initial data (3,4) or (7,8) are given by the functions:

$$\vec{r}=\vec{f}_{\vec{r}}(\vec{R}^0,\tau);\qquad t=f_t(\vec{R}^0,\tau) \qquad (9)$$

We can write the development equations and the initial conditions like (2,4) or (6,8) for the coordinate bodies:

$$\frac{\delta L^{\vec{R}^0}}{\delta q_i^{\vec{R}^0}}=0 \qquad (10)$$

$$q_i^{\vec{R}^0}(\vec{r},0)=\bar{q}_i^{\vec{R}^0}(\vec{r});\quad \dot{q}_i^{\vec{R}^0}(\vec{r},0)=\bar{\dot{q}}_i^{\vec{R}^0}(\vec{r}) \qquad (11)$$

Here $L^{\vec{R}^0}$ is the Lagrangian of the $\vec{R}^0$- coordinate body and $q_i^{\vec{R}^0}$ are the quantities which describe the physical state of this body. Now we can introduce the correct formulation of the Cauchy problem; to determine the solution of the system equations (1,2) or (5,6) which in coordinate system $K(\vec{r},t)$ (that is described by (9,10,11))at the momenium of time $t=0$ satisfies the initial conditions (3,4) or (7,8). The solution of this problem we shall find in following way. If $\tilde{\Psi}\equiv(\tilde{g}_{ik}(\tilde{\vec{r}},\tilde{t}), \tilde{Q}_i(\tilde{\vec{r}},\tilde{t}), \tilde{q}_i^{\vec{R}^0}(\tilde{\vec{r}},\tilde{t}))$ is the solution of the problem (1÷4,10,11) or (5÷8,10,11) in any arbitrary coordinate system $\tilde{K}(\tilde{\vec{r}},\tilde{t})$ we have the world-line $\tilde{\vec{r}}=\vec{\varphi}(\vec{R}^0,\tilde{t})$ of the arbitrary coordinate body -$\vec{R}^0$ in this system, and so, we have relations which connect the variables $(\tilde{\vec{r}},\tilde{t})$ with the variables $(\vec{R}^0,\tau)$:

$$\vec{R}^{\circ} = \vec{R}^{\circ}(\tilde{\vec{r}}, \tilde{t}) \qquad (12)$$

$$\tau(\tilde{\vec{r}}, \tilde{t}) = \int dS(\vec{R}^{\circ}) \qquad (13)$$

We can transform the solution $\tilde{\varphi}$ from the system $\tilde{K}(\tilde{\vec{r}}, \tilde{t})$ into the system $K(\vec{R}^{\circ}, \tau)$ using the relations (12,13). Now making the transformation from $K(\vec{R}^{\circ}, \tau)$ into the $K(\vec{r}, t)$ according to (9) we shall find the unequivocal solution of the Cauchy problem. It is not difficult to verify that this solution does not depend on the choice of coordinate system $\tilde{K}(\tilde{\vec{r}}, \tilde{t})$. Thus, the absence of unequivocal solution in the standard formulation of the Cauchy problem is due to the incorrect introduction of the coordinate system.

Thus, we have shown that we can give the coorect formulation of the Cauchy problem for any covariant theory if we introduce correctly the coordinate system as the certain physical system.

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Վ.Գ. ՀԱՄԲԱՐՁՈՒՄՅԱՆ

ԿՈՇԻ ԽՆԴՐԻ ԿՈՌԵԿՏ ՁԵՎԱԿԵՐՊՈՒՄԸ ԿՈՎԱՐԻԱՆՏ ՏԵՍՈՒԹՅԱՆ ՄԵՋ

Ցույց է տրված, որ Էյնշտեյնի հավասարումների լուծումների սովորական ոչ միարժեքությունը առաջանում է Կոշի խնդրի ոչ կոռեկտ ձևակերպման պատճառով: Ցույց է տրված նաև, որ Կոշի խնդիրը կարելի է ձևակերպել կոռեկտ, եթե կոորդինատական համակարգը մտցվի ուղղակիորեն որպես ֆիզիկական համակարգ:

Երևանի ֆիզիկայի ինստիտուտ