Зависимость поведения KS-энтропии от параметров
нелинейной системы
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(на англ. языке)
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ON THE BEHAVIOUR OF KS-ENTROPY DEPENDING ON PARAMETRES OF NONLINEAR SYSTEMS

A.A.Melkonian

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Նախնատիպ ԵրՖԻ-1452(22)-95

KS-ԷՆՏՐՈՊԻԱՅԻ ՎԱՐՔԻ ԿԱԽՈՒՄԸ ՈՉ ԳԾԱՅԻՆ ՀԱՄԱԿԱՐԳԻ՝ ՊԱՐԱՄԵՏՐԵՐԻՑ

Ա.Ա. Մելքոնյան

Աշխատանքում ներկայացված են Կոլմոգորով-Սինայի էնտրոպիայի հաշվարկները ոչ գծային Φ^4 համիլտոնյան համակարգի համար։ Գտնված է KS-էնտրոպիայի կախումը համակարգի պարամետրերից։

Երևանի ֆիզիկայի ինստիտուտ

ON THE BEHAVIOUR OF KS-ENTROPY DEPENDING ON PARAMETRES OF NONLINEAR SYSTEMS

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The Kolmogorov-Sinai entropy is calculated for the Φ^4 nonlinear Hamiltonian system. The dependence of KS-entropy on the parameters of the system is revealed.

1 Introduction

The aim of the present study is the looking for the dependence of Kolmogorov-Sinai (KS) entropy on the fixed parameters of nonlinear Hamiltonian systems. We carry out this analysis on the example of ϕ^4 models having essential role in field theory and the theory of condensed matter and are known as objects with rich variety of statistical phenomena [1]. These systems are also of particular interest for N-body gravitating systems being rather important for the dynamics of galaxies and star clusters [2].

The subject of our study is the Hamiltonian describing a 1-dimensional system, chain of two-body interacting particles

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + \sum_{i=1}^{N} (Ax_i^4 - Bx_i^2 + \frac{B^2}{4A}) + \frac{C}{2} \sum_{i=1}^{N} (x_{i+1} - x_i)^2.$$

where $A, B, C > 0$ are constants. In [3] the chaotic properties of the system have been studied by means of the calculation of KS-entropy for various values of total energy of the system (integral of the system) and parameter C, while the values of the rest two parameters were fixed: $A, B = 1$. For this case the character of chaotic properties have been studied, namely the KS-entropy and Lyapunov exponents were obtained for several given number of particles N, and hence some general features of the system were revealed. In view of the interesting results obtained in [3], in the present paper we continue the study of the chaotic properties of this system: we are interested on revealing of the the dependence of KS-entropy on the values of the parameters A, B, the relevance of the results to those found in beforementioned paper.

As is well known Lyapunov numbers

$$\lambda = \lim_{t\to\infty} \frac{\ln|\delta x(t)|}{t}$$

are related to *Kolmogorov – Sinai* (KS) entropy via Pesin formula

$$h = \int \sum_{\lambda_i(x)>0} \lambda_i(x) d\mu(x)$$

and are an informative criteria determining the chaotic and regular properties of the dynamical systems. Particularly the integrable systems have zero Lyapunov numbers and hence zero KS-entropy, while the typical systems with mixing have positive KS-entropy and hence at least one non-zero Lyapunov number. However analytical threatment of the non-linear many-dimensional systems as a rule is related with extreme difficulties and numerical studies remain the main tool if not for rigid proofs but at least for revealing highly probable qualitative properties. The method of numerical calculation of Lyapunov exponents has been developed in [4] and later used by many authors. The same method we should use below. At the same time one should clearly understand the principal difficulty of the numerical calculation of Lyapunov exponents - the quantities which are non-local charactersitics of the system and describe the latter in infinite time. Therefore this condition should be carefully fitted on one hand with long enough calulations to reach plataeu-like limits during reasonable time scale, on the other hand with the accumulation of errors during the iterations procedure.

2 Method.

The algorithm of numerical calculations of Lyapunov exponents is described in details in [4,5], therefore we should briefly mention it's main idea only.

First, one has to choose a base of orthonormalized vectors $w_p(t)$ determining the p-dimensional volume element $V_p(t)$, in order to calculate

$$\lambda(x_0, V_p) = \lim_{t\to\infty} \frac{\ln(V_p(x_0,t)/V_p(x_0,0))}{t}$$

The aim is the calculation of the spectrum of Lyapunov numbers for $p = 1, 2, \ldots N$:

$$\lambda^{(p)} = \lambda_1 + \lambda_2 + \ldots + \lambda_p.$$

However this direct procedure only should shortly lead to meaningless results, since during the evolution of the system the angles between vectors exponentially should change, the ortogonality should fail and as a result numerical errors should increase exponentially as well. A way to overcome this difficulty introduced in [4] requires: a) periodic renormalization of vectors; b) periodic orthogonalization. Note, that the new vectors should be situated in the same subset as the old ones. The procedure of the calculation of the evolution of p-dimensional volumes can be realized by means of looking for the evolution of vectors and their orthogonalization via Gram-Schmidt method. If $w_{k-1}(\tau)$ is the evolved tangent vector $w_k(0)$ along the trajectory from $t = (k-1)\tau$ to $t = k\tau$, then one has to calculate first

$$d_k^{(1)} = \|w_{k-1}^{(1)}(\tau)\|,$$
$$w_k^{(1)}(0) = \frac{w_{k-1}^{(1)}(\tau)}{d_k^{(1)}}.$$

By the second step one finds out the values for j=2,...,N:

$$u_{k-1}^{(j)}(\tau) = w_{k-1}^{(j)}(\tau) - \sum_{i=1}^{j-1}[w_k^{(i)}(0)\cdot w_{k-1}^{(j)}(\tau)]w_k^{(i)}(0),$$
$$d_k^{(j)} = \|u_{k-1}^{(j)}(\tau)\|,$$
$$w_k^{(j)}(0) = \frac{u_{k-1}^{(j)}(\tau)}{d_k^{(j)}}.$$

During the $(k-1)$st iterations the volume V_p is being increased by factor $d_k^{(1)}d_k^{(2)}\ldots d_k^{(p)}$. Then by definition

$$\lambda_1^{(p)} = \lim_{n\to\infty}\frac{1}{n\tau}\sum_{i=1}^{n}\ln(d_i^{(1)}d_i^{(2)}\ldots d_i^{(p)}).$$

Substracting $\lambda_1^{(p-1)}$ from $\lambda_1^{(p)}$ and taking into account that $\lambda^{(p)} = \sum_{i=1}^{p}\lambda_i$ one thus finds the p-th Lyapunov number:

$$\lambda_p = \lim_{n\to\infty}\frac{1}{n\tau}\sum_{i=1}^{n}\ln d_i^{(p)}.$$

Lyapunov exponents have been calculated by this expression, whereas the Pesin formula has been used to obtain the KS-entropy.

3 Results of Numerical Experiments.

The results of calculations are represented in Figures 1-4; here we give the minimal number of Figures indicating the main properties of the behaviour of KS-entropy, though extensive calculations have been perfomed for a number of other parameters of the Hamiltonian. Similar robust behaviour of this system at least with respect to the number of particles, has been noticed also during the analysis in [3]. Figures 1(a-c) show the dependence of KS-entropy on the constant A for three various number of particles: $N = 3, 5, 7$; one can see that the qualitatively the behaviour of KS-entropy is really less sensitive to N. Figure 2 indicates the dependence of KS-entropy on the constant B; we give only the case $N = 3$ though this behaviour is the same for other values of N as well. On Figure 3(a,b), the dependence of entropy on the constant C is excibited for $N = 3$ and 7, respectively. In Figure 4 the variation of KS-entropy over energy is shown for fixed values of constants of A and B: $A = 0.5, A = 0.1, C = 1$; similar behaviour fulfills for other values as well.

What are the main conclusions following from these results? First, of all one notes the maximum of KS-entropy is reached at values $A = 0.5$ and $B = 0.5$, while decreasing for both limiting cases: A and B tending to zero and infinity. If the qualitatively this fact is natural, the existence of one and the same maximum value for both cases seems remarkable. Second, one can see that the maximal chaotic state, i.e. with maximal KS-entropy, corresponds to $A = B = 0.5$ and not, say to $A = B = 1$, the case considered in [3].

On the other hand the one-extremum character of dependence of KS-entropy on A and B is quite similar as on C when $A = B = 0.5$ as obtained in [refMB]. This fact seems again remarkable thus indicating the similar contribution in chaotic properties of two quite different members of the Hamiltonian. Their difference is appearing mainly in the value of energy - the bigger ones required for C-dependence.

The study of nonlinear systems by means of similar numerical methods can be helpful in revealing the profound statistical properties of corresponding physical systems and related physical phenomena, though the limitations introduced by the calculation procedure should be evidently taken into account.

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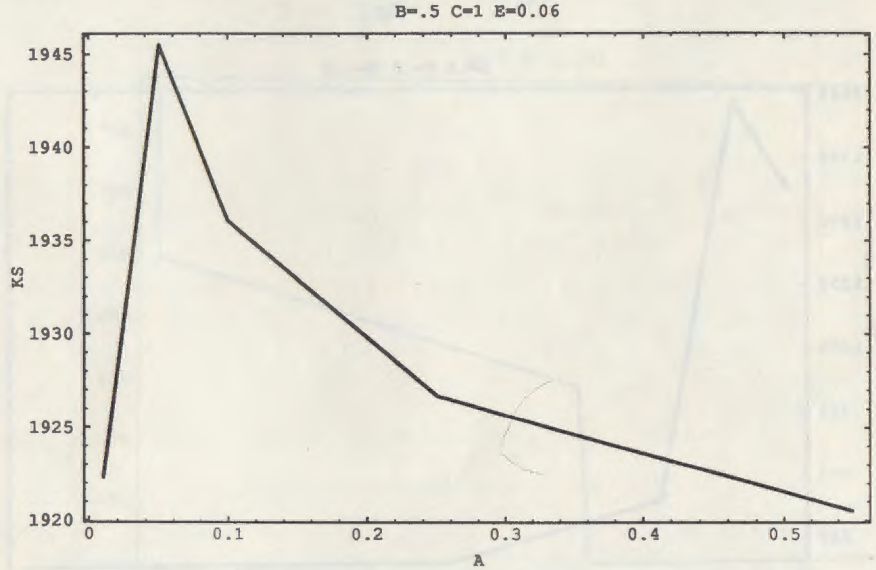

Fig.1

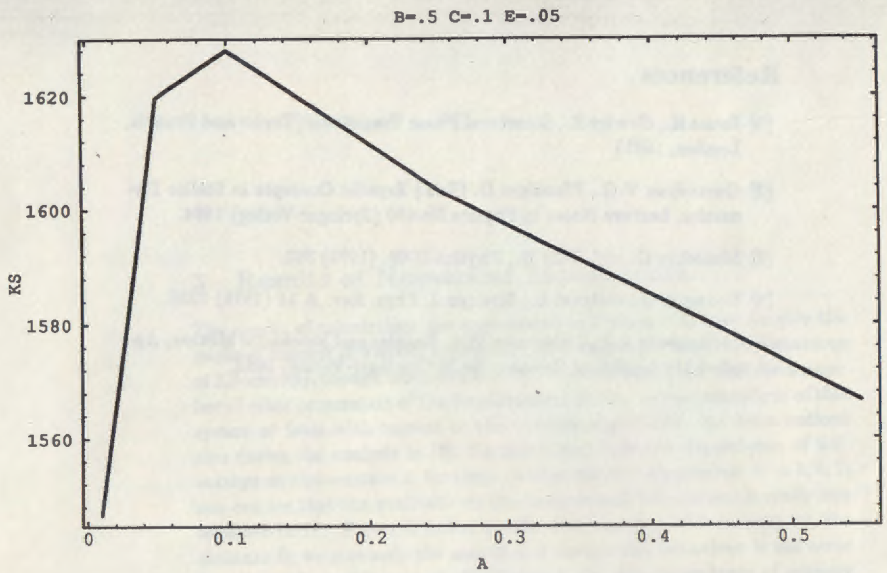

Fig.2

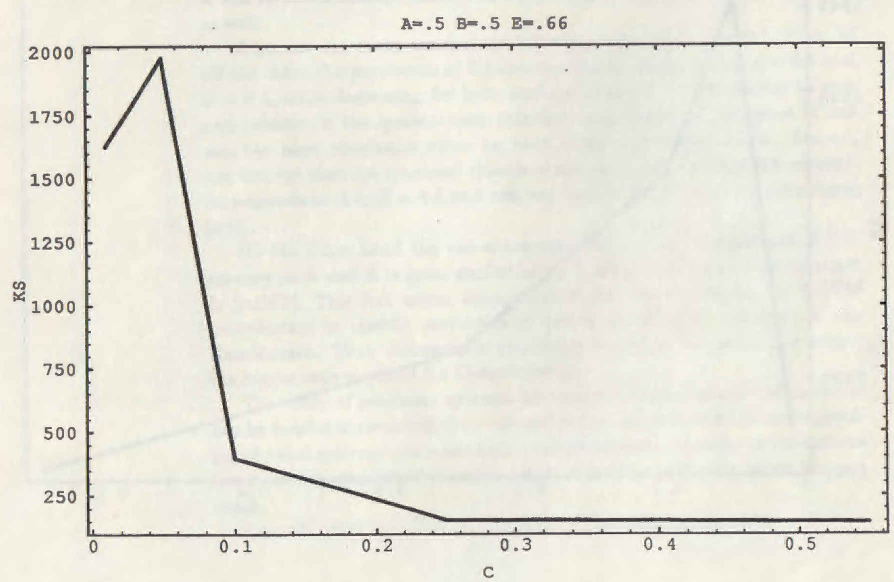

Fig.3

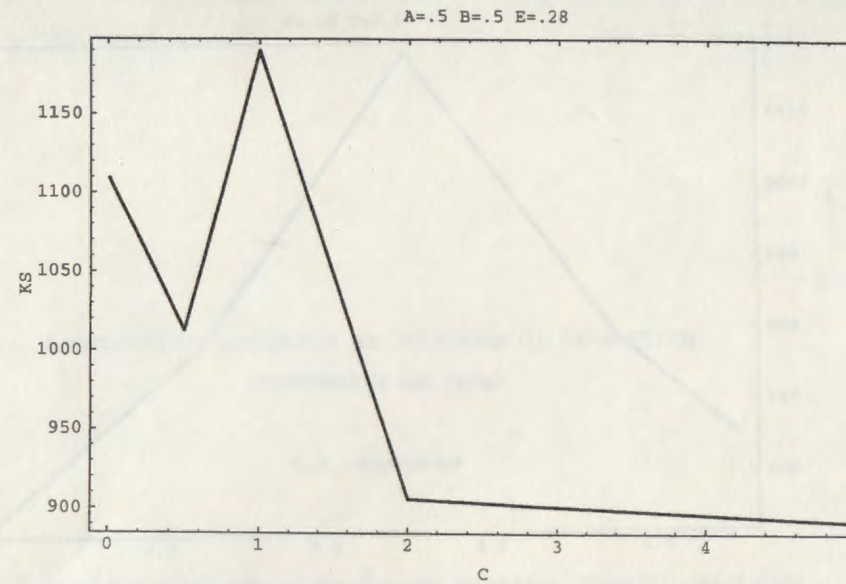

Fig.4

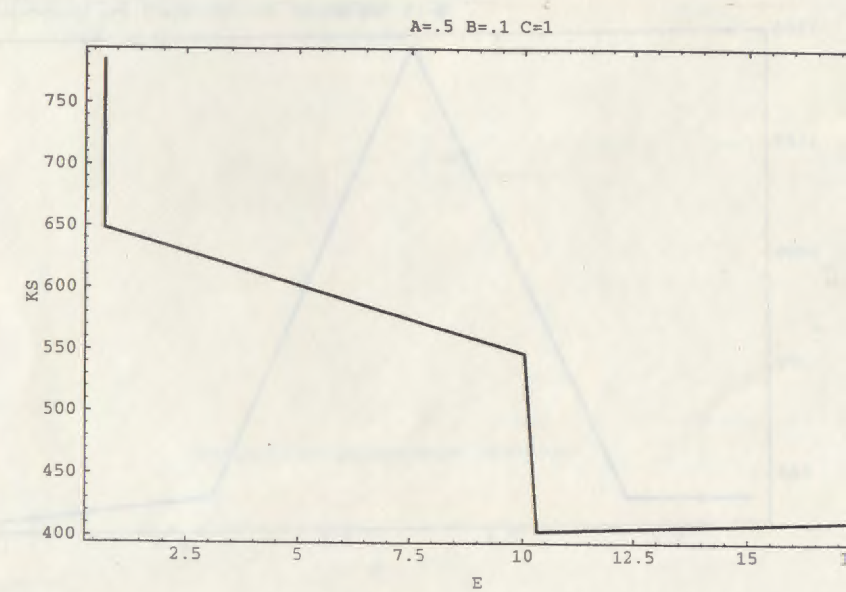

Fig.5

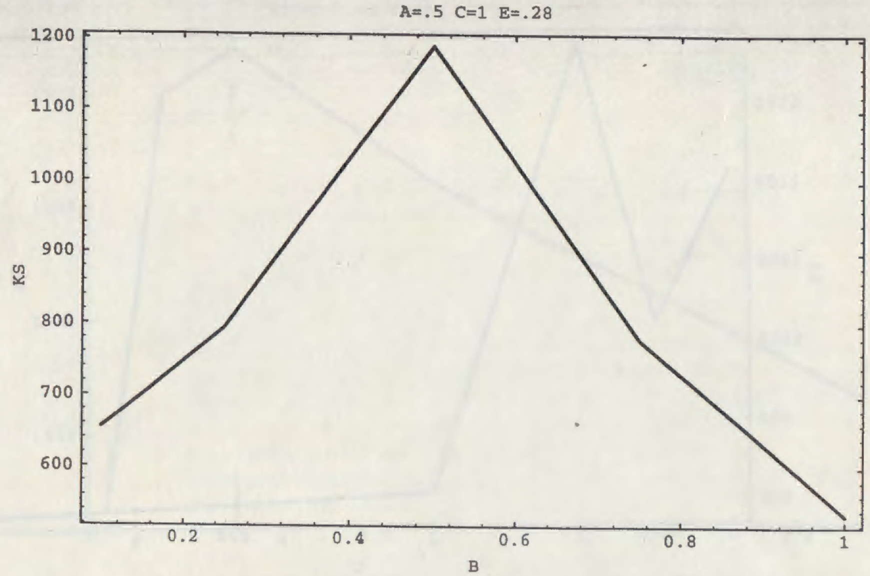

Fig.6

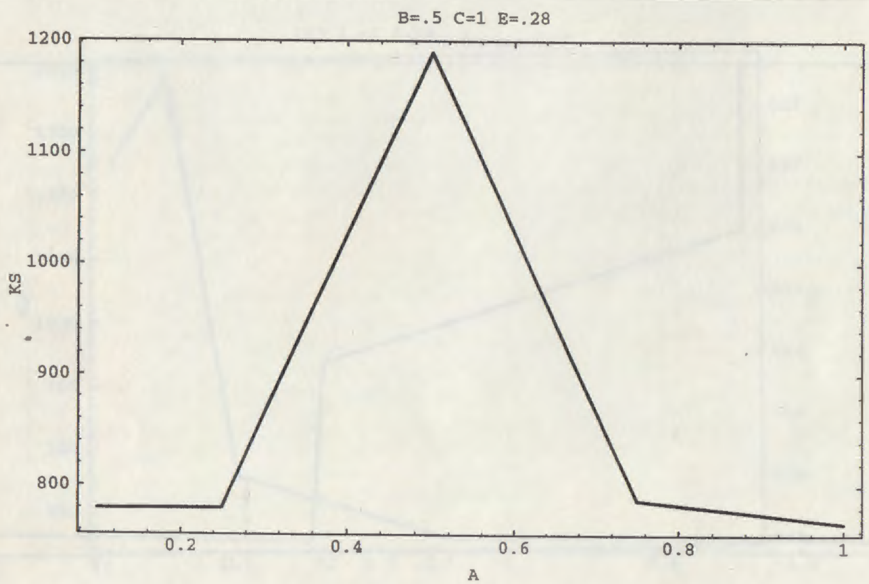

Fig.7

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ЗАВИСИМОСТЬ ПОВЕДЕНИЯ KS-ЭНТРОПИИ ОТ ПАРАМЕТРОВ НЕЛИНЕЙНОЙ СИСТЕМЫ

А. А. Мелконян

В работе представлены вычисления энтропии Колмогорова-Синая для нелинейной гамильтоновой системы Φ^4. Найдена зависимость KS-энтропии от параметров системы.

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