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G.M. Asatryan, A.N. Zaslavsky

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Г.М.АСАТРЯН, А.Н.ЗАСЛАВСКИЙ

ОБ АЛГЕБРАИЧЕСКОЙ РЕАЛИЗАЦИИ СИММЕТРИИ SU(4)

Обсуждается возможность нелинейной реализации симметрии SU (4) с линеаризацией на группе $SU(2)\times Y\times C$. Алгебраические свойства SU (4) восстанавливаются из условия Вайнберга: амплитуды рассеяния голдстоунов на частицах должны иметь разумное (как в теории Редже) асимптотическое поведение. При этом нарушение оказывается минимальным $m^2=m^2_{inv}+m^2_8+m^2_{15}$. Большие значения масс ψ -мезонов приводят в алгебраической реализации SU (4) к высоко лежащим очарованным траекториям D^* и S^* $0<\alpha^{(0)}_{D^*,S^*}<1$.

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G.M.ASATRYAN, A.N.ZASLAVSKY

ON THE ALGEBRAIC REALIZATION OF SU(4) SYMMETRY *)

The possibility of non-linear realization of SU(4) symmetry with its linearization over the SU(2)× Y×C group is discussed. The algebraic properties of the SU(4) group are restored from Weinberg's condition-the amplitudes of the scattering of goldstones on particles must have reasonable asymptotic behaviour (as in Regge theory), the violation being proved to be minimal $m^2 = m^2_{inv} + m^2_8 + m^2_{15}$. The large values of ψ-mesons mass lead in the algebraic realization of the SU(4) symmetry to high lying $\mathcal{D}^*$ and S^* charmed trajectories $0 < \alpha^{(0)}_{\mathcal{D}^*, S^*} < 1$.

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1. The SU(4) group discussed in connection with the interpretation of ψ-mesons [1] is a strongly violated symmetry. The particle mass differences in the vector 16-plet of the SU(4) is of the order of particle masses and the small violation parameter doesn't occur. The assumption of the minimality of SU(4) violation in the mass operator

$$m^2 = m^2_{inv} + m^2_8 + m^2_{15} \tag{1}$$

is considerably worse substantiated here than in the SU(3) case.

In the present work the SU(4) group is considered as a dynamical non-linear symmetry with the linearization over its subgroup $SU(2)\times Y \times C$. The Weinberg's condition [2] about the correct asymptotic behaviour of tree graphs allows one to obtain the relation (1) in the framework of exact dynamical symmetry without the assumption of violation smallness. Large values of ψ-meson masses are compatible with Weinberg's condition on the amplitude asymptotics if charm $C=\pm 1$ trajectories have the $0<\alpha_{D^*,S^*}(0)<1$, i.e. they are gently slanting with $\alpha'_{D^*,S^*} \lesssim 0.2 - 0.25\ (\mathrm{GeV})^{-2}$ slope.

2. Let us consider the non-linear realization of $\frac{SU(4)}{SU(2)\times Y\times C}$. According to the general theory of non-linear realizations [3], it is necessary to introduce the scalar goldstone fields ξ_α with the quantum numbers of non-conserving group generators. Since only the subgroup $SU(2)\times Y\times C$ has algebraic inferences of conservation law type, the particles are classified by the representations of this subgroup. The SU(4) being on the whole a dynamical symmetry, its other conclusions have no algebraic character and don't lead to the

coupling type relations between the processes with definite number of particles. The invariant Lagrangian is constructed from the Ψ fields transforming according to the $SU(2) \times Y \times C$ representations, their covariant derivatives, as well as the covariant derivatives of ξ_a fields, which with an accuracy to the third order terms in ξ_a are:

$$\mathcal{D}_\mu \; _a = \partial_\mu \xi_a + i f_{abc} \xi_b \partial_\mu \xi_c + O(\xi^3), \quad i = 1,2,3,8,15, \qquad (2)$$

$$\mathcal{D}_\mu \Psi = \partial_\mu \Psi + 2i f_{abi} \xi_a \partial_\mu \xi_b T_i \Psi + O(\xi^3), \quad a,b = 4,\dots 7, 9, \dots 14, \qquad (3)$$

where T_i are the generators of $SU(2) \times Y \times C$ subgroup, f_{abi}, f_{abc} are the structure constants of SU(4) group. The hypercharge and charm operators are expressed via T_8 and T_{15} as

$$Y = \frac{2}{\sqrt{3}} T_8 + \frac{1 - 2\sqrt{6}\, T_{15}}{12}; \quad C = \frac{1 - 2\sqrt{6}\, T_{15}}{4}. \qquad (4)$$

Weinberg assumed that the algebraic aspects of a dynamical symmetry restore providing the amplitudes of goldstone scattering from particles in the tree approximation have reasonable asymptotic behaviour [2] (as in the Regge pole theory). Though each tree graph (Figs 1,2) contributing to this process lead to the inappropriate growth of amplitudes with energy, the contributions from different graphs ought to cancel each other. Specifically, in the $SU(2) \times SU(2)$ and SU(3) symmetries this requirement leads to the restoring of particle classification according to their representations [4], as well as to the limitations on the mass operator form.

3. We shall show in what follows that the analogous procedure may be carried out for the non-linear realization of SU(4) symmetry. Let us consider the process of forward scattering of massless goldstone ξ_a on a particle $\xi_a + \alpha \to \xi_b + \beta$

and define the momenta and helicities of α and β particles as P_μ, λ and P'_μ, λ' respectively; and let q_μ, q'_μ be the momenta of the initial and final goldstones. According to Ref. 2, we have chosen the reference frame where all the momenta are parallel. In this frame following relations take place

$$q_\mu = n_\mu \omega, \quad q'_\mu = n_\mu \omega', \quad S = -(P+q)^2 = m_\alpha^2 + 2\mathcal{E}\omega,$$
$$\vec{P} = -\vec{n}|\vec{P}|, \; \vec{P}' = -\vec{n}|\vec{P}'|, \; u = -(P'-q)^2 = m_\beta^2 - 2\mathcal{E}\omega, \tag{5}$$
$$\omega' = \omega + \frac{m_\alpha^2 - m_\beta^2}{2\mathcal{E}}, \quad \lambda = \lambda',$$

where $|\vec{n}| = |n_0| = 1$, ω and ω' are the initial and final energies of the goldstone in this frame, $\mathcal{E} = |\vec{P}| + P^0 = |\vec{P}'| + P'^0$. It follows from the crossing symmetry that the amplitude $M_{\beta b,\alpha a}(\omega,\lambda)$ satisfies the relation

$$M_{\beta b,\alpha a}(\omega,\lambda) = M_{\beta a,\alpha b}(-\omega',\lambda). \tag{6}$$

To analyze the asymptotical behaviour of tree graphs, it is convenient to introduce symmetrical and antisymmetrical (in a, b) amplitudes

$$M^{+}_{\beta b,\alpha a}(\omega,\lambda) = \frac{1}{2}\left[M_{\beta b,\alpha a}(\omega,\lambda) + M_{\beta b,\alpha a}(-\omega',\lambda)\right] \tag{7}$$

$$M^{-}_{\beta b,\alpha a}(\omega,\lambda) = (\omega+\omega')^{-1}\left[M_{\beta b,\alpha a}(\omega,\lambda) + M_{\beta b,\alpha a}(-\omega',\lambda)\right]. \tag{8}$$

The scattering amplitude in the tree approximation could be written as

$$M_{\beta b,\alpha a}(\omega,\lambda) = P_{\beta b,\alpha a}(\omega,\lambda) + \sum_\gamma [(m_\alpha^2 - m_\gamma^2 + 2\omega\mathcal{E})^{-1} Q^\gamma_{\beta b,\alpha a}(\omega,\lambda) + \\ + (m_\alpha^2 - m_\gamma^2 - 2\omega'\mathcal{E})^{-1} Q^\gamma_{\beta b,\alpha a}(-\omega',\lambda)], \tag{9}$$

where $P_{\beta b,\alpha a}(\omega,\lambda)$ corresponds to 1a,b graphs, while the rest of the amplitude corresponds to 2a,b graphs where the γ-particle exchange in S and u-channels takes place. The Lagrangian contains the minimal interaction of the type

$$2 V_i^{\mu} f_{iab} \xi_a \partial_\mu \xi_b + V_a^{\mu} \mathcal{D}_\mu \xi_a , \qquad (10)$$

where V_i^{μ} is a conserving current ($T_i = \int V_i^0 \, d^3x$), and V_a^{μ} is a phenomenological current changing the charm and (or) the strangeness. Then the amplitude reduces to the form:

$$\begin{aligned} M_{\beta b,\alpha a}(\omega,\lambda) = {} & \omega\omega' \widetilde{P}_{\beta b,\alpha a}(\omega,\lambda) + 4i\,\varepsilon(\omega+\omega')\left(f_{abi}T_i + f_{abc}X_c(\lambda)\right)_{\beta\alpha} + \\ & + \omega\omega' \sum_\gamma \Big[(m_\alpha^2 - m_\gamma^2 + 2\omega\varepsilon)^{-1} \widetilde{Q}^{\gamma}_{\beta b,\alpha a}(\omega,\lambda) + \\ & + (m_\alpha^2 - m_\gamma^2 - 2\omega'\varepsilon)^{-1} \widetilde{Q}^{\gamma}_{\beta b,\alpha a}(-\omega',\lambda) \Big] , \end{aligned} \qquad (11)$$

where the $\widetilde{P}_{\beta b,\alpha a}(\omega,\lambda)$ polinomial corresponds to non-minimal interactions and $X_a(\lambda)$ matrices are defined as follows [2] :

$$\eta_\mu \langle \vec{P}', \lambda', \beta | V_a^{\mu} | \vec{P}, \lambda, \alpha \rangle = \frac{4\varepsilon \, [X_a(\lambda)]_{\beta\alpha}}{(2\pi)^3 (4p^0 p'^0)^{1/2}} \, \delta_{\lambda\lambda'} \qquad (12)$$

$a = 4, \dots 7, 9, \dots 14$.

Let us decompose the amplitude into Laurent series in $1/\omega$:

$$\begin{aligned} M^{\pm}_{\beta b,\alpha a}(\omega,\lambda) &= \sum_{n=-\infty}^{N} \omega^n \, \mathcal{A}^{\pm}_{\beta b,\alpha a}(n,\lambda) , \\ \mathcal{A}^{\pm}_{\beta b,\alpha a}(n,\lambda) &= \frac{1}{2\pi i} \int M^{\pm}_{\beta b,\alpha a}(\omega,\lambda)\, \omega^{-n-1} d\omega . \end{aligned} \qquad (13)$$

The decomposition coefficients are independent of non-minimal interactions at $n \leq 0$ and are determined unambiguously as:

$$\mathcal{A}^{+}_{\beta b,\alpha a}(0,\lambda) = 4\left\{ [X_b(\lambda),[m^2, X_a(\lambda)]] + [X_a(\lambda),[m^2, X_b(\lambda)]] \right\}_{\beta\alpha} , \qquad (14)$$

$$\mathcal{A}^{-}_{\beta b,\alpha a}(0,\lambda) = 8\varepsilon \left\{ i f_{abc} X_c(\lambda) + i f_{abi} T_i - [X_a(\lambda), X_b(\lambda)] \right\}_{\beta\alpha} , \qquad (15)$$

where m^2 is the operator of mass square. Let us discuss the asymptotics of $M^{-}_{\beta b,\alpha a}(\omega,\lambda)$ amplitude in the tree approximation. The system of two goldstones antisymmetrical in the a and b indeces could have the following quantum numbers:

$T=1\ , Y=0\ , C=0$	ρ-trajectory
$T=0\ , Y=0\ , C=0$	ω-trajectory
$T=0\ , Y=\pm 2\ , C=0$	exotic exchanges
$T=\frac{1}{2}\ , Y=0\ , C=\pm 1$	$\mathcal{D}^*$-trajectory
$T=0\ , Y=\pm 1\ , C=\pm 1$	S^*-trajectory
$T=\frac{1}{2}\ , Y=\pm 1\ , C=0$	K^*-trajectory
$T=1\ , Y=\pm 1\ , C=\pm 1$	exotic exchanges
$T=\frac{1}{2}\ , Y=\pm 2\ , C=\pm 1$	-"-
$T=0\ , Y=0\ , C=\pm 2$	-"-
$T=\frac{1}{2}\ , Y=\pm 1\ , C=\pm 2.$	-"-
$T=0,1\ , Y=\pm 1\ , C=\mp 1$	-"-

Since $\alpha(0)<1$ for all the given trajectories, the $M^{-}_{\beta b,\alpha a}(\omega,\lambda)$ amplitude reduces and hence the constant term in (15) is zero

$$[X_a(\lambda), X_b(\lambda)] = i f_{abc}\, X_c(\lambda) + i f_{abi}\, T_i\ . \qquad (16)$$

The condition (16) together with the relations:

$$[T_i\, , X_a(\lambda)] = i f_{abi}\, X_b(\lambda), \qquad (17)$$

$$[T_i\, , T_j] = i f_{ijk}\, T_k \qquad (18)$$

following from the SU(2)$\times$ Y $\times$C symmetry, imply that T_i and $X_a(\lambda)$ (i=1,2,3,8,15, a=4,...7,9,...14) constitute the SU(4) algebra. The classification of particles with the given helicities also restore according to the representations of SU(4) group, as the $[X_a(\lambda)]_{\beta\alpha}$ matrices have non-zero matrix elements between different SU(2)$\times$ Y $\times$C multiplets .

To obtain the limitations on the form of mass operator consider the $M^{+}_{\beta b,\alpha a}(\omega,\lambda)$ amplitude to which contribute in the t-channel of the $\xi_a+\alpha\rightarrow\xi_b+\beta$ process the following trajectories:

$T=0,\ Y=0,\ C=0$ the pomeron
$T=1,\ Y=0,\ C=0$ A_2 -trajectory
$T=1,\ Y=\pm 2,\ C=0$ exotic exchanges
$T=\frac{1}{2},\ Y=0,\ C=\pm 1$ D^{**} -trajectory
$T=0,\ Y=\pm 1,\ C=\pm 1$ S^{**} -trajectory
$T=\frac{1}{2},\ Y=\pm 1,\ C=0$ K^{**} -trajectory
$T=1,\ Y=\pm 1,\ C=\pm 1$ exotic exchanges
$T=\frac{1}{2},\ Y=\pm 2,\ C=\pm 1$ -"-
$T=1,\ Y=0,\ C=\pm 2$ -"-
$T=0,\ Y=\pm 2,\ C=\pm 2$ -"-
$T=\frac{1}{2},\ Y=\pm 1,\ C=\pm 2$ -"-
$T=0,1,\ Y=\pm 1,\ C=\mp 1$ -"-

In asymptotics the symmetrical amplitude has the form $M^{+}_{\beta b,\alpha a}(\omega,\lambda)\sim\omega^{\alpha(0)}$. Assuming that for exotic exchanges $\alpha(0)<0$, we find that the part of the $A^{+}_{\beta b,\alpha a}(0,\lambda)$ amplitude corresponding to these exchanges turns to zero

$$\{A^{+}_{\beta b,\alpha a}(0,\lambda)\}_{Y=\pm 2}=0 \qquad a,b=4,\dots 7, \tag{19}$$

$$\begin{array}{ll}\{A^{+}_{\beta b,\alpha a}(0,\lambda)\}_{T=1}=0 & a=4,\dots 7,\ b=9,\dots 12,\\ \{A^{+}_{\beta b,\alpha a}(0,\lambda)\}_{Y=\pm 1, C=\mp 1}=0 & a=9,\dots 12,\ b=4,\dots 7,\end{array} \tag{20}$$

$$\{A^{+}_{\beta b,\alpha a}(0,\lambda)\}_{C=\pm 2}=0 \qquad a,b=9,\dots 14, \tag{21}$$

$$\{A^{+}_{\beta b,\alpha a}(0,\lambda)\}_{Y=\pm 2}=0 \qquad \begin{array}{l}a=13,14,\ b=4,\dots 7,\\ a=4,\dots 7,\ b=13,14.\end{array} \tag{22}$$

The conditions (19) - (22) lead to following relations:

$$[X^a(\lambda),[X^b(\lambda),m^2]]+[X^b(\lambda),[X^a(\lambda),m^2]]= \tag{23}$$
$$a,b=4,\dots 7.$$
$$=\frac{4}{3}f_{a8a'}f_{b8b'}\{[X^{a'}(\lambda),[X^{b'}(\lambda),m^2]]+[X^{b'}(\lambda),[X^{a'}(\lambda),m^2]]\}.$$

$$[X^a(\lambda),[X^b(\lambda),m^2]]+[X^b(\lambda),[X^a(\lambda),m^2]]=$$
$$=4\, f_{a8a'} f_{b8b'}\{[X^{a'}(\lambda),[X^{b'}(\lambda),m^2]]+[X^{b'}(\lambda),[X^{a'}(\lambda),m^2]]\}$$

(24)

$a=4,\ldots 7,\ b=9,\ldots 12,$

$a=9,\ldots 12,\ b=4,\ldots 7.$

$$[X^a(\lambda),[X^b(\lambda),m^2]]+[X^b(\lambda),[X^a(\lambda),m^2]]=$$
$$=\frac{3}{2} f_{a15a'} f_{b15b'}\{[X^{a'}(\lambda),[X^{b'}(\lambda),m^2]]+[X^{b'}(\lambda),[X^{a'}(\lambda),m^2]]\} \qquad (25)$$

$a,\ b=9,\ldots 14.$

$$[X^a(\lambda),[X^b(\lambda),m^2]]+[X^b(\lambda),[X^a(\lambda),m^2]]=$$
$$=2\, f_{a8a'} f_{b8b'}\{[X^{a'}(\lambda),[X^{b'}(\lambda),m^2]]+[X^{b'}(\lambda),[X^{a'}(\lambda),m^2]]\} \qquad (26)$$

$a=13,14,\ b=4,\ldots 7,$

$b=13,14,\ a=4,\ldots 7.$

using which we can show that

$$(K^2)^2 m^2 \equiv \sum_{A,B}[R^A,[R^A,[R^B,[R^B,m^2]]]]= 4K^2 m^2 \equiv 4\sum_A [R^A,[R^A,m^2]], \qquad (27)$$

where $R^A=T^A$ for $A=1,2,3,8,15;$

$R^A=X^A(\lambda)$ for $A=4,\ldots 7,9,\ldots 14$, and K^2 is the quadratic Kasimir operator for the SU(4) group. This implies that K^2 has only the eigenvalues 0 and 4, i.e. the mass operator containes only the SU(4) singlet and the 15-plet:

$$m^2 = m^2_{inv} + m^2_8 + m^2_{15}.$$

Thus, the minimality of the violation is obtained in the algebraic realization of SU(4) without the assumption of its smallness.

4. The violation of the symmetry is connected in the algebraic realization of SU(4) with the existence of A_2, D^{**}, K^{**}, S^{**} - trajectories. If they are absent or have the $\alpha(0)<0$ intercept, then the requirement of correct asymptotics of tree graphs will lead to a partial or the complete restoration of SU(4)

symmetry. In particular, the condition $\alpha_{A_2}(0)<0$ leads to the exact SU(3) symmetry [4], while the relations $\alpha_{S^{**}}(0)<0$ and $\alpha_{D^{**}}(0)<0$ to the relations:

$$\frac{m_{15}^2}{m_8^2}=\frac{5}{2\sqrt{2}}\ , \qquad \text{for} \quad \alpha_{D^{**}}(0)<0\ , \tag{28}$$

$$\frac{m_{15}^2}{m_8^2}=-\frac{2}{\sqrt{2}}\ , \qquad \text{for} \quad \alpha_{S^{**}}(0)<0\ , \tag{29}$$

$$m_{15}^2=m_8^2=0, \qquad \text{for} \quad \alpha_{D^{**}}(0)<0\ \ \alpha_{S^{**}}(0)<0\ , \tag{30}$$

which don't allow to obtain large masses of charmed mesons. Hence, the intercepts of these trajectories must be positive

$$0<\alpha_{D^{**}}(0)<1\ , \qquad 0<\alpha_{S^{**}}(0)<1\ . \tag{31}$$

Under the assumption of exchange degeneracy the same conditions are valid for vector trajectories $0<\alpha_{D^{*}}(0)<1$ and $0<\alpha_{S^{*}}(0)<1$. Taking into account the $n=-2$ terms in the Laurent series decomposition of the amplitudes, one can also obtain the conditions for exotic exchanges $-1<\alpha_{ex}(0)<0$. So, large mass values of charmed mesons lead in the algebraic realization of SU(4) to high lying $C=\pm1$ trajectories. The condition (31) for linear trajectories gives the slope $\alpha'_{D^*,S^*}\lesssim 0.2-0.25\ \mathrm{GeV}^{-2}$.*

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*) In which case the Regge recurrence with $J=3^-$ on the ψ-particle trajectory may have $m\gtrsim(5.0-5.5)$ GeV mass[5].

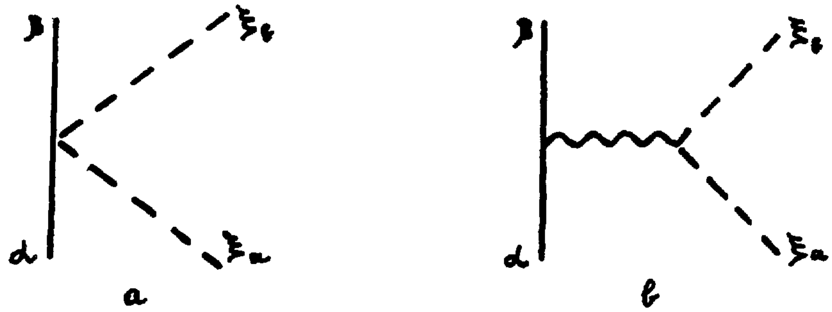

Fig. 1

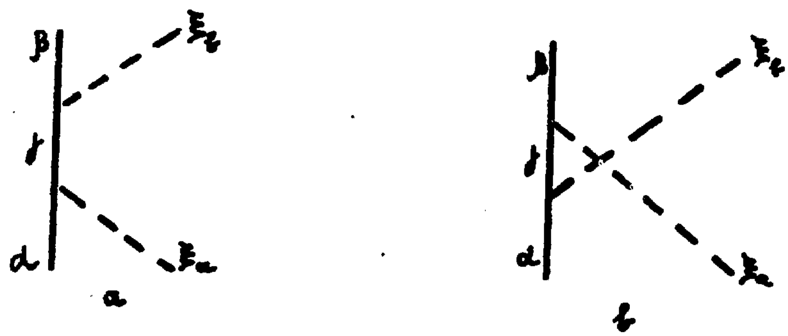

Fig. 2

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Г.М.АСАТРЯН, А.Н.ЗАСЛАВСКИЙ

ОБ АЛГЕБРАИЧЕСКОЙ РЕАЛИЗАЦИИ СИММЕТРИИ SU (4)

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