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ON THE INCLUSIVE ANNIHILATION OF POLARIZED $e^{+}e^{-}$ - PAIR WITH TWO OBSERVED HADRONS*

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1. Although the inclusive e^+e^--annihilation into hadrons has been considered in a number of works (see, e.g. Refs 1 - 8), they all have delt in the main with the investigation of the process

$$e^+e^- \to h\,X, \qquad (1)$$

with the detection of only one hadron. These works contain both the general description of the process [2,3] and different predictions obtained in the frameworks of definite models [1,5-8].

The experimental data [9] on the reaction (1) with the detection of pion available in the energy range of $\sqrt{s}$ = 3 - 7.4 GeV show the scaling behaviour of the $s\frac{d\sigma}{dx}$ cross section in $x \geq 0.45$ range, where $x = 2p/\sqrt{s}$, and the deviation from the scaling at smaller x, that presumably has an effect on the increase of the relation

$$R = \sigma\,(e^+e^- \to \text{hadrons})/\,\sigma\,(e^+e^- \to \mu^+\mu^-)$$

in the mentioned energy range. These problems still require theoretical comprehension.

Still more information on the hadron production mechanism at the annihilation of e^+e^--pair could be gained from the study of the process

$$e^+e^- \to h_1 h_2\, X, \qquad (2)$$

with the detection of two hadrons in final state. First of all, the investigation of the angular distribution of these hadrons would allow one to separate the contributions of both the transversal polarisations of a virtual photon to the decay $\gamma^* \to h_1 h_2 X$, that proved impossible with the type (1) reaction, as well as to find the contribution of the interference between the transversal and the longitudinal components of polarization to this decay. Further, the knowledge of structure functions specifying the transition $\gamma^* \to N h X$ in the reaction (2) allows one to check the crossing relations, which connect this process with the one particle inclusive electroproduction from a nucleon $e^- N \to e^- h X$ analogous to the former process by the structure of hadronic tensor. At last, and seemingly most important is the fact, that the study of the process (2) would allow one to investigate two-particle correlations both the angular and in x_1, x_2, which may serve as the adequate check of the predictions of different models, in particular, the quark parton model [10,11] and the jet picture [12].

In the present work we shall consider in general terms the process (2) with polarized primary particles. In storage rings the positrons and electrons are known (see, e.g., Ref. 13) to acquire the transversal polarization with the degree up to Υ=0.924. Besides, last years the problem of the transformation of transversal polarization into the longitudinal one has been widely discussed [14]. Hence, the consideration of the polarization of annihilating pair appears to be natural. If, however, the presence of polarization leads in the case of the process (1) to the change of cross section form and doesn't give any new information about the structure functions, it is more essential in the case of the process (2). If the polarization of only one of primaries has the longitudinal component, then the fifth structure function is added to four structure functions specifying the process (2) in the case of unpolarized e^+e^--pair.

In Section 2 the hadronic tensor is defined and the cross section of the process (2) is given for arbitrarily polarized electrons and positrons. From the positive definiteness of the hadronic tensor the restrictions on structure functions result. In Section 3 the cross section of the process (2) is expressed in terms of the partial widths of $\gamma^* \to h_1 h_2 X$ decay and the relationship is established between these widths and the structure functions using the density matrix of the virtual photon in the helicity representation as given in Appendix. On the basis of obtained formulae the dependence of cross section on the polarization of primaries is studied and the angular distributions of final hadrons are analyzed. In Appendix we give the expressions for the polarization multipole moments of the virtual photon.

2. Let us find the contribution of one-photon exchange to the two particle inclusive process (2) shown in Fig.1. The differential cross section of this process could be written in the form:

$$d\sigma(e^+e^- \to h_1 h_2 X) = \frac{\alpha^2}{(2\pi)^3} \frac{N_1 N_2}{S^2} L_{\mu\nu} H_{\mu\nu} \frac{d\vec{p}_1}{E_1} \frac{d\vec{p}_2}{E_2} \quad , \tag{3}$$

where N_1 and N_2 are normalizing factors which take on the vales $\frac{1}{2}$ or 1 depending on whether the observed hadrons are bosons or fermions; $S = -(K_1 + K_2)^2$ is the total c.m.s. energy squared and the 4-momenta of corresponding particles are shown in the Fig.1 diagram.

The lepton tensor $L_{\mu\nu}$ specifying the $e^+e^- \to \gamma^*$ transition vertex is

$$L_{\mu\nu} = \frac{1}{2S} Sp\left[(1 - i\hat{a}_1\gamma_5)(m - i\hat{k}_1)\gamma_\mu(1 - i\hat{a}_2\gamma_5)(m + i\hat{k}_2)\gamma_\nu\right] \quad , \qquad (4)$$

where a_1 and a_2 are 4-vectors of electron and positron polarizations respectively, and the hadronic tensor $H_{\mu\nu}$ describing the transition $\gamma^* \longrightarrow h_1 h_2 X$ is defined as

$$H_{\mu\nu} = (2\pi)^3 \sum \langle 0 | J_\mu(0) | P_1, P_2, P_X \rangle \langle P_1, P_2, P_X | J_\nu(0) | 0 \rangle \delta(P_1 + P_2 + P_X - q), \qquad (5)$$

where we assume summation over all the possible states in the beam of non-observed hadrons and the integration over the phase space of these hadrons as well as the summation over the polarizations of all the particles in the final state including the observed ones.

In view of the condition $q_\mu L_{\mu\nu} = q_\nu L_{\mu\nu} = 0$ the leptonic tensor has only space components in the c.m.s., which are determined as

$$L_{ik} = (1 + \vec{\zeta}_1 \vec{\zeta}_2)(\delta_{ik} - v_i v_k) - (\zeta_{1i}^{\perp}\zeta_{2k}^{\perp} + \zeta_{2i}^{\perp}\zeta_{1k}^{\perp}) + i(\zeta_1^{\parallel} + \zeta_2^{\parallel})\epsilon_{ik\ell} v_\ell , \qquad (6)$$

where $\vec{v}$ is the unit vector along the electron momentum, $\vec{\zeta}_1$ and $\vec{\zeta}_2$ are the electron and positron polarization vectors in their own rest frame. Their resolution into the transversal and longitudinal components is relative to the direction of $\vec{v}$. In the coordinate system with z axis along $\vec{v}$, the leptonic tensor has only x and y components [15]

$$L_{mn} = (1 + \vec{\zeta}_1 \vec{\zeta}_2)\delta_{mn} - (\zeta_{1m}^{\perp}\zeta_{2n}^{\perp} + \zeta_{2m}^{\perp}\zeta_{1n}^{\perp}) + (\zeta_1^{\parallel} + \zeta_2^{\parallel})\epsilon_{mnz} , \quad m,n = x,y. \qquad (7)$$

The summation over the polarization of observed hadrons made, the tensor $H_{\mu\nu}$ has to be constructed from three independent 4-momenta q , p_1 and p_2 . We shall make use of the following most general gauge-invariant form of $H_{\mu\nu}$ tensor:

$$H_{\mu\nu} = -\frac{1}{q^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) H_1 + \frac{1}{\lambda_1^2}\bar{p}_{1\mu}\bar{p}_{1\nu} H_2 + \frac{1}{\lambda_2^2}\bar{p}_{2\mu}\bar{p}_{2\nu} H_3 + \\ + \frac{1}{2\lambda_1\lambda_2}\left(\bar{p}_{1\mu}\bar{p}_{2\nu} + \bar{p}_{2\mu}\bar{p}_{1\nu}\right) H_4 + \frac{i}{2\lambda_1\lambda_2}\left(\bar{p}_{1\mu}\bar{p}_{2\nu} - \bar{p}_{2\mu}\bar{p}_{1\nu}\right) H_5 , \qquad (8)$$

where

$$\bar{p}_{1\mu} = p_{1\mu} - \frac{(qp_1)}{q^2} q_\mu , \quad \bar{p}_{2\mu} = p_{2\mu} - \frac{(qp_2)}{q^2} q_\mu ,$$

$$\lambda_1 = \left[(qp_1)^2 + m_1^2 q^2\right]^{1/2}, \quad \lambda_2 = \left[(qp_2)^2 + m_2^2 q^2\right]^{1/2},$$

m_1 and m_2 are the masses of h_1 and h_2 hadrons, and $H_1, \ldots, H_5$ are dimensionless real structure functions which may depend on four independent invariants, e.g.:

$$s = -q^2 , \quad \nu_1 = (qp_1) = -\sqrt{s}\,E_1 , \quad \nu_2 = (qp_2) = -\sqrt{s}\,E_2 , \quad M_X^2 = -P_X^2 . \qquad (9)$$

From the energy-momentum conservation we have the relation

$$M_X^2 = s + m_1^2 + m_2^2 + 2(\nu_1 + \nu_2) - 2(p_1 p_2) , \qquad (10)$$

and instead of M_X^2 one can use $(p_1 p_2)$ as the independent invariant.

The hadronic tensor also has only space components in the c.m.s.

$$H_{ik} = \frac{1}{s}\Big[\delta_{ik} H_1 + n_{1i} n_{1k} H_2 + n_{2i} n_{2k} H_3 + \\ + \frac{1}{2}(n_{1i} n_{2k} + n_{2i} n_{1k}) H_4 + \frac{i}{2}(n_{1i} n_{2k} - n_{2i} n_{1k}) H_5\Big] , \qquad (11)$$

where $\vec{n}_1$ and $\vec{n}_2$ are unit vectors along the $\vec{p}_1$ and $\vec{p}_2$ momenta.

Performing the contraction of this tensor with the leptonic one (6) and substituting the result into the formula (3) , we find for the cross section of this process

$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\vec{p}_1/E_1 \, d\vec{p}_2/E_2} = \frac{\alpha^2 N_1 N_2}{(2\pi)^3 s^3} \Big\{ 2(1 + \xi_1^{\parallel}\xi_2^{\parallel}) H_1 + \big[(1 + \vec{\xi}_1\vec{\xi}_2)(1 - z_1^2) - \\ - 2(\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_1\vec{\xi}_2^{\perp})\big] H_2 + \big[(1 + \vec{\xi}_1\vec{\xi}_2)(1 - z_2^2) - 2(\vec{n}_2\vec{\xi}_1^{\perp})(\vec{n}_2\vec{\xi}_2^{\perp})\big] H_3 + \big[(1 + \vec{\xi}_1\vec{\xi}_2)(z - z_1 z_2) - \\ - (\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_2\vec{\xi}_2^{\perp}) - (\vec{n}_1\vec{\xi}_2^{\perp})(\vec{n}_2\vec{\xi}_1^{\perp})\big] H_4 - (\xi_1^{\parallel} + \xi_2^{\parallel})(\vec{v}[\vec{n}_1\vec{n}_2]) H_5 \Big\} . \qquad (12)$$

Here $z=\cos\theta$, $z_1=\cos\theta_1$ and $z_2=\cos\theta_2$, θ is the angle between $\vec{p}_1$ and $\vec{p}_2$ momenta, θ_1 and θ_2 denote the emission angles of h_1 and h_2 hadrons relative to the direction $\vec{v}$ of primary particles collision.

One can see from (12) that the polarization of primaries not only results in the modification of the cross section corresponding to unpolarized primaries case, but also leads to the appearance of a supplementary term containing H_5, if the polarization of only one of the primaries has longitudinal component, except for the case of trivial kinematical configuration ($\vec{v}\,[\vec{n}_1,\vec{n}_2]$)=0.

As follows from (9) and (10), the structure functions specifying the cross section can depend only on one of the angular variables, viz., on $z=\cos\theta$. Hence, the expression (12) could be integrated at fixed z over the remaining variables. For the convenience of integration we choose two coordinate systems (see Fig.2). We shall determine the solid angle of say, the particle h_1 in the coordinate system with z axis along $\vec{v}$, i.e. $d\Omega_1=dz_1 d\varphi_1$, while the solid angle of another particle (h_2)- in the hatched system with z' axis along $\vec{n}_1$, i.e. $d\Omega_2=dz\,d\varphi_2'$. For definiteness, the azimuthal angles in the initial coordinate system are taken from the ($\vec{v},\vec{\zeta}_1$) plane and in the hatched one from the ($\vec{n}_1,\vec{v}$) plane. z_2 is the dependent variable given by the relation

$$z_2 = z z_1 + \sqrt{(1-z^2)(1-z_1^2)}\cos\varphi_2' . \qquad (13)$$

After the integration of (12) over the azimuthal angle φ_1 we obtain

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{d\omega_1 d\omega_2 dz\, dz_1 d\varphi_2'} = \frac{\alpha^2 N_1 N_2}{(8\pi)^2}\,\frac{x_1 x_2}{s}\Big\{\big[2H_1 + (1-z_1^2)H_2 + (1-z_2^2)H_3 +$$

$$+ (z - z_1 z_2)H_4\big](1+\zeta_1''\zeta_2'') + (\zeta_1''+\zeta_2'')\sqrt{(1-z^2)(1-z_1^2)}\,\sin\varphi_2' H_5\Big\}, \tag{14}$$

where we have introduced the dimensionless variables

$$\omega_j = \frac{2\nu_j}{q^2} = \frac{2E_j}{\sqrt{s}}\ ,\quad x_j = \frac{2P_j}{\sqrt{s}}\ ,\quad j = 1,2,$$

related by the formula

$$x_j = \Big(\omega_j^2 - \frac{4m_j^2}{s}\Big)^{1/2}.$$

The result (14) is obvious: the components of polarization vectors lying in the integration plane, namely, the transversal components, pass away at the integration over the azimuthal angle and the cross section is determined merely by the longitudinal components.

After the integration over z_1 we have the expression

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{d\omega_1 d\omega_2 dz\, d\varphi_2'} = \frac{\alpha^2 N_1 N_2}{3(4\pi)^2}\,\frac{x_1 x_2}{s}\Big\{\big(3H_1 + H_2 + \big[\tfrac{1}{2}(1+z^2) +$$

$$+ (1-z^2)\sin^2\varphi_2'\big]H_3 + zH_4\big)(1+\zeta_1''\zeta_2'') + \frac{3\pi}{8}(\zeta_1''+\zeta_2'')\sqrt{1-z^2}\,\sin\varphi_2' H_5\Big\}, \tag{15}$$

and, at last, the structure with H_5 also disappears at the integration over φ_2' and we get

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{d\omega_1 d\omega_2 dz} = \frac{\alpha^2 N_1 N_2}{24\pi}\,\frac{x_1 x_2}{s}\big(3H_1 + H_2 + H_3 + zH_4\big)(1+\zeta_1''\zeta_2''). \tag{16}$$

This formula reduces in the case of unpolarized primaries to the expression given in Ref. 4 .

It is easy to see on the basis of given results, that one can gain the maximum information about structure functions from the study of the distribution over the remaining angles at fixed z . From the cross section (14) follows, that in the interesting case of transversely polarized primaries (realized in storage rings) it reduces to the case of unpolarized particles. The cross sections (15) and (16) don't allow to separate all the structure functions.

Note that from the positive definiteness of the hadron tensor (5) a number of inequalities imposed on structure functions follows:

$$H_1 \geq 0\,,\quad H_1 + (1-z^2)H_3 \geq 0,$$
$$\left[H_1 + (1-z^2)H_3\right]\left[H_1 + H_2 + z^2 H_3 + z H_4\right] \geq (1-z^2)\left[(zH_3 + \tfrac{1}{2}H_4)^2 + \tfrac{1}{4}H_5^2\right]. \tag{17}$$

These inequalities are necessary and sufficient for the positive definiteness of the third rank tensor (11). The asymmetry of the inequalities (17) with respect to H_2 and H_3 is seeming, as from them follow two inequalities which differ from the given ones by the substitution $H_3 \rightleftarrows H_2$.

3. Let us express the process (2) cross section in terms of the partial widths of $\gamma^* \to h_1 h_2 X$ decay of a timelike virtual photon with given linear polarization. For this decay there exist a chosen $(\vec{n}_1, \vec{n}_2)$ plane in the rest frame of the virtual photon - the plane of h_1 and h_2 particles production, relative to which the orthonormal basis of three linear polarization vectors could be specified. We choose them as:

$$\vec{e}^{\,\ell} = \vec{n}_1\,,\quad \vec{e}^{\,\perp} = \frac{1}{\sin\theta}\left[\vec{n}_1, \vec{n}_2\right]\,,\quad \vec{e}^{\,\parallel} = \left[\vec{n}_1, \vec{e}^{\,\perp}\right], \tag{18}$$

i.e. $\vec{e}^{\,\perp}$ is normal to the $(\vec{n}_1, \vec{n}_2)$ plane and $\vec{e}^{\,\parallel}$ lies in this plane (see Fig.2). The resolving of the virtual photon polarization into the longitudinal ($\vec{e}^{\,\ell}$) and the transversal ($\vec{e}^{\,\perp}$, $\vec{e}^{\,\parallel}$) components is made in the direction of $\vec{n}_1$. This direction is, of course, not the preferential one and one could have chosen instead $\vec{n}_2$ or $(\vec{n}_1 + \vec{n}_2)/|\vec{n}_1 + \vec{n}_2|$.

The partial width of the $\gamma^* \to h_1 h_2 X$ decay with given polarization of the virtual photon is defined as

$$\frac{d\Gamma_{rr'}}{dE_1 dE_2 dz} = \frac{\alpha N_1 N_2}{2\pi^2} \frac{P_1 P_2}{\sqrt{s}} e_i^{r} e_\kappa^{r'} H_{i\kappa} \,, \tag{19}$$

where $\vec{e}^{\,r}$ is one of the vectors (18).

To express the process (2) cross section in terms of the partial widths (19), it is necessary to find the resolution of the leptonic tensor (6) into the basis (18). One can, however, use the density matrix of the virtual photon in helicity representation (see the Appendix) and the relationship between helicity states along $\vec{n}_1$ and the vectors (18)

$$\vec{e}^{\,(1)}(\vec{n}_1) = \frac{1}{\sqrt{2}} e^{i(\gamma+\varphi_1)}(\vec{e}^{\,\parallel} + i\vec{e}^{\,\perp}), \quad \vec{e}^{\,(0)}(\vec{n}_1) = \vec{e}^{\,\ell}, \quad \vec{e}^{\,(-1)}(\vec{n}_1) = -\frac{1}{\sqrt{2}} e^{-i(\gamma+\varphi_1)}(\vec{e}^{\,\parallel} - i\vec{e}^{\,\perp}),$$

where φ_1 is the azimuth angle of $\vec{n}_1$ vector, and $\gamma = \varphi_2' - \pi$ is the angle made by $(\vec{n}_1, \vec{v})$ and $(\vec{n}_1, \vec{n}_2)$ planes and determined from the formulae

$$\sin\gamma = \frac{\sin\theta_2}{\sin\theta} \sin(\varphi_2 - \varphi_1)\,, \quad \cos\gamma = \frac{\cos\theta\cos\theta_1 - \cos\theta_2}{\sin\theta \sin\theta_1}\,. \tag{20}$$

As a result we find for the leptonic tensor in the most general form expressed via the linear polarization vectors (18)

$$\begin{aligned}
L_{i\kappa} = {} & \Big[\tfrac{1}{2}(1+\cos^2\theta_1)(1+\vec{\zeta}_1\vec{\zeta}_2) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp} + (\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp})\Big](e_i^{\parallel}e_\kappa^{\parallel} + e_i^{\perp}e_\kappa^{\perp}) + \\
& + \Big\{\zeta\cos\theta_1\sin 2\gamma - \cos 2\gamma\Big[\tfrac{1}{2}\sin^2\theta_1(1+\vec{\zeta}_1\vec{\zeta}_2) + \frac{1+\cos^2\theta_1}{\sin^2\theta_1}(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp}) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp}\Big]\Big\}(e_i^{\parallel}e_\kappa^{\parallel} - e_i^{\perp}e_\kappa^{\perp}) + \\
& + \Big\{\zeta\cos\theta_1\cos 2\gamma + \sin 2\gamma\Big[\tfrac{1}{2}\sin^2\theta_1(1+\vec{\zeta}_1\vec{\zeta}_2) + \frac{1+\cos^2\theta_1}{\sin^2\theta_1}(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp}) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp}\Big]\Big\}(e_i^{\parallel}e_\kappa^{\perp} + e_i^{\perp}e_\kappa^{\parallel}) + \\
& + i\cos\theta_1(\zeta_1^{\parallel} + \zeta_2^{\parallel})(e_i^{\parallel}e_\kappa^{\perp} - e_i^{\perp}e_\kappa^{\parallel}) + \Big[\sin^2\theta_1(1+\vec{\zeta}_1\vec{\zeta}_2) - 2(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp})\Big] e_i^{\ell}e_\kappa^{\ell} - \\
& - \Big\{\zeta\sin\theta_1\cos\gamma - \cos\theta_1\sin\gamma\Big[\sin\theta_1(1+\vec{\zeta}_1\vec{\zeta}_2) - \frac{2}{\sin\theta_1}(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp})\Big]\Big\}(e_i^{\ell}e_\kappa^{\perp} + e_i^{\perp}e_\kappa^{\ell}) - \\
& - \Big\{\zeta\sin\theta_1\sin\gamma + \cos\theta_1\cos\gamma\Big[\sin\theta_1(1+\vec{\zeta}_1\vec{\zeta}_2) - \frac{2}{\sin\theta_1}(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp})\Big]\Big\}(e_i^{\ell}e_\kappa^{\parallel} + e_i^{\parallel}e_\kappa^{\ell}) - \\
& - i\sin\theta_1(\zeta_1^{\parallel} + \zeta_2^{\parallel})\Big[\cos\gamma(e_i^{\ell}e_\kappa^{\perp} - e_i^{\perp}e_\kappa^{\ell}) + \sin\gamma(e_i^{\ell}e_\kappa^{\parallel} - e_i^{\parallel}e_\kappa^{\ell})\Big]\,,
\end{aligned} \tag{21}$$

where γ is given in the Appendix. Note, that this is the most general form of the tensor containing 6 symmetrical and 3 antisymmetrical combinations of vectors (18). Only 5 of them (4 symmetrical and 1 antisymmetrical combinations) contribute to the cross section of the process under study. For more complicated cases (polarized final particles in reaction (2) or the observation of three hadrons in the final state) the number of contributing combinations will be greater.

Using (21) and the definition (19) we get

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{dE_1dE_2dz\,dz_1d\varphi_1d\gamma}=\frac{\alpha}{2\pi}\frac{1}{s\sqrt{s}}\Big\{\Big[\frac{1}{2}(1+z_1^2)(1+\vec{\xi}_1\vec{\xi}_2)-\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp}+(\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_1\vec{\xi}_2^{\perp})\Big]\frac{d\Gamma_U}{dE_1dE_2dz}+$$
$$+\Big(\gamma z_1\sin 2\gamma-\cos 2\gamma\Big[\frac{1}{2}(1-z_1^2)(1+\vec{\xi}_1\vec{\xi}_2)-\vec{\xi}_1^{\perp}\vec{\xi}_2^{\perp}+\frac{1+z_1^2}{1-z_1^2}(\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_1\vec{\xi}_2^{\perp})\Big]\Big)\frac{d\Gamma_T}{dE_1dE_2dz}+$$
$$+\frac{1}{2}\Big[(1-z_1^2)(1+\vec{\xi}_1\vec{\xi}_2)-2(\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_1\vec{\xi}_2^{\perp})\Big]\frac{d\Gamma_L}{dE_1dE_2dz}- \tag{22}$$
$$-\Big(\gamma\sqrt{1-z_1^2}\sin\gamma+\frac{z_1}{\sqrt{1-z_1^2}}\cos\gamma\Big[(1-z_1^2)(1+\vec{\xi}_1\vec{\xi}_2)-2(\vec{n}_1\vec{\xi}_1^{\perp})(\vec{n}_1\vec{\xi}_2^{\perp})\Big]\Big)\frac{d\Gamma_I^+}{dE_1dE_2dz}-$$
$$-\sqrt{1-z_1^2}\,\sin\gamma\,(\xi_1''+\xi_2'')\frac{d\Gamma_I^-}{dE_1dE_2dz}\Big\},$$

where

$$\frac{d\Gamma_U}{dE_1dE_2dz}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{\sqrt{s}}\frac{1}{2}(e_i''e_k''+e_i^{\perp}e_k^{\perp})H_{ik}=\frac{\alpha N_1N_2}{(2\pi)^2}\frac{P_1P_2}{s\sqrt{s}}\big[2H_1+(1-z^2)H_3\big],$$
$$\frac{d\Gamma_T}{dE_1dE_2dz}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{\sqrt{s}}\frac{1}{2}(e_i''e_k''-e_i^{\perp}e_k^{\perp})H_{ik}=\frac{\alpha N_1N_2}{(2\pi)^2}\frac{P_1P_2}{s\sqrt{s}}(1-z^2)H_3,$$
$$\frac{d\Gamma_L}{dE_1dE_2dz}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{\sqrt{s}}e_i^{\ell}e_k^{\ell}H_{ik}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{s\sqrt{s}}(H_1+H_2+z^2H_3+zH_4), \tag{23}$$
$$\frac{d\Gamma_I^+}{dE_1dE_2dz}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{\sqrt{s}}\frac{1}{2}(e_i^{\ell}e_k''+e_i''e_k^{\ell})H_{ik}=-\frac{\alpha N_1N_2}{(2\pi)^2}\frac{P_1P_2}{s\sqrt{s}}\sqrt{1-z^2}(2zH_3+H_4),$$
$$\frac{d\Gamma_I^-}{dE_1dE_2dz}=\frac{\alpha N_1N_2}{2\pi^2}\frac{P_1P_2}{\sqrt{s}}\frac{i}{2}(e_i^{\ell}e_k''-e_i''e_k^{\ell})H_{ik}=\frac{\alpha N_1N_2}{(2\pi)^2}\frac{P_1P_2}{s\sqrt{s}}\sqrt{1-z^2}H_5.$$

As was to be expected, the process cross section is expressed in terms of five widths depending on the same invariants as the structure functions. These widths are: Γ_U is the partial width of unpolarized transverse virtual photons decay; Γ_T is the difference of widths with corresponding transversal polarizations; Γ_L is the decay width of longitudinally polarized photons, and $\Gamma_I^{\pm}$ allows for the interference (both symmetrical and antisymmetrical) between the longitudinal and transversal components of polarization. Note, that the positivity of longitudinal as well as of each separate transversal components of the width follows immediately from the conditions (17).

As in the right hand side of formula (22) the partial widths only are E_1, E_2 and z -dependent, we can integrate the formula (22) over these variables. Hence, the study of z_1, φ_1 and χ angular distributions will allow one to find the differential as well as the total partial widths depending on the conditions of an experiment.

Let us pass to the study of the polarization dependence of the angular distribution (22) integrated over E_1, E_2 and z (all the reasoning below will also be valid for the differential characteristics if not otherwise stated).

In the case of longitudinally polarized particles the angular distribution has simple form

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{dz_1d\chi} = \frac{d\sigma^0}{dz_1d\chi}(1+\zeta_1''\zeta_2'') - \frac{\alpha}{s\sqrt{s}}\sqrt{1-z_1^2}\,\sin\chi(\zeta_1''+\zeta_2'')\Gamma_I^-, \quad (24)$$

where

$$\frac{d\sigma^0}{dz_1d\chi} = \frac{\alpha}{2s\sqrt{s}}\left[(1+z_1^2)\Gamma_U - (1-z_1^2)(\cos 2\chi\,\Gamma_T - \Gamma_L) - 2z_1\sqrt{1-z_1^2}\cos\chi\,\Gamma_I^+\right]$$

is the cross section of unpolarized particles. It follows from (24) that at $\zeta_1''=-\zeta_2''=\pm 1$ the cross section turns into zero (due to the helicity conservation) and one could judge about the value of Γ_I^- by the difference of cross sections with $\zeta_1''=\zeta_2''=\pm 1$.

In the interesting case of antiparallel transversely polarized primaries the cross section has the form

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{dz_1\,d\varphi_1\,d\gamma}=\frac{\alpha}{2\pi s\sqrt{s}}\Big\{\Gamma_U+\zeta_1\zeta_2\Big[(\cos 2\varphi_1\cos 2\gamma - z_1\sin 2\varphi_1\sin 2\gamma)\Gamma_T+ \tag{25}$$
$$+\sqrt{1-z_1^2}\,\sin 2\varphi_1\sin\gamma\,\Gamma_I^{\,+}\Big]-\tfrac{1}{2}(1+\zeta_1\zeta_2\cos 2\varphi_1)\sqrt{1-z_1^2}\Big[\sqrt{1-z_1^2}(\Gamma_U+\cos 2\gamma\,\Gamma_T-\Gamma_L)+2z_1\cos\gamma\,\Gamma_I^{\,+}\Big]\Big\}.$$

It follows from the formulae (22) and (25) that the polarization of primaries essentially affects the angular distribution of detected hadrons. At the same time, to gain maximum information - the definition of all the partial widths- it is sufficient to study the angular distribution of final state hadrons in the case when at least one of the primaries has the longitudinal component of polarization vector.

Integrating the cross section (22) over all the variables of h_2 particle (including E_2) we arrive at the expression of Ref. 3 for the angular distribution of h_1 hadron in reaction (1)

$$\frac{d\sigma(e^+e^-\to h_1X)}{d\Omega_1}=\frac{\alpha}{2s\sqrt{s}}\Big\{2(1+\zeta_1^{\parallel}\zeta_2^{\parallel})\Gamma_U+\Big[(1-z_1^2)(1+\vec{\zeta}_1\vec{\zeta}_2)-2(\vec{n}_1\vec{\zeta}_1^{\perp})(\vec{n}_1\vec{\zeta}_2^{\perp})\Big](\Gamma_L-\Gamma_U)\Big\}. \tag{26}$$

For antiparallel transversely polarized primaries we immediately have from (25)

$$\frac{d\sigma(e^+e^-\to h_1X)}{d\Omega_1}=\frac{\alpha}{2s\sqrt{s}}\Big[\Gamma_U+\Gamma_L+z_1^2(\Gamma_U-\Gamma_L)-\zeta_1\zeta_2(1-z_1^2)\cos 2\varphi_1(\Gamma_U-\Gamma_L)\Big], \tag{27}$$

that exactly coincides with the expression of Ref.9 if the azimuthal angle is taken from the polarization plane, not the orbit plane.

Let us see what information could be gained from the cross sections integrated over φ_1 or z_1 . As was said above, the transversal components of polarization pass away at the integration of (22) over φ_1 and we come to the expression (24). The integration of (22) over z_1 gives us

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{d\varphi_1 d\gamma}=\frac{\alpha}{3\pi s\sqrt{s}}\Big\{(2\Gamma_U-\cos 2\gamma\,\Gamma_T+\Gamma_L)(1+\zeta_1''\zeta_2'')+\zeta_{1x}(\zeta_{2x}\cos 2\varphi_1+\zeta_{2y}\sin 2\varphi_1)\times$$
$$\times(\Gamma_U-2\cos 2\gamma\,\Gamma_T-\Gamma_L)-\frac{3\pi}{4}\sin\gamma\big[\zeta_{1x}(\zeta_{2x}\sin 2\varphi_1-\zeta_{2y}\cos 2\varphi_1)\Gamma_I^{+}+(\zeta_1''+\zeta_2'')\Gamma_I^{-}\big]\Big\} \qquad (28)$$

At last, the further integrations result in

$$\frac{d\sigma(e^+e^-\to h_1h_2X)}{d\gamma}=\frac{2\alpha}{3s\sqrt{s}}\Big[(2\Gamma_U-\cos 2\gamma\,\Gamma_T+\Gamma_L)(1+\zeta_1''\zeta_2'')-\frac{3\pi}{4}\sin\gamma(\zeta_1''+\zeta_2'')\Gamma_I^{-}\Big], \qquad (29)$$

$$\frac{d\sigma(e^+e^-\to h_1X)}{dz_1}=\frac{\pi\alpha}{s\sqrt{s}}\big[(1+z_1^2)\Gamma_U+(1-z_1^2)\Gamma_L\big](1+\zeta_1''\zeta_2''), \qquad (30)$$

$$\frac{d\sigma(e^+e^-\to h_1X)}{d\varphi_1}=\frac{2\alpha}{3s\sqrt{s}}\big[(2\Gamma_U+\Gamma_L)(1+\zeta_1''\zeta_2'')+\zeta_{1x}(\zeta_{2x}\cos 2\varphi_1+\zeta_{2y}\sin 2\varphi_1)(\Gamma_U-\Gamma_L)\big],$$

$$\sigma(e^+e^-\to X)=\frac{4\pi\alpha}{3s\sqrt{s}}(2\Gamma_U+\Gamma_L)(1+\zeta_1''\zeta_2''). \qquad (31)$$

It may seem at first that the expression (29) refers to the process $e^+e^-\to h_2X$, and not to the process (2). The investigation of the γ-dependence of cross section implies that detecting the hadron h_1 within all the solid angle (as the cross section was integrated over z_1 and φ_1) and fixing the hadron h_2 (characterized by the parameters z and $\varphi_2'=\gamma+\pi$) relative to it, we study, in essence, the dependence of the cross section on the orientation of ($\vec{n}_1,\vec{n}_2$) plane making sense exactly for the process (2). This point is supported also by the presence of Γ_T and Γ_I^- quantities in (29). The relations (26), (27) and (30) refer to the process (1) if the integration is over E_2 and z, whereas (31) refers to the process $e^+e^-\to X$ if the integration over E_1 is performed as well. Otherwise, the mentioned formulae establish the relationship between the differential cross sections of the process (2) and the corresponding differential widths.

We have described the process (2) both in terms of structure functions and in terms of partial widths. Although these approaches are completely equivalent, as it is evident from (23), the description by means of partial widths is more instructive from the physical view point . The study of the process $e^+e^- \to X$ with no chosen direction in the final state allows one to find only the combination $2\Gamma_U + \Gamma_L$, corresponding to the unpolarized virtual photon. The process (1) has already the predetermined direction and the width Γ_L , specified by the polarization vector in this direction, could be separated from the unpolarized transverse width Γ_U as it directly follows from (26). And, at last, the study of the angular distribution of final state hadrons in the process (2) with the chosen plane will allow one to find also the contribution of the transversal component of photon polarization and the interference of transversal and longitudinal components and to investigate the dependence of widths on corresponding invariants, specifically, the correlations in ω_1 and ω_2 , and to check the predictions of different models [10 - 12].

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APPENDIX

The density matrix of a virtual timelike photon produced at the annihilation of arbitrarily polarized electron-positron pair is determined in its rest frame as

$$\rho_{\lambda\lambda'} = L_{ik}\, e_i^{(\lambda')} e_k^{(\lambda)*}, \qquad (A.1)$$

where L_{ik} is the leptonic tensor, which for $S \gg m^2$ has the form (6). Taking as the resolving basis the spin 1 helicity states along an arbitrary direction $\vec{n}$, characterized by θ and φ angles in the coordinate system with z axis along $\vec{v}$, we obtain for the elements of the density matrix

$$\begin{aligned}
\rho_{\pm1\pm1} &= \tfrac{1}{2}(1+\cos^2\theta)(1+\vec{\zeta}_1\vec{\zeta}_2) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp} + (\vec{n}\vec{\zeta}_1^{\perp})(\vec{n}\vec{\zeta}_2^{\perp}) \pm \cos\theta(\zeta_1^{\parallel}+\zeta_2^{\parallel}),\\
\rho_{00} &= \sin^2\theta(1+\vec{\zeta}_1\vec{\zeta}_2) - 2(\vec{n}\vec{\zeta}_1^{\perp})(\vec{n}\vec{\zeta}_2^{\perp}),\\
\rho_{\pm10} &= \mp\frac{e^{\mp i\varphi}}{\sqrt{2}}\sin\theta\Big\{\cos\theta(1+\vec{\zeta}_1\vec{\zeta}_2) - \Big[2\frac{\cos\theta}{\sin^2\theta}(\vec{n}\vec{\zeta}_1^{\perp})(\vec{n}\vec{\zeta}_2^{\perp}) \pm i\zeta\Big] \pm (\zeta_1^{\parallel}+\zeta_2^{\parallel})\Big\},\\
\rho_{0\pm1} &= \mp\frac{e^{\pm i\varphi}}{\sqrt{2}}\sin\theta\Big\{\cos\theta(1+\vec{\zeta}_1\vec{\zeta}_2) - \Big[2\frac{\cos\theta}{\sin^2\theta}(\vec{n}\vec{\zeta}_1^{\perp})(\vec{n}\vec{\zeta}_2^{\perp}) \mp i\zeta\Big] \pm (\zeta_1^{\parallel}+\zeta_2^{\parallel})\Big\},\\
\rho_{\pm1\mp1} &= \tfrac{1}{2}e^{\mp 2i\varphi}\Big\{\sin^2\theta(1+\vec{\zeta}_1\vec{\zeta}_2) + 2\Big[\frac{1+\cos^2\theta}{\sin^2\theta}(\vec{n}\vec{\zeta}_1^{\perp})(\vec{n}\vec{\zeta}_2^{\perp}) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp} \pm i\zeta\cos\theta\Big]\Big\},
\end{aligned} \qquad (A.2)$$

where

$$\zeta = (\zeta_{1x}\zeta_{2x} - \zeta_{1y}\zeta_{2y})\sin 2\varphi - (\zeta_{1x}\zeta_{2y} + \zeta_{2x}\zeta_{1y})\cos 2\varphi. \qquad (A.3)$$

It follows from these expressions that the resolving made into the helicity states along the direction of $\vec{v}$ (i.e., $\theta=\varphi=0$), the matrix elements with $\lambda=\lambda'=0$ turn to zero [15]. This means that to the accuracy of terms containing the electron mass, the virtual photons are always transversal with respect to the direction of intersecting beams which is due to the helicity conservation of ultrarelativistic electrons and positrons.

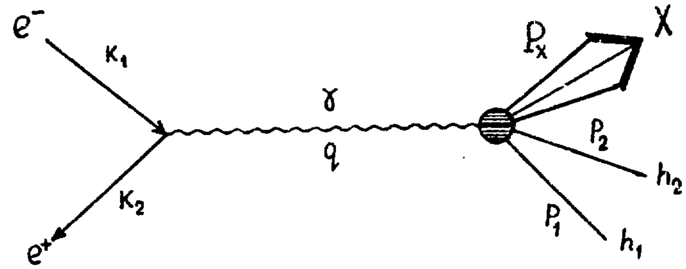
Using (A.2) it is easy to calculate the polarization multipole moments q^L_M of the virtual photon, namely, the polarization vector (q^1_M) and the alignment (q^2_M), which are determined as:

$$q^L_M = \frac{\sqrt{3}}{2}\left(1+\zeta_1^{\parallel}\zeta_2^{\parallel}\right)^{-1} a^L_M , \qquad \text{(A.4)}$$
$$a^L_M = \sum_{\lambda\lambda'} (-1)^{1-\lambda'} C^{LM}_{1,\lambda;1,-\lambda'} \rho_{\lambda\lambda'} .$$

The values of a^L_M are as follows:

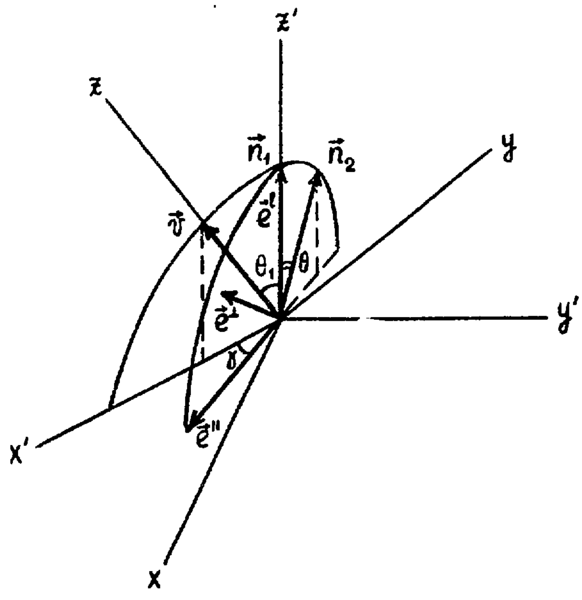
$$a^0_0 = \frac{2}{\sqrt{3}}\left(1+\zeta_1^{\parallel}\zeta_2^{\parallel}\right),$$
$$a^1_{\pm1} = \pm e^{\mp i\varphi}\sin\theta\left(\zeta_1^{\parallel}+\zeta_2^{\parallel}\right), \quad a^1_0 = \sqrt{2}\cos\theta\left(\zeta_1^{\parallel}+\zeta_2^{\parallel}\right),$$
$$a^2_{\pm1} = \pm e^{\mp i\varphi}\sin\theta\left\{\cos\theta\left(1+\vec{\zeta}_1\vec{\zeta}_2\right) - \left[2\,\frac{\cos\theta}{\sin^2\theta}\left(\vec{n}\vec{\zeta}_1^{\perp}\right)\left(\vec{n}\vec{\zeta}_2^{\perp}\right) \pm i\zeta\right]\right\}, \qquad \text{(A.5)}$$
$$a^2_{\pm2} = \frac{1}{2}e^{\mp 2i\varphi}\left\{\sin^2\theta\left(1+\vec{\zeta}_1\vec{\zeta}_2\right) + 2\left[\frac{1+\cos^2\theta}{\sin^2\theta}\left(\vec{n}\vec{\zeta}_1^{\perp}\right)\left(\vec{n}\vec{\zeta}_2^{\perp}\right) - \vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp} \pm i\zeta\cos\theta\right]\right\},$$
$$a^2_0 = \frac{1}{\sqrt{6}}\left[\left(3\cos^2\theta-1\right)\left(1+\vec{\zeta}_1\vec{\zeta}_2\right) - 2\left(\vec{\zeta}_1^{\perp}\vec{\zeta}_2^{\perp}\right) + 6\left(\vec{n}\vec{\zeta}_1^{\perp}\right)\left(\vec{n}\vec{\zeta}_2^{\perp}\right)\right].$$

At $\theta=\varphi=0$ they pass into the corresponding expressions of the Ref. 15 . Note, that the nature of the dependence of virtual photon polarization and its alignment on the polarization of primary electrons and positrons [15] is qualitatively unchanged in the considered general case.



e^-
K_1
γ
q
K_2
e^+
P_x
X
P_2
h_2
P_1
h_1

Fig.1



z'
z
y
$\vec{n}_1$
$\vec{n}_2$
$\vec{v}$
$\vec{e}'$
θ_1
θ
$\vec{e}^\perp$
y'
γ
$\vec{e}''$
x'
x

Fig.2

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ОБ ИНКЛЮЗИВНОЙ АННИГИЛЯЦИИ ПОЛЯРИЗОВАННОЙ
e^+e^- -ПАРЫ С ДВУМЯ ВЫДЕЛЕННЫМИ АДРОНАМИ
(на английском языке)
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