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ON THE INCLUSIVE REACTION $e^+e^- \to VX$ WITH REGARD FOR POLARIZATION STATES OF GENERATED VECTOR MESON

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Г.Н.ХАЧАТРЯН,Ю.Г.ШАХНАЗАРЯН

ОБ ИНКЛЮЗИВНОЙ РЕАКЦИИ $e^+e^- \to VX$ С УЧЕТОМ ПОЛЯРИЗАЦИОННЫХ СОСТОЯНИЙ ОБРАЗУЮЩЕГОСЯ ВЕКТОРНОГО МЕЗОНА

Рассмотрен инклюзивный процесс $e^+ e^- \to VX$ с учетом поляризационных состояний векторного мезона. Построен тензор, описывающий вершину перехода $\gamma^* \to VX$, который в общем случае содержит восемь структурных функций. В спиральном представлении вычислены элементы матрицы плотности образующегося векторного мезона, которые выражаются через указанные структурные функции и векторы поляризации аннигилирующих частиц. Показано, что на основе изучения углового распределения продуктов распада векторного мезона на псевдоскалярные частицы ($\rho \to 2\pi$, $\omega \to 3\pi$, $\varphi \to 2K$) и лептон-антилептонную пару ($\psi, \psi' \to \ell^+ \ell^-$) можно определить эти структурные функции.

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ON THE INCLUSIVE REACTION $e^+e^- \to VX$ WITH REGARD FOR POLARIZATION STATES OF GENERATED VECTOR MESON

The inclusive process $e^+e^- \to VX$ with regard for polarization states of vector meson is considered. The tensor describing the transition vertex $\gamma^* \to VX$, that comprises eight structure functions in general case, is constructed. In the helicity representation the elements of the density matrix of the generated vector meson are expressed through the mentioned structure functions and polarization vectors of the annihilating particles. It is shown, that studying the angular distribution of products of vector meson decay into pseudoscalar particles ($\rho \to 2\pi$, $\omega \to 3\pi$, $\varphi \to 2K$) and a lepton-antilepton pair($\psi, \psi' \to l^+l^-$) it is possible to determine these structure functions.

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1. It is well known, that in the case, when we are not interested in the detected hadron polarization, the inclusive process $e^+e^- \to VX$ is described by two structure functions, and the cross-section of such a process has the universal form, independent of the spin of h (see,e.g., [1]). For concrete hadrons the difference is in the structure functions themselves. It's obvious, that taking into account the polarization of detected particle in the final state leads to the increase in the number of structure functions, describing the corresponding inclusive process, and to the modification of the cross-section, which appearence will depend upon the spin of h. For stable hadrons the polarization measuring in the final state involves considerable experimental difficulties. In the case, when the unstable hadron is the detected particle, the situation differs. It's well known that the study of the decay products angular distribution of the latter permits one to find cirtain combinations of density matrix elements of this particle that in their turn are connected with its polarization states.

The aim of our present work is to consider that case of the inclusive process

$$e^+ + e^- \to V + X \qquad (1)$$

when the summation isn't performed over vector meson polarizations, and to clear up the question, of what characteristics of process (1) the study of the angular distribution of decay products of the vector particle may give information. For these purposes we'll use the main decay modes of the well known vector mesons ($\rho \to 2\pi$, $\omega \to 3\pi$, $\varphi \to 2K$, $\psi, \psi' \to \ell^+\ell^-$). For decays into pseudoscalar particles, as it'll be shown below, the angular distribution is determined only by the vector meson alignment. And it is only the angular distribution of the longitudinally polarized leptons in the case of $V \to \ell^+\ell^-$ decay that the vecton polarization vector contributes to.

2. Now let's pass to the description of process (1), pictured in the diagram (see figure). Let's represent the differential cross-section of the process in the form

$$d\sigma = \frac{\alpha^2}{2S^2} L_{\mu\nu} T_{\mu\nu} \frac{d\vec{p}}{P_0} , \qquad (2)$$

where $S = -(K_1 + K_2)^2$ is the total c.m.s.* energy square; 4-momenta of particles are shown in the diagram.

The lepton tensor, describing the possible polarization of the electrons and positrons in the storage rings[2] is

$$L_{\mu\nu} = \frac{1}{2S} Sp\left[(1 - i\hat{a}_1\gamma_5)(m - i\hat{K}_1)\gamma_\mu(1 - i\hat{a}_2\gamma_5)(m + i\hat{K}_2)\gamma_\nu\right], \qquad (3)$$

*) In this work the metric $a_\mu b_\mu = \vec{a}\vec{b} - a_0 b_0$, $a_\mu = (\vec{a}, i a_0)$ is used.

where a_1 and a_2 are 4-vectors of electron and positron polarization.

The tensor $T_{\mu\nu}$ characterizing the transition $\gamma^* \to VX$ is defined as

$$T_{\mu\nu} = (2\pi)^3 \sum_X \langle \tau, p; P_X | J_\mu(0) | 0 \rangle^* \langle \tau, p; P_X | J_\nu(0) | 0 \rangle \delta(q - p - P_X), \quad (4)$$

where τ is the polarization index of vector meson, and the summation is carried out over all the possible states in the beam of non-observed hadrons. It must be constructed out of independent 4-momenta q and p, must comprise the part independent of polarization and depend linearly on vector particle 4-vector polarization a_ρ and tensor $D_{\alpha\beta}$, that characterize its alignment.

Taking into account the conditions [3]

$$p_\rho a_\rho = 0, \; p_\alpha D_{\alpha\beta} = p_\beta D_{\alpha\beta} = 0, \; D_{\alpha\beta} = D_{\beta\alpha}, \; D_{\alpha\alpha} = 0$$

and setting requirements of P and T invariance, and also hermitian condition following from the difinition (4), for the most general gauge-invariant form of $T_{\mu\nu}$ tensor we get the expression

$$\begin{aligned} T_{\mu\nu} = {} & \bar{\delta}_{\mu\nu} T_1 + \frac{1}{m_V^2} \bar{P}_\mu \bar{P}_\nu T_2 + \frac{1}{m_V} a_\rho e_{\mu\nu\gamma\sigma} \left(\delta_{\rho\gamma} q_\sigma T_3 + \frac{1}{m_V^2} q_\rho q_\gamma P_\sigma T_4 \right) + \\ & + D_{\alpha\beta} \left[\frac{1}{m_V^2} q_\alpha q_\beta \left(\bar{\delta}_{\mu\nu} T_5 + \frac{1}{m_V^2} \bar{P}_\mu \bar{P}_\nu T_6 \right) + \bar{\delta}_{\mu\alpha} \bar{\delta}_{\nu\beta} T_7 + \right. \\ & \left. + \frac{1}{m_V^2} q_\alpha \left(\bar{P}_\mu \bar{\delta}_{\nu\beta} + \bar{P}_\nu \bar{\delta}_{\mu\beta} \right) T_8 \right]. \end{aligned} \quad (5)$$

Here

$$\bar{\delta}_{\mu\nu} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \bar{P}_\mu = P_\mu - \frac{(qP)}{q^2} q_\mu,$$

m_V is the vector meson mass, and $T_1, \dots, T_8$ are demensionless real structure functions, dependent on two independent invariants: $s = -q^2$ and $\nu = (qp)$ or $M_X^2 = s + 2\nu + m_V^2$.

So in the general case of arbitrarily polarized initial particles the process(1) with regard for polarization states of the vector meson is characterized by eight structure functions. In case of unpolarized or transversely polarized leptons it's only symmetrical in μ and ν indexes part of tensor $T_{\mu\nu}$, comprising six structures, that make contribution (the two structures, comprising the polarization vector a_ρ don't make contribution).

Writing the current of the transition $\gamma^* \to VX$ in (4) as

$$\langle \tau, p; P_X | J_\nu(0) | 0 \rangle = \epsilon_\beta^{(\tau)*}(\vec{p}) \langle p; P_X | J_\nu^\beta(0) | 0 \rangle, \tag{6}$$

we'll also determine the tensor

$$t_{\mu\nu}^{\alpha\beta} = (2\pi)^3 \sum_X \langle p; P_X | J_\mu^\alpha(0) | 0 \rangle^* \langle p; P_X | J_\nu^\beta(0) | 0 \rangle \delta(q - p - P_X), \tag{7}$$

that we'll need later.

The tensor $t_{\mu\nu}^{\alpha\beta}$ should be constructe of 4-momenta q and p. It should be gauge-invariant in μ and ν ndexes, and based on condition $p_\rho \epsilon_\rho^{(\tau)}(\vec{p}) = 0$ shouldn't comprise p_α an p_β . P and T invariant form of this tensor, satisfing the hermit , is following:

$$\begin{aligned} t_{\mu\nu}^{\alpha\beta} = {} & \delta_{\alpha\beta}\left(\bar{\delta}_{\mu\nu} t_1 + \frac{1}{m_V^2} \bar{p}_\mu \bar{p}_\nu t_2\right) + \frac{1}{m_V^2} q_\alpha q_\beta \left(\bar{\delta}_{\mu\nu} t_3 + \frac{1}{m_V^2} \bar{p}_\mu \bar{p}_\nu t_4\right) + \\ & + \left(\bar{\delta}_{\mu\alpha} \bar{\delta}_{\nu\beta} t_5 + \bar{\delta}_{\mu\beta} \bar{\delta}_{\nu\alpha} t_6\right) + \frac{1}{m_V^2}\left(\bar{p}_\mu \bar{\delta}_{\nu\alpha} q_\beta + \bar{p}_\nu \bar{\delta}_{\mu\beta} q_\alpha\right) t_7 + \\ & + \frac{1}{m_V^2}\left(\bar{p}_\mu \bar{\delta}_{\nu\beta} q_\alpha - \bar{p}_\nu \bar{\delta}_{\mu\alpha} q_\beta\right) t_8 . \end{aligned} \tag{8}$$

As it was to be expected, it also contained eight independent structures with real factors, dependent on two invariants.

The tensors $T_{\mu\nu}$ and $t^{\alpha\beta}_{\mu\nu}$ are connected by the condition

$$T_{\mu\nu} - \mathcal{P}_{\alpha\beta}\, t^{\alpha\beta}_{\mu\nu}\,, \tag{9}$$

where $\mathcal{P}_{\alpha\beta}$ is the covariant density matrix for spin 1 particles (see, e.g.,[3]), that in our metric looks as

$$\mathcal{P}_{\alpha\beta} = \epsilon^{(\tau)}_{\alpha}(\vec{s})\epsilon^{(\tau)*}_{\beta}(\vec{p}) = \frac{1}{3}\Big(\mathcal{E}_{\alpha\beta} + \frac{p_\alpha p_\beta}{m_v^2} + \frac{3}{2m_v}\, e_{\alpha\beta\rho\sigma} a_\rho p_\sigma - \frac{3}{2} D_{\alpha\beta}\Big). \tag{10}$$

Using the condition (9), one can establish the following relation between the structure functions forming (5) and (8):

$$\begin{aligned}
T_1 &= t_1 + \frac{1}{3m_v^2}\Big(\frac{\nu^2}{m_v^2} - s\Big) t_3 + \frac{1}{3}(t_5 + t_6),\\
T_2 &= t_2 + \frac{1}{3m_v^2}\Big(\frac{\nu^2}{m_v^2} - s\Big) t_4 + \frac{1}{3}(t_5 + t_6) + \frac{2}{3}\,\frac{\nu}{m_v^2}(t_7 + t_8),\\
T_3 &= -\frac{\nu}{2s}(t_5 - t_6) + \frac{1}{2s}\Big(\frac{\nu^2}{m^2} - s\Big)(t_7 - t_8),\\
T_4 &= -\frac{m_v^2}{2s}(t_5 - t_6) + \frac{\nu}{2s}(t_7 - t_8),\\
T_5 &= -\frac{1}{2} t_3\,, \qquad T_6 = -\frac{1}{2} t_4\,,\\
T_7 &= -\frac{1}{2}(t_5 + t_6), \quad T_8 = -\frac{1}{2}(t_7 + t_8).
\end{aligned} \tag{11}$$

Let's also find the relation between these values and the structure functions $\overline{W}_1$ and $\overline{W}_2$, determining the cross section of the process(1) in the case, when the summation over the vector particle polarization is made

$$d\sigma = \frac{\alpha^2}{s^2} R\, \frac{d\vec{p}}{p_0}\,.$$

$$R = (1 + \xi_1^{\parallel}\xi_2^{\parallel})\,\overline{W}_1 + \frac{\vec{p}^{\,2}}{2m_v^2}\Big[(1 + \vec{\xi}_1\vec{\xi}_2)\sin^2\theta - 2(\vec{n}\vec{\xi}_1^{\perp})(\vec{n}\vec{\xi}_2^{\perp})\Big]\overline{W}_2\,, \tag{12}$$

where $\vec{n}$ is the unit vector along the vector meson momentum $\vec{p}$ in c.m.s., $\vec{\xi}_1$ and $\vec{\xi}_2$ are the polarization vectors of electron and positron in their own rest frame, and their resolution into the transversal and longitudinal components is relative to the electron momentum.

Performing the summation over the vector particle polarizations in (2), we find

$$\bar{W}_1 = 3T_1 \, , \quad \bar{W}_2 = 3T_2 \, , \tag{13}$$

that, according to (11), permits one to devide the $\bar{W}_1$ and $\bar{W}_2$ into the composite parts, that can be measured by means of the angular distributions of the vecton decay products.

3. Let's determine the polarization density matrix of the vector meson in the helicity representation

$$\rho_{\lambda_v \lambda'_v} = \frac{M_{if}^{(\lambda_v)} M_{if}^{(\lambda'_v)+}}{\sum\limits_{\lambda_v} |M_{if}^{(\lambda_v)}|^2} \, , \tag{14}$$

where $M_{if}^{(\lambda_v)}$ is the amplitude of the process (1) of the generation of the vector meson with the helicity λ_v. The summation over all the possible states in X beam is implied in the numerator, as well as the summation over the vecton polarization in denominator.

It's easy to see, that $\rho_{\lambda_v \lambda'_v}$ is expressed through tensor $t_{\mu\nu}^{\alpha\beta}$ and looks like

$$\rho_{\lambda_v \lambda'_v} = R_{\lambda_v \lambda'_v} / R \, , \tag{15}$$

$$R_{\lambda_v \lambda'_v} = \frac{1}{2} L_{\mu\nu} t_{\mu\nu}^{\alpha\beta} \epsilon_\alpha^{(\lambda'_v)}(\vec{p}) \, \epsilon_\beta^{(\lambda_v)*}(\vec{p}) , \tag{16}$$

where $\epsilon_\beta^{(\lambda_v)}(\vec{p})$ is the helicity state of the vector particle, and the quantity

$$R = \sum_{\lambda_v} R_{\lambda_v\lambda_v} = \frac{1}{2} L_{\mu\nu} t^{\alpha\beta}_{\mu\nu} (\delta_{\alpha\beta} + \frac{P_\alpha P_\beta}{m_v^2})$$

determines the process (1) cross-section in the case of unpolarized vectons and has the form (12).

We calculate the density matrix elements $\rho_{\lambda_v\lambda_v'}$ according to formulae (15) and (16) in c.m.s., where lepton tensor has only spatial components and is determined by the expression [4]

$$L_{ik} = (1+\vec{\zeta}_1\vec{\zeta}_2)(\delta_{ik} - v_i v_k) - (\zeta^\perp_{1i}\zeta^\perp_{2k} + \zeta^\perp_{2i}\zeta^\perp_{1k}) + i(\zeta_1'' + \zeta_2'') e_{ik\ell} v_\ell, \tag{17}$$

$\vec{v}$ is a unit vector along the ele..ron momentum.

For the corresponding elements of density matrix (15) we get

$$\begin{aligned}
\rho_{\pm1\pm1} &= \frac{1}{R}\Big\{\Big[t_1 + \frac{\vec{p}^2}{2m_v^2} t_2 \sin^2\theta + \frac{1}{4}(t_5 + t_6)(1+\cos^2\theta)\Big](1+\zeta_1''\zeta_2'') - \\
&- \frac{1}{4}\Big[2\frac{\vec{p}^2}{m_v^2} t_2 - (t_5 + t_6)\Big]\sin^2\theta\cos(\varphi_1+\varphi_2)\zeta_1^\perp\zeta_2^\perp \pm \frac{1}{2}(t_5 - t_6)\cos\theta(\zeta_1'' + \zeta_2'')\Big\}, \\
\rho_{00} &= \frac{1}{R}\Big\{(t_1 + \frac{s\vec{p}^2}{m_v^4} t_3)(1+\zeta_1''\zeta_2'') + \frac{\vec{p}^2}{2m_v^2}\Big[t_2 + \frac{s\vec{p}^2}{m_v^4} t_4 + \frac{p_0^2}{\vec{p}^2}(t_5 + t_6) - \\
&- \frac{2\sqrt{s}\, p_0}{m_v^2}(t_7 + t_8)\Big]\sin^2\theta\Big[1 + \zeta_1''\zeta_2'' - \cos(\varphi_1+\varphi_2)\zeta_1^\perp\zeta_2^\perp\Big]\Big\}, \\
\rho_{\pm10} &= \rho^*_{0\pm1} = \mp\frac{1}{2\sqrt{2}R}\sin\theta\Big\{\Big[\frac{p_0}{m_v}(t_5 + t_6) - \frac{\sqrt{s}\,\vec{p}^2}{m_v^3}(t_7 + t_8)\Big]\times \\
&\times\Big[\cos\theta\big(1 + \zeta_1''\zeta_2'' - \cos(\varphi_1+\varphi_2)\zeta_1^\perp\zeta_2^\perp\big) \pm i\sin(\varphi_1+\varphi_2)\zeta_1^\perp\zeta_2^\perp\Big] \pm \\
&\pm\Big[\frac{p_0}{m_v}(t_5 - t_6) + \frac{\sqrt{s}\,\vec{p}^2}{m_v^3}(t_7 - t_8)\Big](\zeta_1'' + \zeta_2'')\Big\}, \\
\rho_{1-1} &= \rho^*_{-11} = \frac{1}{4R}(t_5 + t_6)\Big\{\sin^2\theta(1+\zeta_1''\zeta_2'') + \Big[(1+\cos^2\theta)\cos(\varphi_1+\varphi_2) - \\
&- 2i\cos\theta\sin(\varphi_1+\varphi_2)\Big]\zeta_1^\perp\zeta_2^\perp\Big\},
\end{aligned} \tag{18}$$

where θ is the angle between $\vec{v}$ and $\vec{n}$, and φ_1 and φ_2 are the angles made by $\vec{\zeta}_1^{\perp}$ and $\vec{\zeta}_2^{\perp}$ with the reaction plane $(\vec{v},\vec{n})$ measured in the plane, normal to $\vec{v}$.

Though the matrix elements (18) are calculated in c.m.s., it's easy to show using [7], that they determine the angular distribution of the decay products of the vector meson in its own rest frame, with z axis along the vecton momentum $\vec{n}$ in c.m.s. of reaction (1) (helicity system).

At first, let's consider the vector meson decay into the pseudoscalar particles. Denote by θ' and φ' the polar and azimuth angles (the latter is measured from the plane of reaction $(\vec{v},\vec{n})$) of the unit vector $\vec{r}$, characterizing the flight direction of one of the decay particles in the case of two particle decay ($\rho \to 2\pi$, $\varphi \to 2K$), and the direction of the normal to the decay plane in the case of three particle decay ($\omega \to 3\pi$). The normalized angular distribution for mentioned decays looks like[5]

$$W(\theta',\varphi') = \frac{3}{4\pi}\Big[\frac{1}{2}(\rho_{11}+\rho_{-1-1})\sin^2\theta' + \rho_{00}\cos^2\theta' - \frac{1}{\sqrt{2}}(Re\,\rho_{10} - Re\,\rho_{-10})\sin 2\theta'\cos\varphi' + \frac{1}{\sqrt{2}}(Im\,\rho_{10} + Im\,\rho_{-10})\sin 2\theta'\sin\varphi' - Re\,\rho_{1-1}\sin^2\theta'\cos 2\varphi' + Im\,\rho_{1-1}\sin^2\theta'\sin 2\varphi'\Big]. \tag{19}$$

Using (18), let's write out the density matrix elements (the combinations of elements) that can be found by means of comparison of the distribution (19) with the experiment:

$$\frac{1}{2}(\rho_{11}+\rho_{-1-1}) = \frac{1}{R}\Big\{\Big[t_1 + \frac{\vec{p}^2}{2m_v^2}t_2\sin^2\theta + \frac{1}{4}(t_5+t_6)(1+\cos^2\theta)\Big](1+\zeta_1^{\parallel}\zeta_2^{\parallel}) - \frac{1}{4}\Big[2\frac{\vec{p}^2}{m_v^2}t_1 - (t_5+t_6)\Big]\sin^2\theta\cos(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp}\Big\},$$

$$\rho_{00} = \frac{1}{R}\Big\{(t_1 + \frac{s\vec{p}^{\,2}}{m_v^4}t_3)(1+\zeta_1^{\parallel}\zeta_2^{\parallel}) + \frac{\vec{p}^{\,2}}{2m_v^2}\Big[t_2 + \frac{s\vec{p}^{\,2}}{m_v^4}t_4 + \frac{p_0^2}{\vec{p}^{\,2}}(t_5+t_6) -$$
$$- \frac{2\sqrt{s}\,p_0}{m_v^2}(t_7+t_8)\Big]\sin^2\theta\Big[1+\zeta_1^{\parallel}\zeta_2^{\parallel} - \cos(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp}\Big]\Big\}, \qquad (20)$$

$$Re\rho_{10} - Re\rho_{-10} = -\frac{1}{2\sqrt{2}R}\Big[\frac{p_0}{m_v}(t_5+t_6) - \frac{\sqrt{s}\,\vec{p}^{\,2}}{m_v^3}(t_7+t_8)\Big]\sin 2\theta \times$$
$$\times\Big[1+\zeta_1^{\parallel}\zeta_2^{\parallel} - \cos(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp}\Big],$$

$$\mathcal{Y}m\rho_{10} = \mathcal{Y}m\rho_{-10} = -\frac{1}{2\sqrt{2}R}\Big[\frac{p_0}{m_v}(t_5+t_6) - \frac{\sqrt{s}\,\vec{p}^{\,2}}{m_v^3}(t_7+t_8)\Big]\sin\theta\sin(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp},$$

$$Re\rho_{1-1} = \frac{1}{4R}(t_5+t_6)\Big[\sin^2\theta(1+\zeta_1^{\parallel}\zeta_2^{\parallel}) + (1+\cos^2\theta)\cos(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp}\Big],$$

$$\mathcal{Y}m\rho_{1-1} = -\frac{1}{2R}(t_5+t_6)\cos\theta\sin(\varphi_1+\varphi_2)\zeta_1^{\perp}\zeta_2^{\perp}.$$

The measuring of the cross-section (12) at the given angle θ permits one to find R , and the study of the cross-section dependence on θ gives one the possibility to **separate** $\bar{W}_1$ and $\bar{W}_2$, that, according to (13) and 11), are exspressed as

$$\bar{W}_1 = 3t_1 + \frac{s\vec{p}^{\,2}}{m_v^4}t_3 + (t_5+t_6),$$
$$\bar{W}_2 = 3t_2 + \frac{s\vec{p}^{\,2}}{m_v^4}t_4 + (t_5+t_6) - \frac{2\sqrt{s}\,p_0}{m_v^2}(t_7+t_8). \qquad (21)$$

As it follows from (20), some of the structure function combinations entering (18), viz. $t_5 - t_6$ and $t_7 - t_8$, don't enter the matrix element combinations, that can be found by the study of the decay angular distribution of vector meson into the pseudoscalar particles. From the other side, one can see from (11) and (5) that the mentioned combinations of the structure functions determine the contribution of the vecton polarization vector, therefore one can say, that the polarization vector of the vector particle doesn't contribute to the considered decay. It agrees with the statement [7] , that the study of the angular distribution of vector meson decay into the pseudoscalar particles permits us to find only the vector meson alignment.

There are six independent quantities $t_1,\dots,t_4$, t_5+t_6 and t_7+t_8 in the expressions (20) and (21), that should be defined. Even in the case of unpolarized initial particles the equations (20) and (21) allow to determine these structure functions already at any fixed θ angle, and if one takes into account the possibility of studying the matrix elements dependence on θ, then we'll get some additional relations, that can be used for check up. In the case, when lepton polarisation vectors have the transversal components, one can also study the dependence of matrix elements (20) on the azimuth angles φ_1 and φ_2 .

In the interesting case of antiparallel transversely polarized primaries it's necessary to put $\zeta_1^{\parallel}=\zeta_2^{\parallel}=0$ and $\varphi_1=\varphi$, $\varphi_2=\varphi+\pi$ in (20). If the annihilating particles have only the longitudinal polarization, then the non-vanishing combinations (20) contain the factor $1+\zeta_1^{\parallel}\zeta_2^{\parallel}$, expressing the helicity conservation of relativistic leptons. As it follows from (19) and (20) the polarization of primaries essentially affects the angular distribution of the vector meson decay products. So in the case of unpolarized or just longitudinally polarized particles the terms containing $\mathcal{Im}\rho_{10}$ and $\mathcal{Im}\rho_{1-1}$ don't enter the distribution (19) at all.

Finishing this section we note, that the ratios

$$\frac{\mathcal{Im}\rho_{10}+\mathcal{Im}\rho_{-10}}{Re\,\rho_{10}-Re\,\rho_{-10}}=\frac{\sin(\varphi_1+\varphi_2)\,\zeta_1^{\perp}\zeta_2^{\perp}}{\cos\theta\left[1+\zeta_1^{\parallel}\zeta_2^{\parallel}-\cos(\varphi_1+\varphi_2)\,\zeta_1^{\perp}\zeta_2^{\perp}\right]}\;, \qquad (22)$$

$$\frac{\mathcal{Im}\,\rho_{1-1}}{Re\,\rho_{1-1}}=\frac{-2\cos\theta\,\sin(\varphi_1+\varphi_2)\,\zeta_1^{\perp}\zeta_2^{\perp}}{\sin^2\theta(1+\zeta_1^{\parallel}\zeta_2^{\parallel})+(1+\cos^2\theta)\cos(\varphi_1+\varphi_2)\,\zeta_1^{\perp}\zeta_2^{\perp}}$$

don't comprise the unknown parameters and are determined by the scattering angle θ and the polarization of the primaries. This means that at the fixed lepton polarization and at the given angle θ the mentioned ratio mustn't depend on the energy of reaction $\sqrt{S}$ and the detected vecton energy P_0.

4. Now, let's consider the decay of vector meson into the lepton pair. Note, that for ψ and ψ' resonances the decays into the electron- positron and muonic pair compose the noticable part and are used for identification of these resonances.

Let the unit vector $\vec{r}(\theta',\varphi')$ determine the direction of flight of electron (μ^--meson) in the helicity system. Then for the angular distribution of the decay products in the vector meson rest frame we get an expression

$$\frac{dN}{d\cos\theta' d\varphi'}=\alpha^2\frac{m_V}{2f_V^2}\sqrt{1-\frac{4m^2}{m_V^2}}\;W(\theta',\varphi'), \tag{23}$$

$$W(\theta',\varphi')=\frac{1}{4}L_{iK}\sum_{\lambda_V\lambda_V'}\rho_{\lambda_V\lambda_V'}\,\epsilon_i^{(\lambda_V)}(0)\,\epsilon_K^{(\lambda_V')*}(0),$$

where m is the lepton mass, f_V is the coupling constant, characterizing $V-\gamma^*$ transition, and L_{iK} is the lepton tensor, describing the decay products.

We'll also take onto account the longitudinal polarization of decayed leptons, because, as it's known, the further decay of μ-mesons is the analizer of their polarization. Accordingly we can write down the lepton tensor as

$$L_{ik}=(\delta_{ik}-r_i r_k)(1+\xi_1''\xi_2'')+\frac{4m^2}{m_V^2}r_i r_k(1-\xi_1''\xi_2'')+ \qquad (24)$$

$$+i(\xi_1''+\xi_2'')e_{ik\ell}r_\ell\,,$$

where ξ_1'' and ξ_2'' determine the degree of ℓ^- and ℓ^+ longitudinal polarization. To include in the consideration the vector mesons, that might have decayed into the pair of heavy leptons, we preserve in (24) the terms, comprising lepton mass. Substituting (24) into (23), we get the expression for angular distribution

$$\begin{aligned}W(\theta',\varphi')=\frac{1}{4}\Big\{&\frac{1}{2}(\rho_{11}+\rho_{-1-1})\Big[(1+\cos^2\theta')(1+\xi_1''\xi_2'')+\frac{4m^2}{m_V^2}\sin^2\theta'(1-\xi_1''\xi_2'')\Big]+\\&+\rho_{00}\Big[\sin^2\theta'(1+\xi_1''\xi_2'')+\frac{4m^2}{m_V^2}\cos^2\theta'(1-\xi_1''\xi_2'')\Big]+\Big[\Big(\frac{1}{\sqrt{2}}(Re\rho_{10}-Re\rho_{-10})\cos\varphi'-\\&-\frac{1}{\sqrt{2}}(Im\rho_{10}+Im\rho_{-10})\sin\varphi'\Big)\sin 2\theta'+(Re\rho_{1-1}\cos 2\varphi'-Im\rho_{1-1}\sin 2\varphi')\sin^2\theta'\Big]\times\\&\times\Big[1+\xi_1''\xi_2''-\frac{4m^2}{m_V^2}(1-\xi_1''\xi_2'')\Big]+\Big[(\rho_{11}-\rho_{-1-1})\cos\theta'+\Big(\sqrt{2}(Re\rho_{10}+Re\rho_{-10})\cos\varphi'-\\&-\sqrt{2}(Im\rho_{10}-Im\rho_{-10})\sin\varphi'\Big)\sin\theta'\Big](\xi_1''+\xi_2'')\Big\} \qquad (25)\end{aligned}$$

In particular case, when we are not interested in the polarization of the decay leptons, we get from (25)

$$\begin{aligned}W^0(\theta',\varphi')=&\frac{1}{2}(\rho_{11}+\rho_{-1-1})\Big(1+\cos^2\theta'+\frac{4m^2}{m_V^2}\sin^2\theta'\Big)+\rho_{00}\Big(\sin^2\theta'+\frac{4m^2}{m_V^2}\cos^2\theta'\Big)+\\&+\Big[\Big(\frac{1}{\sqrt{2}}(Re\rho_{10}-Re\rho_{-10})\cos\varphi'-\frac{1}{\sqrt{2}}(Im\rho_{10}+Im\rho_{-10})\sin\varphi'\Big)\sin 2\theta'+\\&+(Re\rho_{1-1}\cos 2\varphi'-Im\rho_{1-1}\sin 2\varphi')\sin^2\theta'\Big]\Big(1-\frac{4m^2}{m_V^2}\Big). \qquad (26)\end{aligned}$$

Substituting (26) into (23) and integrating over the angles with regard for condition $Sp\rho=1$, we come to the well-known expression for $V\to \ell^+\ell^-$ decay width

$$\Gamma(V\to\ell^+\ell^-)=\frac{1}{3}\frac{\alpha^2}{f^2/4\pi}m_v\left(1+\frac{2m^2}{m_v^2}\right)\sqrt{1-\frac{4m^2}{m_v^2}}\ . \qquad (27)$$

So, if one doesn ... e the longitudinal polarization of the final leptons, then, though the $V\to\ell^+\ell^-$ decay angular distribution (26) differs from (19), it permits one to find the same combinations of $\rho_{\lambda_v\lambda_v'}$, that characterize the decay to pseudoscalar particles, and $V\to\ell^+\ell^-$ decay is also determined by the **alignment** of the vector meson.

If the longitudinal polarization of the decay leptons is measured in the experiment, then, as it follows from (25), the angular distribution contains new combinations of density matrix elements, that will permit one to determine t_5-t_6 and t_7-t_8. The latter, as it was shown earlier, stipulate the contribution of the polarization vector of the vecton to the angular distribution of decay products.

These new combinations of density matrix elements is most easy to fin by means of studying the angular dependence of the difference of distributions (25), corresponding to the helicities $\xi_1''=\xi''=\pm 1$

$$W^{+1+1}(\theta',\varphi')-W^{-1-1}(\theta',\varphi')=(\rho_{11}-\rho_{-1-1})\cos\theta'+\left[\sqrt{2}(Re\rho_{10}+Re\rho_{-10})\cos\varphi'-\right.$$

$$\left.-\sqrt{2}(\Im m\rho_{10}-\Im m\rho_{-10})\sin\varphi'\right]\sin\theta'. \qquad (28)$$

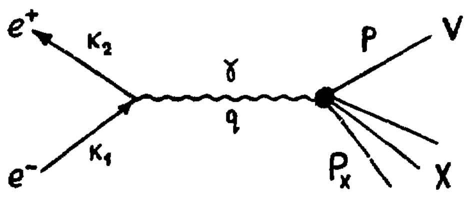
For the combinations in question we get from (18)

$$\rho_{11}-\rho_{-1-1}=\frac{1}{R}(t_5-t_6)\cos\theta(\zeta_1''+\zeta_2''),$$

$$Re\rho_{10}+Re\rho_{-10}=-\frac{1}{\sqrt{2}R}\left[\frac{p_0}{m_V}(t_5-t_6)+\frac{\sqrt{s}\,\vec{p}^{\,2}}{m_V^3}(t_7-t_8)\right]\sin\theta(\zeta_1''+\zeta_2''), \quad (29)$$

$$\mathcal{Im}\rho_{10}-\mathcal{Im}\rho_{-10}=0.$$

As it follows from written above, using the $V\to\ell^+\ell^-$ decay one canseparate all the structure functions, determining the transition vertex $\gamma^*\to VX$, if in the production reaction (1) the annihilating electron or positron has the longitudinal component of polarization and if one also measures the longitudinal polarization of one of the final leptons.



e+
K2
e-
K1
γ
q
P
V
Px
X

R E F E R E N C E S

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