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Г.К.САВВИДИ

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И АСИМПТОТИЧЕСКАЯ СВОБОДА

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INFRARED INSTABILITY OF THE VACUUM STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

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INFRARED INSTABILITY OF THE VACUUM STATE OF GAUGE THEORIES AND ASYMPTOTIC FREEDOM

Yerevan, 1977

D.J.Gross, F.Wilezek and H.D.Politzer have found that non-Abelian gauge theories are "asymptotically free"[1]. Physical application of this discovery needs a solution of the problems connected first with infrared singularities of this theory and, secondly, with quarks confinement. A number of authors consider the solution of the infrared problem to be a natural explanation of quarks confinement.

It is shown in this paper that infrared singularities of Yang-Mills' massless theory cause the vacuum state instability. We come to this conclusion through analysis of one-loop correction to Yang-Mills classical action due to vacuum polarization. Such an instability during the interaction of the gauge fields and quarks seems to cause their confinement.

Let us consider the effective action Γ for a pure Yang-Mills theory.

$$\Gamma = \sum \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \, \Gamma^{(n)\, a_1 \ldots a_n}_{\mu_1 \ldots \mu_n} A^{a_1}_{\mu_1} \ldots A^{a_n}_{\mu_n} =$$

$$= S^{cl}_{Y.M.} + W^{(1)} + W^{(2)} + \ldots \qquad (1)$$

where S^{cl}_{YM} is the classical action, $W^{(1)}$ is the one-loop correction and $W^{(2)}$ is the two-loop correction, etc. It is evident from the generalized Word identities that the effective action is a gauge-invariant value for fields satisfying source-free fields equations

$$\nabla^{ab}_{\mu} G^{b}_{\mu\nu} = 0 \tag{2}$$

It is necessary to use expansion in powers of momenta instead of expansion in powers of field A. In position space such on expansion looks like:

$$\Gamma = \int d^4x \left\{ \bar{\mathcal{L}} + \tilde{\mathcal{L}} + \tilde{\tilde{\mathcal{L}}} + \cdots \right\} \tag{3}$$

From the standpoint of the gauge invariance it follows that $\bar{\mathcal{L}}$ function depends only on invariances $\mathcal{F} = \frac{1}{4} G^{a}_{\mu\nu} G^{a}_{\mu\nu}$ and $\mathcal{G} = \frac{1}{4} G^{a}_{\mu\nu} G^{*a}_{\mu\nu}$ but does not depend on covariant derivatives $G^{a}_{\mu\nu}$. $\tilde{\mathcal{L}}$ function depends on invariant combinations composed by using one-step covariant derivative $G^{a}_{\mu\nu}$ etc. To calculate $\bar{\mathcal{L}}$ function it is necessary to set renormalization condition. As in manifest covariant formulation of Yang-Mills theory used in this paper the counter term has the following universal form

$$Z \cdot S^{cl}_{Y.M.} \tag{4}$$

the usual renormalization conditions are

$$\frac{\partial \bar{\mathcal{L}}}{\partial \mathcal{F}} \bigg|_{t = \ln\left(\frac{\mathcal{F}}{\mu^2}\right) = \mathcal{G} = 0} = -1 \tag{5}$$

The function $-\bar{\mathcal{L}}$, in some sense, is analogous with Coleman-Weinberg effective potential [2] and represents the negative sum of all nonderivative (covariant) terms in the Lagrange density.

Let us consider a covariant constant field

$$\nabla^{ab}_{\rho} G^{b}_{\mu\nu} = 0 \tag{6}$$

for which $\tilde{\mathcal{L}} = \tilde{\tilde{\mathcal{L}}} = \cdots = 0$. The common gauge-invariant solution of equation (6) is $\hat{A}_{\mu} = -\frac{1}{2} \hat{G}_{\mu\nu} x_{\nu} - g^{-1} S^{-1} \partial_{\mu} S$ where S is an arbitrary matrix in the adjoint representation of group. Having set the gauge condition $S^{-1}\partial_{\mu} S$ for $SU(2)$ group we get [3]:

$$A^{a}_{\mu} = -\frac{1}{2} F_{\mu\nu} x_{\nu} \cdot n^{a} \tag{7}$$

where $F_{\mu\nu}$ and n^a do not depend on x and $n^a n^a = 1$. Invariances $\mathcal{F}$ and $\mathcal{G}$ become equal to $\frac{1}{2}(\vec{H}^2 - \vec{E}^2)$ and $\bar{E} \cdot \bar{H}$ respectively, where $\bar{E}$ and $\bar{H}$ are defined in terms of $F_{\mu\nu}$ as it is the case in QED.

One - loop correction to classical action for the field (7) has the form [3].

$$\mathcal{L}^{(1)} = 2 \cdot \frac{1}{16\pi^2} \int \frac{ds}{s^3} \frac{g f_1 s \cdot g f_2 s}{sh(g f_1 s) \sin(g f_2 s)} + \tag{8}$$

$$+ \frac{1}{4\pi^2} \int \frac{ds}{s^3} g f_1 s \cdot g f_2 s \left\{ \frac{\sin(g f_1 s)}{sh(g f_2 s)} - \frac{\sin(g f_2 s)}{sh(g f_1 s)} \right\}$$

where $f_1^2 = \mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$ and $f_2^2 = -\mathcal{F} + (\mathcal{F}^2 + \mathcal{G}^2)^{1/2}$
In case of "magnetic" field $\mathcal{F} > 0$, $\mathcal{G} = 0$ (8) is

$$\mathcal{L}^{(1)} = \frac{1}{8\pi^2}\int \frac{ds}{s^3}\,\frac{gHs}{sh(gHs)} + \frac{1}{4\pi^2}\int \frac{ds}{s^3}(gHs)\,Sin\,(gHs) \qquad (9)$$

To renormalize (9) it is necessary to use (4) and (5). The result of the replacement of the variable S by $\frac{x}{\mu^2}$ will be:

$$\mathcal{L}^{(1)} = \mu^4 \frac{1}{8\pi^2}\int_0^\infty \frac{dx}{x^3}\left\{ \frac{ax}{sh\,(ax)} - 1 - \frac{a^2x}{2}\left(\frac{1}{shx} - \frac{x\,chx}{sh^2x}\right)\right\} + \mu^4\frac{1}{4\pi^2}\int_0^\infty \frac{dx}{x^3}\left\{ax\,Sin(ax) - \frac{a^2x}{2}(\sin x + x\cos x)\right\} \qquad (10)$$

where $a = \frac{H}{\mu^2}$. After the integration over X we have

$$\mathcal{L}^{(1)} = -\frac{11(gH)^2}{48\pi^2}\left[\ln\left(\frac{H}{\mu^2}\right) + \frac{1}{22}\right] \qquad (11)$$

Thus the energy density looks like

$$\mathcal{E} = \frac{H^2}{2} + \frac{11(gH)^2}{48\pi^2}\left[\ln\left(\frac{gH}{\mu^2}\right) + \frac{1}{22}\right] \qquad (12)$$

having its new minimum outside the point $H = 0$ - that is to say the one loop corrections have generated spontaneous symmetry breaking.

From the standpoint of renormalization group we have the following result [3]

$$\frac{\partial\bar{\mathcal{L}}}{\partial\mathcal{F}} = -\frac{g^2}{\bar{g}^2} \qquad (13)$$

where $\frac{d\bar{g}}{dt} = \bar{\beta}(\bar{g})$ In one-loop approximation the equation (13) represents the leading logarithmic term in (11) and, vice versa, it is possible to calculate the function β in one-loop approximation[1]. At the same time equation(13) makes sure that multiloop (three or more) corrections will not change this situation[2].

In conclusion it can be noted that on the basis of the above effect it is possible to develop a phenomenological picture leading to quarks confinement. Details will be given in the next paper.

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1) $\bar{\beta} = -\frac{g}{2}\cdot\frac{\partial\left(\frac{\partial\bar{\mathcal{L}}}{\partial\mathcal{F}}\right)}{\partial t}\Big|_{t=0}$

2) It's easy to see from (13) and from the result obtained in (4), that two-loop correction doesn't change this situation.

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