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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ЕФИ-226-(19)-77

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ON THE STABILITY OF TWO-DIMENSIONAL TIRRING MODEL

ԵՐԵՎԱՆ 1977 ЕРЕВАН

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Р.П. ГРИГОРЯН

К ВОПРОСУ О СТАБИЛЬНОСТИ ДВУМЕРНОЙ МОДЕЛИ ТИРРИНГА.

Рассмотрен вопрос устойчивости двумерной массивной модели Тирринга, включающей составные поля типа $\bar{\psi}^i\psi^i$ (ψ^i N-компонентный спинор в изотопическом пространстве; $i=1,2,..N$) Найдено выражение для асимптотики эффективного потенциала составного поля и показано, что при N =1,2 при достаточно малых g (g- константа взаимодействия) теория является устойчивой. Рассмотрена эта же теория в пределе больших N $(N\sim\frac{1}{g^2})$; приведены аргументы, показывающие, что для значений $N>2$ теория является неустойчивой.

Ереванский физический институт
Ереван 1977

ЕФИ-226-(19)-77

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ON THE STABILITY OF TWO-DIMENSIONAL TIRRING MODEL

The question of the stability of two-dimensional massive Tirring model, including composite fields of the type $\bar{\psi}^i \psi^i$ (ψ^i – N-component spinor in the isotopic space) is considered. The expression for the asymptotics of the effective potential of composite field is found and it's shown that for $N=1,2$ with g being rather small, (g is the interaction constant) the potential is stable. The same theory is considered in the limit of large N $\left(N \sim \frac{1}{g^2}\right)$; the arguments showing that for values $N>2$ the theory is unstable, are given.

Yerevan Physics Institute

Yerevan 1977

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ЕФИ-226-(I9)-77

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Interaction models, including composite fields are intensively discussed lately. Discussion of such models seems to be useful, as the requirement of the stability with respect to excitations, corresponding to composite fields operators, can lead to additional limitations on the models (by stability we mean a limitation of a potential from below). It's convinient to study the problems of stability of theories by investigating the asymptotics of the solutions of renormalization group equations for effective potential, that appear to be inhomogeneous for the potentials of composite fields[1].

In the present paper the problem of the stability of two-dimensional massive Tirring model, including composite fields of the type $\bar{\psi}^i\psi^i$ (ψ^i is N-component spinor in the isotopic space; $i=1,\dots N$) are considered. This model is asymptotically free. In item 2 the equation of the renormalization group for generating functional of Green's functions is derived and its asymptotics is found. The expression for asymptotics of the effective potential of the composite field is also found. In item 3 the same model is considered in the limit of large $N(N\sim g^2)$; in this case all the values are calculated precisely. In item 4 the comparison of the obtained results with the results of the paper[2] are given.

2. So, let's consider the theory described by the langrangian

$$\mathcal{L} = i\bar{\psi}^i\hat{\partial}\psi^i - M_0\bar{\psi}^i\psi^i + \frac{1}{2}g_0^2(\bar{\psi}^i\psi^i)^2 \qquad (1)$$

The generality functional for renormalized Green's functions (for both elementary and composite fields) has the form [1]

$$e^{iW_R} = C\int d\bar{\psi}^i d\psi^i e^{i\int dx\left[\mathcal{L}_R(g) + \eta^i\psi^i + \bar{\psi}^i\bar{\eta}^i + KZ_2\bar{\psi}^i\psi^i + KZ_1 + KZ_3K\right]} \qquad (2)$$

where $\mathcal{L}_R$ is given by the expression

$$\mathcal{L}_R = Z_\psi i\bar{\psi}^i\hat{\partial}\psi^i - Z_M M\bar{\psi}^i\psi^i + \frac{1}{2}Zg^2(\bar{\psi}^i\psi^i)^2 \qquad (3)$$

(Z_ψ, Z — renormalization constants of ψ - field and the vertex $(\bar{\psi}^i\psi^i)^2$ correspondingly, the value $(1-Z_M)\bar{\psi}^i\psi^i$ is the mass counterterm ; η is the sourse to the elementary field and K is the sourse to the composite field).

Let's limit ourselves to the case when $\eta^i = \bar{\eta}^i = 0$ and K does not depend on coordinates. Introducing $W(K)$ by means of the relation $W_R(K) = \Omega^{(2)}W(K)$ ($\Omega^{(2)}$ is two-dimensional volume) and following the papaer [1] we deduce the renormalization group equation for $W(K)$. It has the following form

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma K\frac{\partial}{\partial K}\right)W(K) = \gamma_1 K + \gamma_2 K^2 \qquad (4)$$

where

$$\beta(g) = \mu\frac{\partial}{\partial\mu}g\Big/_{g_0} = -\frac{N}{2\pi}g^3,$$

$$\gamma = -\mu\frac{\partial}{\partial\mu}\ln Z_2 Z_4^{-1}\Big/_{g_0} = -\frac{N-1}{\pi}g^2,$$

$$\gamma_1 = \left(\mu\frac{\partial}{\partial\mu}Z_1 - Z_1\frac{\partial}{\partial\mu}\ln Z_2 Z_4^{-1}\right)\Big/_{g_0} = -\frac{N}{\pi}M, \tag{5}$$

$$\gamma_2 = \left(\mu\frac{\partial}{\partial\mu}Z_3 - 2Z_3\mu\frac{\partial}{\partial\mu}\ln Z Z_4^{-1}\right)\Big/_{g_0} = \frac{N}{2\pi},$$

μ is on auxiliary mass, and the results in (5) are given in the lowest order approximation over the charge. (Note, that in the one-loop approximation Z_4 is limited; besides, since the renormalization constants can be chosen independent of mass parameters of the theory,[3] the values Z_M and Z_2 can be chosen equal to each other. Since the value $\gamma \leq 0$, then according to analysis in paper[1], the asymptotics $W(K)$ at $K\to\infty$ is proportional to K^2 (the limit $W(K)$ is supposed to exist at $g\to 0$). Presenting $W(K)$ in the form $W(K) = aK^2$ we find the equation for a:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + 2\gamma\right)a = \gamma_2 \tag{6}$$

(the term $\gamma_1 K$ at $K\to\infty$ is omitted). For the renormalization prescription described in[1] the asymptotics $W(K)$ and consequently a, does not depend on mass parameters and μ. Solving the equation (6) we get the equation for asymptotics of $W(K)$:

$$W(K) = -\frac{N}{2(N-2)}g^{-2}\left[1 + C_1 g^{\frac{4-2N}{N}}\right]K^2; \quad N \neq 2 \tag{7}$$

The expression for asymptotics of the effective potential $V(\Sigma)$ determined by the relation

$$V(\Sigma) = -W(K) + K\Sigma\,, \qquad \Sigma = \frac{\partial W}{\partial K}\,; \quad K = \frac{\partial V}{\partial \Sigma}\,, \tag{8}$$

has the following form:

$$V(\Sigma) = -\frac{N-2}{2N} g^2 \left[1 + C_1 g^{\frac{V-2N}{N}}\right]^{-1} \Sigma^2; \quad N \neq 2. \tag{9}$$

In the case $N=2$ for $V(\Sigma)$ we have

$$V(\Sigma) = \frac{1}{Y} g^2 \left(C_2 - \ln g\right)^{-1} \Sigma^2 \tag{10}$$

From formulae (9) and (10) one can see that when $N=1,2$ and g is rather small the potential is stable irrespective of the quantities and the signs of unknown constants C_1 and C_2. As one can see from formulae (9), the second summond dominates for $N>2$ and at small g, and in this case one can't say anything definite about the sign of $V(\Sigma)$ as the sign of constant C_1 is unknown. However, the supposition that $C_1 = 0$ for any value of N seems to be reasonable. The first argument in favour of it is that the exponent of g is fractional for arbitrary N in formula (9). Secondly, the exact solution of the model at large N $\left(N \sim \frac{1}{g^2}\right)$ as it will be shown in item 3, leads to equality $C_1 = 0$.

The considered situation may change if we'll take values $g>1$. However, in this case the expressions (7), (9) and (10) are in general improper and to solve the question of the stability of the system one should know the behaviour of functions $\beta(g)$, γ and γ_2 at large g without the perturbation theory.

3. Now let's consider the same model at $N \sim \frac{1}{g^2}$. For such values of N one can calculate coefficient C_1 in expressions (7) and (9).

In this case the main contribution to the expression for $W(K)$ comes from diagrams, proportional to the value $N(Ng^2)^n$ (n is the diagram order). Let's present $W(k)$ in the form

$$W(K) = W_1(K) + W_2(K) \qquad (11)$$

where $W_1(K)$ corresponds to the sum of the series of diagrams, the general term of which can be presented in the form:

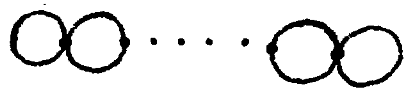

Fig. 1.

All other diagrams we'll relate to $W_2(K)$. The series of these diagrams will begin with the diagram of the third order in Ng^2. The first two terms of this series are shown in Fig.2:

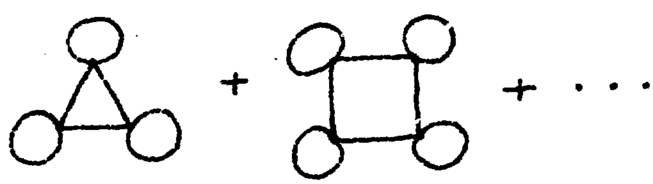

Fig.2

(Diagrams, corresponding to counterterms are obvious to contribute to $W_1(K)$ and $W_2(K)$).

The asymptotics of $W_1(K)$ is easy to calculate (W_1 represents the sum of the series of the geometric progression)

$$W_1(K) = -\frac{K^2}{2g^2}; \quad K \gg \mu \tag{12}$$

The values $\beta(g)$, γ and γ_2 are calculated exactly and are equal correspondingly to

$$\beta(g) = -\frac{N}{2\pi} g^3, \quad \gamma = -\frac{N}{\pi} g^2, \tag{13}$$

$$\gamma_2 = \frac{N}{2\pi}.$$

Now let's substitute the relation (11), with the value $W_1(K)$ determined by the expression (12), into equation (4) for $K \gg \mu$ in which the values $\beta(g)$, γ and γ_2 are given by the relations (13). We'll get the homogeneous equation for $W_2(K)$:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma K\frac{\partial}{\partial K}\right) W_2(\rho K) = 0 \tag{14}$$

(ρ is a scaling coefficient), for which the following expression is the solution:

$$W_2(\rho K) = \exp\left\{-\int_0^t \frac{2\,dt'}{\gamma(G(t'))-2}\right\} W_2(K; G(t), \mu) \tag{15}$$

where $G(t)$ satisfies the equation

$$\dot{G}(t) = \frac{\beta(G)}{2-\gamma(G)}; \quad t = \ln\rho. \tag{16}$$

Since the theory is asymptotically free, we can insert $G^2 \sim \frac{2\pi}{Nt}$ and then the expression (15) will take the form:

$$W_2(pK) \sim p^2[\ln p]^{-2} W_2(K,0,\mu) \tag{17}$$

Comparing the formulae (17) and (12) we see, that $W_1(K)$ is dominant at $K \gg \mu$ and the asymptotics of $W(K)$ is defined by

$$W(K) = -\frac{K^2}{2g^2}\ ; \quad K \gg \mu \tag{18}$$

Comparing expression (18) and (7) we conclude, that $C_1 = 0$.

4. In paper [2] the model described by the langrangian

$$\mathcal{L}_\sigma = i\bar{\Psi}^i\hat{\partial}\psi^i - \frac{1}{2}\sigma^2 + g_0\bar{\Psi}^i\psi^i\sigma \tag{19}$$

was considered for values $N \sim \frac{1}{g_0^2}$, which is equivalent to model (1) at $M_0 = 0$ for the same values of N. The plot of the effective potential $V(\sigma_c)$ for the renormalized model (19) pointed out the stability of the theory ($\sigma_c = \frac{\partial W_\sigma}{\partial J} = \langle 0|\sigma|0\rangle_J$; W_σ is the generating functional of renormalized Green's functions of the model (19), J is the external source to the field σ). However based on improper written formulae (5.11) and (5.12) in paper [2], the erroneous conclusion about the coincidence of values $V(\sigma_c)$ and $V(\Sigma_c)$ was drawn ($\Sigma_c = \frac{\partial W_{\bar{\Psi}\psi}}{\partial J} = \langle 0|\bar{\Psi}\psi|0\rangle_J$; $W_{\bar{\Psi}\psi}$ is the generating functional of model (1) with $M_0 = 0$, J is the sourse of the composite field $\bar{\Psi}\psi$).

The correct analogies of the formulae (5.11) and (5.15) in paper[2] look correspondingly like

$$V(\Sigma_c) = V(\sigma_c) - \frac{1}{2}\mathcal{J}^2$$
$$\Sigma_c = \sigma_c - \mathcal{J} \qquad (20)$$

and lead to the following value for the asymptotics of $V(\Sigma_c)$:

$$V(\Sigma_c) \sim -a\Sigma_c^2 , \quad a>0 \qquad (21)$$

So, the model (19) at $N \sim \frac{1}{g_0^2}$ being stable with respect to excitation of the type σ_c appears to be unstable with respect to excitations of the type Σ_c.

The author is indebted to I.V.Tyutin and A.D.Linde for valuable discussion.

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The manuscript was received on the 22th of March,1977

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(на английском языке)
Ереванский физический институт

Тех.редактор А.С.Абрамян

Заказ 1158	ВФ-03403	Тираж 299

Подписано к печати 8/XII-77г. Формат издания 30х40
0,8 уч.изд.л. Ц. 6 к.

Издано Отделом научно-технической информации
Ереванского физического института ,Ереван-36 пер.Маркаряна 2

индекс 3624