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# RENORMALIZATION GROUP EQUATION FOR COMPOSITE FIELDS

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I. Recently, the so-called effective potentials-the generating functionals of the vertex functions at zero external momenta-play the more important role when studying the gauge theories. They are convenient for the investigation of the spontaneous breakdown of symmetry in gauge theories [I] as well as for studying the possible restrictions on the models, connected with the stability requirements of the theory [2]. In the present paper we discuss some properties of the effective potential for the composite field of the type $\varphi^2$ ( $\varphi$- is the Bose field). The discussion of such potentials seems to be useful too. First, the stability requirements of the theory with respect to such excitations could lead to some restrictions. Secondly, in analogy with the superconductivity model of BCSh [3] or Nambu-Jona-Lasinio field theory model [4], one can expect, that if the vacuum expectation of such an operator is different from zero, then it'd lead to dynamical addition to particle masses, and in a massless theory to the appearance of masses of fields (in particular of gauge ones). In this paper the renormalization group equation that turns out to be inhomogeneous is derived for such potentials. These equations can be solved exactly, and different asymptotics can be investigated with the help of these solutions. In section 2 the asymptotics of such potentials is found in asymptotically free theories. In section 3, for the case of Yang-Mills field theory interacting with the fermion field, it is shown that the stability requirements of theory (just as in[2] it's understood as the restriction of potential from below) lead to restrictions on multiplete contents of fermions. The arguments, showing that in a stable, asymptotically free gauge theory with zero bare masses of all particles the dynamical mass of some particles should appear, are given in section 4.

2. The renormalization group equations for the effective potential (as defined below) are derived in the ususal way, based on the relations between the renormalized and non- renormalized Green's functions.

Let's consider an arbitrary theory of scalar, vector and spinor fields $\varphi_i$ , $A^a_\mu$ , $\psi_i$ , $\bar{\psi}_i$ , the set of which we'll define as $\omega_i$ . Besides we introduce $\varphi_i(x)\varphi_j(x)$, $\varphi_i(x)A^a_\mu(x)$ , $A^a_\mu(x)A^b_\nu(x)$ type composite fields $\sigma_i(x)$ . We'll designate the set of all the fields $\omega_i$ and $\sigma_i$ is $\Omega_i$ . Let's introduce the sources to the fields $\omega_i$ and $\sigma_i$ ($\mathcal{J}_i$ and $K_i$ ), the set of which is designated as $N_i$ . he generating functional of Green's functions of all the operaors $\Omega_i$ has the form:

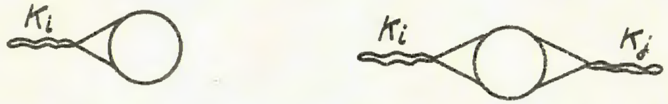


$$e^{iW_1(N)} = \int d\omega_i \, e^{i\int dx(L_R + N_i\Omega_i)} \qquad (1)$$

where $L_R$ is the renormalized langrangian (allowing for a gauge fixing term and for the ghost particles), the integration over ghost fields being incorporated in $d\omega_i$ also. $W_1$ gives the finite Green's functions of the elementary fields $\omega_i$. Green's functions of the composite fields are still divergent. These are the vertex function divergences of the type

(2)

and one-particle irreducible diagram divergences of the type

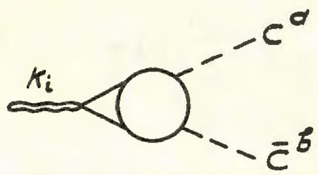


(3)

corresponding to the vacuum expectations $\langle\sigma_i\rangle$ and $\langle\sigma_i\sigma_j\rangle$. These divergences are eliminated by counterterms

$$K(Z_{K\sigma}-1)\sigma + KZ_{K\omega}\omega + \partial_\mu KZ_{\mu K\omega}\omega + Z^{(1)}K + KZ^{(2)}K \qquad (4)$$

where for brevity the indices are omitted. If we confine ourselves to gauges of the type $\partial_\mu A_\mu + \beta\varphi$ (the gauge fixing term is equal to $\sim(\partial_\mu A_\mu + \beta\varphi)^2$ then the diagrams of the type (2) with the substitution $\omega \to C$ ( $C^a$ and $\bar{C}^a$ are ghost particles) are not divergent. Indeed, because of the continuity of C-particle line the only diagram that could diverge is




(5)

However, C-fields have $V_1 \sim \partial_\mu \bar{C} A_\mu C$ and $V_2 \sim \bar{C} C \varphi$ interaction vertexes. $\bar{C}$-field line can join the vertex function either through the vertex $V_1$ (in this case the vertex function is proportional to the momentum) or through the superrenormalizable vertex $V_2$. In both cases the divergence index is at least - 1, and the diagram is conventionally convergent. So, the generating functional of finite Green's functions of any field is

$$e^{iW_R(N;\lambda,\mu)} = \int d\omega \exp\Big\{ i \int dx \big[ L_R + J\omega + K Z_{K\sigma}\sigma + K Z_{K\omega}\omega + \partial_\mu K Z_{\mu K\omega}\omega + Z^{(1)}K + K Z^{(2)}K \big] \Big\} \tag{6}$$

where the dependence of $W_R$ on all the renormalized parameters $\lambda_i$ and the "renormalization point" $\mu$ is given in the explicit form. Note, that according to the theorem[5] on divergence structure, all the values Z depend on mass parameters just in a trivial way following from the dimentional counting. To derive the renormalization group equation, it's convenient to express the renormalized parameters $\lambda_i$ in (6) through the non-renormalized ones $\lambda_i^{(0)}$ and to make substitution of the variables $\omega_i = Z_{ij}^{-\frac{1}{2}}\omega_j$, where $Z_{ij}$ is the matrix of renormalization constants of elementary fields wave functions. The generating functional takes the form



$$e^{iW_R(N;\dots)} = \int d\omega\, e^{i\int dx\, [L + NS\Omega + Z^{(1)}K + K Z^{(2)}K]} \tag{7}$$

$$S_{y\omega} = Z^{-\frac{1}{2}},\quad S_{y\sigma} = 0,\quad S_{K\omega} = Z_{K\omega} Z^{-\frac{1}{2}},\quad S_{K\sigma} = Z_{K\sigma} Z^{-\frac{1}{2}} Z^{-\frac{1}{2}} \tag{8}$$

where $L$ (and certainly $N_i$) is now independent of $\mu$ at fixed $\lambda_i^{(0)}$. We differentiate (8) with respect to $\mu$ at fixed $\lambda_i^{(0)}$:

$$\mu \frac{\partial}{\partial\mu} W(N;\dots)\Big/_{\lambda^{(0)}} = -N\gamma \frac{\partial}{\partial N} W + \gamma^{(1)}K + K\gamma^{(2)}K \tag{9}$$

$$W_R = \Omega^{(4)} W, \qquad \Omega^{(4)} \text{ – 4-volume} \tag{10}$$

or on passing to the differentiation with respect to $\mu$ at fixed renormalized parameters we have

$$\Big(\mu\frac{\partial}{\partial\mu} + \beta_i \frac{\partial}{\partial\lambda_i} + N_i \gamma_{ij} \frac{\partial}{\partial N_j}\Big) W(N;\lambda,\mu) = \gamma_i^{(1)} K_i + K_i \gamma_{ij}^{(2)} K_j \tag{11}$$

where

$$\beta_i = \mu\frac{\partial}{\partial\mu}\lambda_i\Big/_{\lambda^{(0)}}, \qquad \gamma_{ij} = -\Big(\mu\frac{\partial}{\partial\mu} S_{iK}\Big)\Big/_{\lambda^{(0)}} S_{Kj}^{-1} \tag{12}$$

$$\gamma_i^{(1)} = \Big[S_{K\sigma}\,\mu\frac{\partial}{\partial\mu}\big(S_{K\sigma}^{-1} Z^{(1)}\big)\Big]_i\Big/_{\lambda^{(0)}} \tag{13}$$

$$\gamma_{ij}^{(2)} = \Big[S_{K\sigma}\,\mu\frac{\partial}{\partial\mu}\big(S_{K\sigma}^{-1} Z^{(2)} [S_{K\sigma}^{T}]^{-1}\big) S_{K\sigma}^{T}\Big]_{ij}\Big/_{\lambda^{(0)}}$$

Using (11), one can get the renormalization group equations for the effective potential determined as follows:

$$-V(\Omega;\cdots) = W(N;\cdots) - \Omega_i N_i \tag{14}$$

$$\Omega_i = \frac{\partial W}{\partial N_i}\,; \qquad N_i = \frac{\partial V}{\partial \Omega_i}\,. \tag{15}$$

Using (11) we get

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} - \Omega_j\gamma_{ij}\frac{\partial}{\partial\Omega_i}\right)V(\Omega;\lambda,\mu) = -\gamma_i^{(1)}\frac{\partial V}{\partial\sigma_i} - \frac{\partial V}{\partial\sigma_i}\gamma_{ij}^{(2)}\frac{\partial V}{\partial\sigma_j} \tag{16}$$

One should keep in mind, that the potential $\Sigma$, corresponding to the composite field, is to be determined as follows

$$\frac{\partial W}{\partial K_i} = \sigma_i = \Sigma_i + \omega_{\sigma_i}^{(1)}\,\omega_{\sigma_i}^{(2)} \tag{17}$$

where $\omega_{\sigma_i}^{(1)}\omega_{\sigma_i}^{(2)}$ is the product of potentials, corresponding to the elementary fields, the product of which is $\sigma_i$.

The effective potential $V(\Omega)$ plays a role analogous to that of the effective potential $V(\Phi)$. In particular, the values of vacuum expectations of $\Omega_i$-fields are determined from the condition

$$\frac{\partial V}{\partial \omega_i} = 0, \qquad \frac{\partial V}{\partial \Sigma_i} = 0. \tag{18}$$



In the case when the solution of (18) is $\Sigma_i \neq 0$, we get the dynamical addition to the particle masses[3,4]. In particular, if in the theory with $\Sigma_i = 0$ the masses are equal to zero, then in the theory with $\Sigma_i \neq 0$ they become massive (see also section 4).

Since the value of $V$- potential in the point satisfying the condition (18) is the energy density of the system, the potential $V$ must be limited from below. The renormalization group equation allows one to investigate this problem. In general, it's more convenient first to pass to the other generating functional $\overline{W}(N;\cdots)$:

$$\overline{W}(N) = W(N) - \tilde{\gamma}_i^{(1)}N_i - N_i\tilde{\gamma}_{ij}^{(2)}N_j \tag{19}$$

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i}\right)\tilde{\gamma}_j^{(1)} + \gamma_{ji}\,\tilde{\gamma}_i^{(1)} = \gamma_j^{(1)} \tag{20}$$

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i}\right)\tilde{\gamma}_{jk}^{(2)} + \gamma_{je}\,\tilde{\gamma}_{ek}^{(2)} + \tilde{\gamma}_{je}^{(2)}\gamma_{ek} = \gamma_{jk}^{(2)}, \tag{21}$$

$\gamma_j^{(1)}$ and $\gamma_{jk}^{(2)}$ are certainly different from zero only, for the values of indices, corresponding to $\sigma_i$ - fields. It's easy to veryfy, that $\overline{W}$ satisfies the homogeneous renormalization group equation

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\lambda_i} + N_i\gamma_{ij}\frac{\partial}{\partial N_j}\right)\overline{W} = 0 \tag{22}$$

Further one should find the solution (22) for the case, when some of sources are exposed to a homogeneous extension in analogy with Ref[2], and then use the formula (19). We shall not deal with the cumbersome general expression, but show how to find the asymptotics $W$ at large $N$ (an also $V$ at large $\Omega$ ) by a simple example.

Suppose that $J_i = \omega_i = 0, \quad \gamma_{ij} = \gamma \delta_{ij},$ $\gamma_{ij}^{(2)} = \gamma^{(2)} \delta_{ij}$ and denote the homogeneous extension factor of sources as $\rho \, (t = \ln \rho)$. Then $W(\rho K; \dots)$ satisfies the equation (where $W$ is a homogeneous function of 4-dimensional mass parameters and $K$ has a dimensionality equal to 2)

$$\left(\frac{\partial}{\partial t} - \frac{\beta_i - n_{(i)}\lambda_i}{2-\gamma}\frac{\partial}{\partial \lambda_i} - \frac{4}{2-\gamma}\right) W(\rho K; \dots) = -\frac{\rho}{2-\gamma}\gamma_i^{(1)} K_i - \frac{\rho^2}{2-\gamma}\gamma^{(2)} K_i K_i \qquad (23)$$

where $n_i$ is the dimensionality of $\lambda_i$-parameter.

The corresponding equation for $V(\rho\Sigma; \dots)$ is

$$\left(\frac{\partial}{\partial t} - \frac{\beta_i - n_{(i)}\lambda_i}{2+\gamma}\frac{\partial}{\partial \lambda_i} - \frac{4}{2+\gamma}\right) V(\rho\Sigma; \dots) =$$

$$= \frac{\rho}{2+\gamma}\gamma_i^{(1)} \frac{\partial V(\rho\Sigma; \dots)}{\partial(\rho\Sigma_i)} + \frac{\rho^2}{2+\gamma}\gamma^{(2)} \frac{\partial V(\rho\Sigma; \dots)}{\partial(\rho\Sigma_i)} \frac{\partial V(\rho\Sigma; \dots)}{\partial(\rho\Sigma_i)} \qquad (24)$$

As was said above, the equation (23) can be solved exactly. However the asymptotics $W(\rho K; \dots)$ and $V(\rho\Sigma; \dots)$ at $\rho \to \infty$ may be found without recourse to the exact solutions of (20), (21) and (22). Remember that we have confined ourselves to the asymptotically free theories only.

One can conclude from the equation (23), that $W(\rho K; \dots)$



behaves asymptotically at $\gamma^{(2)} \neq 0$ either as $\rho^2$ or stronger than $\rho^2$ (the term $\rho\gamma_i^{(1)} K_i$ in the right-hand side of (23) can be neglected).

Let's consider these cases separately.

I. $W \sim K^2$ at $K \to \infty$

In this case we asymptotically have

$$W(K) \sim aK^2 \qquad (25)$$

Substituting (25) into (23) or (11) we get:

$$\left(\mu\frac{\partial}{\partial \mu} + \beta_i \frac{\partial}{\partial \lambda_i} + 2\gamma\right) a = \gamma^{(2)} \qquad (26)$$

In fact, since the asymptotics is independent of dimensional parameters the coefficient $a$ is also independent of dimentional parameters and $\mu$ (under the renormalization procedure described in Ref.[5]). In this case the potential has the asymptotics at large $\Sigma$ :

$$V(\Sigma; \dots) \sim \frac{1}{4a}\Sigma^2 \qquad (27)$$

This result can be also obtained by substituting (27) into (16) directly and neglecting the term with $\gamma^{(1)}$ . Then the equation for $a$ is (26) again.

2. $W > K^2$ at $K \to \infty$

In this case neglecting the right-hand side of the equation (23) we have the usual homogeneous equation of the renormalization group (one could have neglected the right-hand side of the equation (24), as in this case $V$ would increase slower than $\Sigma^2$ ). Adopting the usual procedure we get

$$W(pK;\lambda,\mu) = \exp\left\{\int_0^t \frac{4dt'}{2-\gamma(\Lambda(t'))}\right\} W(K;\Lambda(t),\mu) \tag{28}$$

$$\dot{\Lambda}_i = \frac{1}{2-\gamma(\Lambda)}\left(\beta_i(\Lambda) - \Lambda_i n_{(i)}\right) \tag{29}$$

Taking that in asymptotically free theories $f \sim h^2 \sim g^2$ $G^2 \to \frac{2}{6t}$, $\gamma \sim \gamma_\Sigma g^2$ [5] ( $g$ are Yang-Mills constants, $h$ are Jukava coupling constants, $f$ are the scalar coupling constants of $f\varphi^4$- interaction, we have

$$\gamma(\Lambda) \longrightarrow \frac{2\gamma_\Sigma}{6t} \tag{30}$$

$$W(pK;\lambda,\mu) \sim p^2[\ln p]^{\frac{2\gamma_\Sigma}{6}} W(K;0,\mu) \tag{31}$$

under the assumption that the limit $W$ at $\lambda \to 0$ is different from zero.

So, the case 2 is realized at $\gamma_\Sigma > 0$ . The case 1 is realized at $\gamma_\Sigma \leq 0$ (and $\gamma^{(2)} \neq 0$ ). For the asymptotics of $V$ we get

$$V(p\Sigma;\lambda,\mu) \sim p^2[\ln p]^{-\frac{2\gamma_\Sigma}{6}} V(\Sigma;0,\mu) \tag{32}$$

If $\gamma^{(2)}$ is zero, then the asymptotics is expressed by means of formulae (31) and (32).

To make the theory stable, it's necessary that the value of $W(K;0,\mu)$ be positive in the second case, and $a$ to be positive in the first case.



3. As an example let's consider the Yang-Mills field theory interacting with the fermion field. As the composite field we take $\sigma = A_\mu^a A_\mu^a$ and put $K \gg \mu^2$. The value $W(K;0,\mu)$ is equal to

$$W(K;0,\mu) = -(3+\alpha^2)\frac{n}{16\pi^2} K^2 \ln\frac{K}{\mu^2} \tag{33}$$

Here for convenience the gauge fixing term is chosen to be $-\frac{1}{2\alpha}(\partial_\mu A_\mu^a)^2$ and the propagator of the gauge field (in momentum representation) is

$$\mathcal{D}_{\mu\nu}^{ab}(p) = -i\delta^{ab}\left[\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2} + \alpha\frac{p_\mu p_\nu}{p^4}\right] \tag{34}$$

and $\alpha = 0$ corresponds to the transverse gauge.

The value $W(K;0,\mu)$ turns out negative in any gauge and therefore to make the theory stable it's necessary that $\gamma_\Sigma < 0$

In the theory under consideration there are two dimensionless parameters $g$ and $\alpha$. The corresponding $\beta$ - functions are

$$\beta_g = -\frac{g^3}{16\pi^2}\left[\frac{11}{3}s - \frac{4}{3}\tau\right]; \quad \beta_\alpha = \frac{g^2 s}{16\pi^2}\alpha(\alpha(\infty)-\alpha) \tag{35}$$

$$\alpha(\infty) = \frac{13}{3} - \frac{8}{3}\frac{\tau}{s}$$

where $f^{abc}f^{dbc} = s\delta^{ad}$, $\operatorname{tr}\tau^a\tau^b = \tau\delta^{ab}$, $\tau^a$ are the generators of the gauge transformation of the spinor field. One can see that the differential equation for the effective parameter $\alpha(t)$ has two fixed points: $\alpha = 0$ and $\alpha = \alpha(\infty)$ (one of which is unstable) so in asymptotics the theory is effective described by one of the gauges $\alpha = 0$ and $\alpha = \alpha(\infty)$. Dealing further with these gauges only, we get $\beta_\alpha = 0$. The simple calculation gives for the values $\gamma_\Sigma$ and $\gamma^{(2)}$:

$$\gamma_\Sigma = -\frac{1}{16\pi^2}\left(\frac{35}{6}s + \frac{1}{2}s\alpha - \frac{8}{3}\tau\right), \quad \gamma^{(2)} = (3+\alpha^2)\frac{n}{8\pi^2}, \tag{36}$$

where $n$ is the group dimension.

The condition $\gamma_\Sigma \leq 0$ leads to the following limitations

$$\begin{aligned} \frac{35}{16}s &\geq \tau \quad \text{at} \quad \alpha = 0 \\ 2s &\geq \tau \quad \text{at} \quad \alpha = \alpha(\infty) \end{aligned} \tag{37}$$

that appear to be stronger, than the condition of asymptotic freedom $\frac{11}{4}s > \tau$. The condition (37) look like gauge dependent. However, since $s$ and $\tau$ take on discrete values, these conditions may coincide (it is checked for the case of SU(2) group). Thus, in the stable Yang-Mills field theory the potential behaves as $V \sim \frac{1}{4a}\Sigma^2$, where $a$ is obtained from the equation

$$\left[-\frac{1}{2}\left(\frac{11}{3}s - \frac{4}{3}\tau\right)g^3\frac{\partial}{\partial g} - \left(\frac{35}{6}s + \frac{1}{2}s\alpha - \frac{8}{3}\tau\right)g^2\right]a = (3+\alpha^2)n \tag{38}$$

the solution of which is

$$\frac{1}{a} = \frac{\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha}{(3+\alpha^2)n}\, g^2\left(1 + Cg^{2\cdot\frac{\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha}{\frac{4}{3}s - \frac{4}{3}\tau}}\right)^{-1} \tag{39}$$

where $C$ is an arbitrary constant. It should be remembered, that the coefficients $\beta_g$, $\gamma$ and $\gamma^{(2)}$ are obtained in the framework of perturbation theory in the charge.



Let's consider the structure of $a$ quantity in detail. At $\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha > 0$ $a$ is positive for sufficiently small $g$ (irrespective of the value of constant $C$) and the corrections to this value (taking in $\beta$, $\gamma$ and $\gamma^{(2)}$ the account of higher order terms in $g$) are small at small $g$. At $\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha < 0$ the term with $C$ in parentheses dominates for small $g$, and nothing could be concluded about the sign of $a$. At large $g$ (when strictly speaking the expression (39) is inapplicable) the term with C becomes small and $a$ becomes negative. If this conclusion were strict, it would mean that at $\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha < 0$ the theory is unstable.

However, the $a$ sign conclusion is obtained for a range, where the expression (39) is, generally speaking, incorrect, and, so, the solution of the stability problem requires the knowledge of $\beta_g$, $\gamma$ and $\gamma^{(2)}$ at large $g$ outside the perturbation theory frameworks. This situation is similar to that of "zero-charge" problem, where the conclusion about the presence of a fictions pole was based on the utilization of perturbation theory in the domain where it was inapplicable. So, if we confine ourselves to the case $\frac{4}{3}\tau - \frac{13}{6}s - \frac{1}{2}s\alpha > 0$, then we can definitely predict the value and what's more important the sign of $a$. In this case the theory turns out to be stable. There arises the additional restriction on the fermion representation:

$$\tau > \frac{13}{8}s \tag{40}$$

as at $\alpha = 0$, so at $\alpha = \alpha(\infty)$.

The following multiplete composition of fermions (the same in both gauges) satisfies the inequalities (37) and (40) in SU(2) group case: 7 or 8 doublets; 2 triplets; 1 triplet and 3 or 4 doublets. Thus the study of the effective potentials for composite fields could be seen to impose additional restrictions on the theory structure following from the boundedness of the potential from below, and here also the renormalization group methods may prove useful.

4. Let's consider the gauge theory with zero bare masses of particles. We'll show that if the physical masses of particles are zero, then the vacuum expectation of the field $\sigma = A_\mu^a A_\mu^a$ is equal to zero. On the other hand, the effective potential $V(\Sigma)$ will be shown to have the stationary point at $\Sigma \neq 0$ (additional to the stationary point at $\Sigma = 0$ with $V(\Sigma) = 0$ ) with the negative potential value in this point. Using these results we draw the conclusion, that the dynamical mass should appear in the theory. It's easy to see, that the vacuum expectation of the field $\sigma$ satisfies the self-consistent equation:

$$i\Sigma = \mathcal{D}_{\mu\mu}^{aa}(x,x;\Sigma) = \int \frac{d^4p}{(2\pi)^4} \mathcal{D}_{\mu\mu}^{aa}(p;\Sigma) \tag{41}$$

where we assume that the vacuum is translation- invariant (i.e. $\Sigma = const$ , and the propagator of the gauge field depends on the coordinate difference) and the $\Sigma$-dependence of the propagator is explicit. To eliminate the divergences we'll use the method of dimensional regularization[6]. Since we have supposed that all physical masses are equal to zero, the gauge field propagator would have the form:



$$\mathcal{D}_{\mu\mu}^{aa}(p;\Sigma) = \frac{1}{p^2}\sum_k a_k(\Sigma)\, \ln^k\left(-\frac{p^2}{\mu^2}\right) \tag{42}$$

where $a_k$ are numerical coefficients, that depend on $\Sigma$ and also on theory parameters. Since in the dimensional regularization

$$\int dp\, \frac{\ln^\alpha p^2}{p^2} = 0 \tag{43}$$

takes place, we get from (41) - (43)

$$\Sigma = 0 \tag{44}$$

Let's pass to the second step. Consider the following range of $K$-source values:

$$1 \gg g^2 \ln\frac{K}{\mu^2} \gg g^2 \tag{45}$$

This condition could always be satisfied at sufficiently small $g$ . In this range of $K$-values the function $W(K)$ is determined by a zero-order term, i.e. by the formula (33). Now, the effective potential is

$$V(\Sigma) \approx -\frac{4\pi^2}{(3+\alpha^2)\,n}\cdot\frac{\Sigma^2}{\ln\frac{\Sigma}{\mu^2}}\,, \quad \Sigma \gg \mu^2 \tag{46}$$

So, there exists the range of $\Sigma$ - values, in which $V(\Sigma)$ is negative and decreases with growth of $\Sigma$ . Since we have supposed, that the theory is stable, then $V(\Sigma)$ would begin to grow at sufficiently large $\Sigma$ . Hence, $V(\Sigma)$ has a minimum at $\Sigma \neq 0$ point. In the view of the aforesaid this means that dynamical mass should appear in the theory.

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УРАВНЕНИЕ ГРУППЫ ПЕРЕНОРМИРОВОК ДЛЯ СОСТАВНЫХ ПОЛЕЙ

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