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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ

ԳԻՏԱԿԱՆ ՀԱՂՈՐԴՈՒՄ НАУЧНОЕ СООБЩЕНИЕ

ЕФИ-237(30)-77

SU 7809660

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ԵՐԵՎԱՆ 1977 ЕРЕВАН

YEREVAN PHYSICS INSTITUTE

Scientific Report ЕФИ-237(30)-77

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Yerevan, 1977

I. Introduction

The interpretation of ψ and χ's as the bound states of new heavy quarks naturally leads to the SU_3 singlet assignment of these mesons. This results in a number of relations between different decay modes of the new mesons, which are extensively tested on experiment. (see e.g. [1-4]).

When comparing the theoretical predictions with the experimental data the well known problem of taking into account the SU_3 violating mass differences inside a multiplet arises. The usual procedure is to factor out an explicit mass dependence due to the phase space integration of the simplest matrix element (e.g. the factors $K^{2\ell+1}$ for two-body decays). We would like to emphasize, however, that for the particles with spin the process is described by several amplitudes and the corrections due to the mass differences depend essentially on the relative values of these amplitudes. So the theoretical predictions form a rather wide interval the usual phase space results being inside this interval.

Relatively unambiguous results from SU_3 can be obtained if the decay matrix element is described with one independent structure only. If there are two independent amplitudes (e.g. in the decay $\psi \to B\bar{B}$, see below) one new parameter appears. The value of this parameter can be found through angular distribution or polarization measurements or comparison of probabilities of two different decays. The situation becomes far more complicated when the number of independent amplitudes is larger than two. In this case we as a rule did not analyse the mass dependence of decay probabilities. We will return to this problem with further accumulation of experimental data.

It is noteworthy that there is some ambiguity because of the arbitrariness of the choice of independent amplitudes for which SU_3 is assumed to be valid. Indeed, one set of the amplitudes is usually connected with the another set through an explicit mass dependence. There is also a source of ambiguity due to the spin formalism used. We will not, however, dwell on these problems here.

The following notation will be used: $B_8 = (N, \Lambda, \Sigma, \Xi)$ is the baryon octet, $B_{10} = (\Delta, \Sigma^*, \Xi^*, \Omega)$ is the baryon decuplet with $s = 3/2$, $P = (\pi, K, \eta)$ is the pseudoscalar meson octet, $V = (\rho, K^*, V_8 = \omega/\sqrt{3} + 2\varphi/\sqrt{6})$ is the vector meson octet and $T = (A_2, K^{**}, T_8 = f/\sqrt{3} + 2f'/\sqrt{6})$ is the tensor meson octet. We will assume zero mixing for $(\eta - \eta')$ and ideal mixing for $(\omega - \varphi)$ and $(f - f')$. The 3P charmonium levels with the masses 3.415 GeV, 3.508 GeV and 3.550 GeV and $J^{PC} = 0^{++}, 1^{++}$ and 2^{++} are denoted through χ_0, χ_1 and χ_2 respectively.

In Sec.II we discuss two-and quasi-two-body decays of ψ and χ's .Namely, in the first subsection the decays into $B_8\bar{B}_8$ and in the second one into $B_{10}\bar{B}_{10}$ are considered. Then we analyse the decays of ψ into PV and VT and the decays of χ's into 2P and 2V. At the end of this section we give a short discussion of $\chi \to 2T$ and $\chi \to PT$ decays.

In Sec.III the three-body decays $\chi \to 2PV$, $\chi \to 2PT$, $\chi \to 3P$ and $\psi \to 2PV$ are considered. It is worth noting that while comparing the SU_3 predictions for the decays $\chi \to 2PV$ with experiment[2-4] the incorrect SU_3 factor was used (4/3 instead of 1) and the mass differences were not taken into account. Both corrections are of the same sign and essential.

II. Two-Body Decays

1. The decays into $B_8\bar{B}_8$ SU_3-symmetry gives equal widths for all the final states:

$$\gamma(N\bar{N}) = \gamma(\Lambda\bar{\Lambda}) = \gamma(\Sigma\bar{\Sigma}) = \gamma(\Xi\bar{\Xi}) \qquad (1)$$

Due to the mass diferences, however, γ is not the decay width but so called reduced width:

$$\gamma = \Gamma/\beta \qquad (2)$$

where β depends on the masses of the final particles and can be calculated through the phase space integration of the decay amplitude squared. In what follows we present values of β for different decays.

a) $\psi \to B_8\bar{B}_8$. We write the decays amplitude in the form

$$A(\psi \to B_8 \bar{B}_8) = \psi_\mu \bar{u} \left(f_1 \gamma_\mu - \frac{f_2}{m_\psi} \sigma_{\mu\nu} \mathcal{P}_\nu \right) u \tag{3}$$

where $\mathcal{P} = p_1 + p_2$ is the ψ four momentum.

If we assume that SU_3 symmetry is valid for the form factors $f_{1,2}$ the kinematical corrections due to the mass differences are easily found to be

$$\beta(\psi \to B_8 \bar{B}_8) = (1-4\mu^2)^{1/2} \left[1 + 2\mu^2 + 12\mu X + 2X^2(1+8\mu^2) \right] \tag{4}$$

where $\mu = m_B/m_\psi$, $X = f_2/2f_1$.

The relative correction factors $R = \beta(\psi \to B_8 \bar{B}_8) / \beta(\psi \to N\bar{N})$ (for $B_8 = \Lambda, \Sigma, \Xi$) are presented in Fig.1 as the functions of X (see also Table I).

The existing experimental data[3] give the following values for the partial widths

$$B(p\bar{p}) = (0{,}198 \pm 0{,}015)\%, \quad B(\Lambda\bar{\Lambda}) = (0{,}16 \pm 0{,}08)\%, \quad B(\Xi\bar{\Xi}) \sim 0{,}04\%$$

The large suppression of $\psi \to \Xi\bar{\Xi}$ decay implies either strong violation of SU_3 symmetry, or $X \approx -0.5$. Within a 20% variation of x around this point the agreement of the SU_3 - predictions with the experiment can be obtained.

To resolve the problem whether SU_3 is valid not only total width but also the angular distribution measurements should be done extracting separately f_1 and f_2.

b) $\underline{\chi \to B_8 \bar{B}_8}$. These decays are not yet registered. Looking on ψ one could expect them on the level of around 0.1% and so accessible to experimental measurement.

A single invariant amplitude contributes to the decays $\chi_{0,1} \to B_8 \bar{B}_8$ and consequently β can be determined unambiguously. In both cases it is equal to $(1 - 4m_B^2/m_\chi^2)^{3/2}$. The relative correction factors are presented in Table 1. It is noteworthy that the $\Xi\bar{\Xi}$ rate is relatively weakly suppressed and it's measuring is very interesting.

The decay $\chi_2 \to B_8 \bar{B}_8$ is described with two invariant amplitudes.

$$A(\chi_2 \to B_8 \bar{B}_8) \sim \chi_{\mu\nu} \bar{u}_1 \left[\frac{\tau_\mu \tau_\nu}{m_\chi} + x(\tau_\mu \gamma_\nu + \tau_\nu \gamma_\mu) \right] u_2 \tag{5}$$

where $\tau = p_1 - p_2$ .The factor β is easy to find

$$\beta(\chi_2 \to B_8 \bar{B}_8) = K^3 \left[K^4 - 8 x K^2 \frac{m_B}{m_\chi} + 10 x^2 \left(1 - \frac{2}{5} K^2\right) \right] \tag{6}$$

where $K = (1 - 4 m_B^2 / m_\chi^2)^{1/2}$.

In the Table I the bounds for $R = \beta(\chi_2 \to B\bar{B}) / \beta(\chi_2 \to N\bar{N})$ as the functions of x are presented.

<u>2. The decays into $B_{10}\bar{B}_{10}$</u> SU_3 predicts the equality of all reduced widths

$$\gamma(\Delta\bar{\Delta}) = \gamma(\Sigma^* \bar{\Sigma}^*) = \gamma(\Xi^* \bar{\Xi}^*) = \gamma(\Omega\bar{\Omega}) \tag{7}$$

Due to the high value of the baryon spin (s=3/2) the calculation of β is essentially more complicated. Indeed the decays $\psi \to B_{10}\bar{B}_{10}$ and $\chi_2 \to B_{10}\bar{B}_{10}$ are determined with four and five invariant amplitudes respectively. But the decays $\chi_{0,1} \to B_{10}\bar{B}_{10}$ are described with two amplitudes only, so the calculation of β in the latter case is not too complicated.

The amplitude of the decay $\chi_0 \to B_{10}\bar{B}_{10}$ can be written as follows

$$A(\chi_0 \to B_{10}\bar{B}_{10}) \sim \bar{u}_\mu \left[g_{\mu\nu} + x \frac{P_\mu P_\nu}{m_\chi^2} \right] u_\nu \tag{8}$$

where u_μ is the Rarita-Schwinger spinor for s = 3/2 and P is the χ four-momentum. The correction factor β is equal to

$$\beta(\chi_0 \to B_{10}\bar{B}_{10}) = K^3 \left[1 + \frac{K^2}{10\mu^4}(1 - 2\mu^2) + \frac{x K^2}{10\mu^2}(\mu^{-2} - 3) + \frac{x^2 K^4}{40\mu^4} \right] \tag{9}$$

where $\mu = m_B/m_\chi$ and $K = (1-4\mu^2)^{1/2}$.

The amplitude of the decay $\chi_1 \to B_{10}\bar{B}_{10}$ can be written in the form

$$A(\chi_1 \to B_{10}\bar{B}_{10}) \sim \chi_\alpha \bar{u}_\mu \gamma_\alpha \gamma_5 \left(g_{\mu\nu} + x\frac{P_\mu P_\nu}{m_\chi^2}\right) u_\nu \qquad (10)$$

After somewhat lengthy calculation we find for β

$$\beta(\chi_1 \to B_{10}\bar{B}_{10}) = K^3(\mu^{-4} - 4\mu^{-2} + 10) + xK^5\mu^{-2}(\mu^{-2} - 2) + \frac{x^2}{4}K^7\mu^{-4} \qquad (11)$$

The notations are the same as in Eq.(9).

The bounds on relative correction factors R(x) are presented in Table 2.

3. The decays $\psi \to VT$ SU_3 symmetry with the experimental fact[1] of the $\psi \to \omega f'$ and $\psi \to \phi f$ decays suppression predict the equality $\gamma(\rho^0 A_2^0) = \gamma(K^{0*}\bar{K}^{0**}) = \gamma(\omega f) = \gamma(\phi f')$.

As is known from the experiment [1]

$$\Gamma(\rho^0 A_2^0) : \Gamma(K^{0*}\bar{K}^{0**}) : \Gamma(\omega f) : \Gamma(\phi f') = (1{,}0 \pm 0{,}5) : (1{,}8 \pm 0{,}5) : (0{,}7 \pm 0{,}3) : (0{,}3 \pm 0{,}2)$$

The production of strange particles being usually suppressed, the enhancement of the K* K** final state seems rather surprising.

To connect γ with the decay width Γ we need the kinematical correction factors β. Unfortunately the calculation is rather tedious because the decay $\psi \to VT$ is described with 5 independent invariant amplitudes:

$$A(\psi \to VT) = a_1(\psi V T) + a_2(\psi V)(PTP) + a_3(PV)(\psi TP) + a_4(K\psi)(VTP) + a_5(K\psi)(PV)(PTP) \qquad (14)$$

where P and K are the four momenta of ψ and V respectively and e.g. $(PTV) \equiv T_{\alpha\beta}P_\alpha V_\beta$.

Terrified with computational difficulties we have not calculated β as a function of all the four parameters a_i/a_1. The following estimates however have been done. First β has been calculated when only one of a_i is nonvanishing. In this case the value e.g. of $R = \beta(K^* K^{**})/\beta(A_2 \rho)$ varies from 0.5 (when $a_1 \neq 0$) to 0.2 (when $a_5 \neq 0$). Such a large deviation from the usual s-wave phase space calculations presented in the second line of Table II is due to the factor $m_T^{-4} m_V^{-2}$ The latter appears in the expression for the decay probability because of the summation on the polarisation states. As noticed L.B.Okun such factors would not appear if the matrix element was transverse:

$$A(\psi \to VT) = a\, \mathcal{F}^{\psi}_{\alpha\beta} \mathcal{F}^{V}_{\mu\nu} R^{T}_{\alpha\beta\mu\nu} \tag{13}$$

where $\mathcal{F}_{\alpha\beta}$ and $R_{\alpha\beta\mu\nu}$ are the usual field strength tensors for vector and tensor fields respectively.

The values of R for such a choice of the amplitude are presented in the fourth line of Table III.

Let us note that for the massive particles is in principle impossible to find out whether an interaction proceeds with conserved current or not because of the evident relation.

$$V_\alpha X_\alpha = m_V^{-2} \mathcal{F}^{V}_{\alpha\beta} k_\beta X_\alpha$$

In the fourth line of Table III the bounds for R(x) are presented if the amplitude has the form

$$A(\psi \to VT) = 2 m_\psi^{-2} \{ [(K\psi)(\varphi T V) + (\varphi V)(\psi T \varphi)](\varphi q) - (\varphi q)^2 (VT\psi) - (K\psi)(\varphi V)(\varphi T \varphi)\} + x\,(\varphi T \varphi)(V\psi) \tag{14}$$

Here $\mathcal{P}$, K and q are the four momenta of ψ, V and T respectively. In Fig.3 R as the functions of x are presented.

The first term in expression (14) describes the transverse interaction of the tensor meson and does not give the factors m_T^{-4}.

As is seen from Table III R varies in rather wide region even for this special choice of the amplitude (eq.(14)). It is evident that for R considered as a function of all the four ratios a_i/a_1 the bounds are wider. However, the maximum values of R(K*K**) and $R(\phi f')$ are reached for about the same values of a_i/a_1. This is why we failed to find such a form of the amplitude that R(K*K**) would be greater than 1 and simultaneously $R(\phi f')$ would be less than 1 in accordance with the central values of the experimental data. Probably with the more accurate measurements an agreement between the experiment and the wide interval of the theoretical predictions will be reached.

4. The decays $\chi \to 2P$ SU_3 symmetry gives the following predictions

$$\gamma(\pi^+\pi^-) = 2\gamma(\pi^0\pi^0) = \gamma(K^+K^-) = \gamma(K^0\bar{K}^0) = 2\gamma(\eta\eta)$$

The decay amplitudes are unambiguous in this case and the factors β can be easily found. For the decay $\chi_0 \to 2P$ β is equal to $k = (1 - 4m_P^2/m_\chi^2)^{1/2}$ and for the decay $\chi_2 \to 2P$ $\beta = k^5$. The decay $\chi_1 \to 2P$ is forbidden by P - conservation. Experiment [2] gives $\Gamma(\pi^+\pi^-) \approx \Gamma(K^+K^-)$ for χ_0.

5. The decays $\chi \to 2V$ SU_3 symmetry with the ideal $(\omega-\phi)$ mixing gives the following results

$$\gamma(\rho^0\rho^0)=\frac{1}{2}\gamma(\rho^+\rho^-)=\frac{1}{2}\gamma(K^*\bar{K}^*)=2\gamma(\phi\phi)-\gamma(\omega\omega)+\frac{1}{2}\gamma(\omega\phi)$$

These decays have not yet been registered.[4] This seems to be rather strange because the decays $\chi\to 2\pi\rho$ and $\chi\to K\pi K^*$ have noticeable probabilities and, say, the 2π-system in the $\chi\to 2\pi\rho$ decay has the same quantum numbers as ρ. A Possible explanation of this fact could be the dominance of the decay $\chi\to\pi\pi'\to 2\pi\rho$ where π' is the radial excitation of π.

Let us note that the absence of the decays $\chi\to 2\rho, 2K^*$ at the level of the decays $\chi\to 2\pi, 2K$ implies a considerable violation of $SU_6(SU_{6W})$ symmetry. The same is true for ψ-meson because the decay $\psi\to\pi\rho$ which experimentally has rather big probability is forbidden in the $SU_6(SU_{6W})$ symmetry limit.

The amplitude of the decay $\chi_1\to 2V$ is written in the form

$$A(\chi_1\to 2V)\sim\varepsilon_{\alpha\beta\gamma\delta}\chi_\alpha V_{1\beta}V_{2\gamma}(K_{1\delta}-K_{2\delta}) \tag{15}$$

and the correction factors β can be found unambiguously: $\beta = K^3$.

Their ratios are presented in Table IV.

The amplitude of the decay $\chi_0\to 2V$ is described with two invariant structures

$$A(\chi_0\to 2V)\sim V_{1\alpha}V_{2\beta}\left(g_{\alpha\beta}+X\frac{P_\alpha P_\beta}{m_\chi^2}\right) \tag{16}$$

This gives fc β

$$\beta = K\left[3+\frac{1}{4}K^2\lambda_1^2\lambda_2^2+\frac{1}{4}XK^2\lambda_1^2\lambda_2^2(1-\lambda_1^2-\lambda_2^2)+\frac{X^2}{16}K^4\lambda_1^2\lambda_2^2\right] \tag{17}$$

where $\lambda_i = m_\chi/m_i$, $m_{1,2}$ are the masses of the vector mesons and $K^2=[1-(\lambda_1^{-1}+\lambda_2^{-1})^2][1-(\lambda_1^{-1}-\lambda_2^{-1})^2]$

The ranges of variation of R's as the functions of x are presented in the second line of Table IV. In the next line the S-wave phase space results are given for comparison.

As for the decay $\chi_2 \to 2V$ it's amplitude is determined with four invariant structures and the calculation of β is rather tedious. We postpone it till the experimental data will appear.

6. The decays $\chi \to PT$ The SU_3 predictions are the following

$$\gamma(\pi A_2) = \gamma(K K^{**}) = 3\gamma(\eta f) = \frac{3}{2}\gamma(\eta f')$$
$$\gamma(\eta' f) = 2\gamma(\eta' f')$$

The decay $\chi_0 \to PT$ is forbidden due to the parity conservation.

The decay $\chi_1 \to PT$ is described with two invariant structures

$$A(\chi_1 \to PT) \sim \chi_\alpha T_{\alpha\beta} \mathcal{P}_\beta + x(\chi_\alpha q_\alpha)(\mathcal{P}_\mu \mathcal{P}_\nu T_{\mu\nu}) m_\chi^{-2} \qquad (18)$$

where $\mathcal{P}$ and q are the four momenta of χ and T and $T_{\alpha\beta}$ is the spin function of the tensor meson.

From expression (18) it follows

$$\beta = K^3 \lambda_T^2 \left[1 + \frac{K^2 \lambda_T^2}{10} + \frac{x}{10} K^2 \lambda_T^2 (1 + \lambda_T^{-2} - \lambda_P^{-2}) + \frac{x^2}{40} \lambda_T^2 K^4\right] \qquad (19)$$

The notations are the same as in Eq. (17)

We find e.g. from eq.(19) that $R(K^*K^{**}) = \beta(K^*K^{**})/\beta(\pi A_2)$ varies in limits 0.45-0.75, $R(\eta f)$ varies in limits 0.9-1.0 and $R(\eta f')$ varies in limits 0.25-0.6. The minimum values of $R's$ are reached at almost the same point, so is true also for the maximum values.

The amplitude of the decay $\chi_2 \to PT$ is written in the form

$$A(\chi_2 \to PT) \sim \varepsilon_{\alpha\beta\gamma\delta} \chi_{\alpha\mu} T_{\beta\nu} P_\gamma q_\delta \left(g_{\mu\nu} + x \frac{q_\mu P_\nu}{m_\chi^2} \right) \qquad (20)$$

and accordingly the corrections factor β is equal to

$$\beta = K^3 \left[1 + \frac{K^2 \lambda_T^2}{20} + \frac{x}{20} K^2 \lambda_T^2 \left(1 + \lambda_T^{-2} - \lambda_P^{-2} \right) + x^2 \frac{K^4 \lambda_T^2}{80} \right] \qquad (21)$$

The notations are the same as in Eq. (17)

For these decays $R = \beta(\chi_2 \to PT) / \beta(\chi_2 \to \pi A_2)$ is bounded for K*K** by (0.55 - 0.85), for ηf by (09 - 0.94) and for $\eta f'$ by (0.38 - 078)

Concluding this section we note that the decays $\chi \to TV$ are forbidden by SU_3 and C-invariance. As for the decays $\chi \to 2T$ we did not consider them because of computational difficulties and because the experimental measurement of these decays is more complicated than, say, of $\chi \to 2V$.

III. Three-body decays

1. The decays $\chi \to 2PV$ The discussion given in the previous section shows that even for the two-body decays SU_3 symmetry gives very uncertain predictions. Of course the three-body decays should be in the worse position. However there exist experimental data on the decays $\chi \to 2PV$ and that is why we will try to clarify the theoretical situation.

First of all we would like to note that the relation

$$\left[\gamma(K^{0*} K^- \pi^+) + c.c. \right] : \gamma(\rho^0 \pi^+ \pi^-) = 4 : 3$$

used in literature [2-4] is erroneous.

The correct SU_3 predictions are the following:

$$(\rho^0\pi^+\pi^-)=1,\quad (\rho^0K^+K^-)=(\rho^0K^0\bar{K}^0)=0{,}25,$$
$$(\rho^+\pi^0\pi^-+c.c.)=2,\quad (\rho^+K^0K^-+c.c.)=1,$$
$$(K^{0*}K^-\pi^++c.c.)=1,\quad (K^{0*}\bar{K}^0\pi^0+c.c.)=0{,}5,$$
$$(K^{0*}\bar{K}^0\eta+c.c.)=1{,}5,\quad (K^{+*}K^-\pi^0+c.c.)=0{,}5, \tag{22}$$
$$(K^{+*}\bar{K}^0\pi^-+c.c.)=1,\quad (K^{+*}K^-\eta+c.c.)=1{,}5,$$
$$(\omega K^+K^-)=(\omega K^0\bar{K}^0)=0{,}25,\quad (\phi K^+K^-)=(\phi K^0\bar{K}^0)=0{,}5.$$

In particular $[(K^{0*}K^-\pi^+)+c.c.]:(\rho^0\pi^+\pi^-)=1:1.$
The ratio 4:3 would be obtained if one assumes the following cascade mechanism: $\chi\to 2V\to 2PV$. But in the decay $V\to 2P$ SU_3 is badly violated because of the mass differences (e.g. $\rho\to 2\pi$ but $\rho\not\to 2K$).

Only decays into $\rho^0\pi^+\pi^-$ and $K^{0*}K^-\pi^+$ are measured on experiment [2,4] (unfortunately the accuracy is not high):

$$\frac{\Gamma(\chi_0\to K^{*0}K^-\pi^++c.c.)}{\Gamma(\chi_0\to\rho^0\pi^+\pi^-)}=1{,}4\pm0{,}8.$$
$$\frac{\Gamma(\chi_1\to K^{0*}K^-\pi^++c.c.)}{\Gamma(\chi_1\to\rho^0\pi^+\pi^-)}=1{,}2\pm1{,}2.$$
$$\frac{\Gamma(\chi_2\to K^{0*}K^-\pi^++c.c.)}{\Gamma(\chi_2\to\rho^0\pi^+\pi^-)}=1{,}1\pm0{,}9.$$

We consider in this subsection only decays into these final states.

The amplitude of the decay $\chi_0\to 2PV$ is equal to
$$A(\chi_0\to 2PV)=f_1(\tau V)+f_2(PV) \tag{23}$$
where V is the spin-vector of the corresponding vector meson, $\tau=p_1-p_2$, $P=p_1+p_2$, and $p_{1,2}$ are the four momenta of the pseudoscalar mesons. We make the simplest assumption that f_1 = const and $f_2=\bar{f}_2(\tau k)m_\chi^{-2}$ where $\bar{f}_2$= const and

K is the vector meson four momentum. Note, that the amplitude should be antisymmetric under the interchange $p_1 \leftrightarrow p_2$.

The relative kinematical correction factor $R =$ $= \beta(K^* K\pi)/\beta(\rho\pi\pi)$ is presented in Fig.2. Taking R into account the theoretical prediction is the following

$$\frac{\Gamma(\chi_0 \to K^{0*} K^- \pi^+ + c.c.)}{\Gamma(\chi_0 \to \rho^0 \pi^+ \pi^-)} = 0{,}32 \div 0{,}73. \qquad (24)$$

The accuracy of the theoretical prediction is not very high but it is rather difficult to increase the upper bound in eq. (24). So if the experimental data were reliable one could conclude that the strange channel in the decay $\chi_0 \to 2PV$ was enchanced as compared to SU_3. Usually strange channels are suppressed.

The consideration of the decay $\chi_1 \to 2PV$ is mooe difficult because its amplitude is described with five invariant structures. Under assumption of constant formfactors only two survive and the amplitude has the form

$$A(\chi_1 \to 2PV) = \varepsilon_{\alpha\beta\gamma\delta}\, \chi_\alpha V_\beta (g_1 \tau_\gamma k_\delta + g_2 \tau_\gamma p_\delta)$$

For this case the correction factor R is changed as function of g_1/g_2 inside the bounds 0,45-0,8.. Taking into consideration another formfactors makes of course these bounds wider but as an estimate they are useful.

The decay $\chi_2 \to 2PV$ is in analogous position. The calculation with the simplest amplitude $\sim \chi_{\mu\nu} V_\mu \tau_\nu$ gives R = 0.6. Taking into account another ferms we can make R roughly speaking two times larger or smaller.

2. The decays $\chi \to 2PT$ SU_3 symmetry relates the probabilities of different decays. The difference between this case and the decay into 2PV is that the charge parity of the tenso· meson is positive and so the two pseudoscalar mesons form a symmetric octet. It is doubtful that these decays will be registered in the nearest time but sooner or later it will be done. So we give the SU_3 Predictions for these decays

$$
\begin{aligned}
&(A^0\pi^0\eta)=1;\ (A^0K^+K^-)=(A^0K^0\bar{K}^0)=0{,}75;\\
&(A^+\pi^-\eta+\text{c.c.})=2;\ (A^+K^-K^0+\text{c.c.})=3;\\
&(K^{**+}K^-\pi^0+\text{c.c.})=1{,}5;\ (K^{+**}K^-\eta+\text{c.c.})=0{,}5;\\
&(K^{**+}\bar{K}^0\pi^-+\text{c.c.})=3;\ (K^{**0}K^-\pi^++\text{c.c.})=3;\ (K^{**0}\bar{K}^0\pi^0+\text{c.c.})=1{,}5;\\
&(K^{**0}\bar{K}^0\eta+\text{c.c.})=0{,}5;\ (f\pi^+\pi^-)=2(f'K^+K^-);\\
&(f'K^+K^-)+(f'\pi^+\pi^-)-(fK^+K^-)=0{,}75.
\end{aligned}
\qquad (25)
$$

These relations as usually should be corrected because of mass differences. The rough estimate of correction factors e.g. for the decay $\chi_0 \to 2PT$ gives

$$
\begin{aligned}
&\beta(f\pi\pi)=1,\ \beta(A_2\eta\pi)=0{,}5,\ \beta(A_2K\bar{K})=0{,}3,\ \beta(K^{**}K\pi)=0{,}35,\\
&\beta(K^{**}K\eta)=0{,}15,\ \beta(fK\bar{K})=0{,}35,\ \beta(f'\pi\pi)=0{,}35,\ \beta(f'K\bar{K})=0{,}1.
\end{aligned}
$$

These numbers were obtained with the simplest form of the dec y amplitude and their uncertainty is rather big. If experimer al data on these decays appear, more detailed calculation wi.l be needed to make the comparison with SU_3.

3. The decays $\chi \to 3P$ SU_3 gives the following relations

$$
\begin{aligned}
&(\pi^0K^+K^-)=(\pi^0K^0\bar{K}^0)=1{,}5;\ (2\pi^0\eta)=1;\ (\pi^+\pi^-\eta)=2;\\
&(3\eta)=1/3;\ (\pi^+K^-K^0)+\text{c.c.}=3;\ (K^+K^-\eta)=(K^0\bar{K}^0\eta)=0{,}5.
\end{aligned}
$$

It seems that an experimental registration of these decays will be rather difficult.

We hoped that the corrections due to mass differences in this case would not be so essential as in the former cases. However, the matrix elements of these decays depends essentially on the four momenta of the pseudoscalar mesons and this leads to rather large mass corrections.

The decay $\chi_0 \to 3P$ is forbidden by parity conservation.

The amplitude of the decay $\chi_1 \to 3P$ has the form

$$A(\chi_1 \to 3P) = \chi_\mu \left[p_{1\mu} f(p_1; p_2, p_3) + \text{cyclic permutation} \right] \tag{26}$$

where $f(p_1; p_2, p_3) = f(p_1; p_3, p_2)$ and the power series expansion of f starts from the quadratic on momenta terms. We take into account the first term only:

$f(p_1; p_2, p_3) = f_0 (\mathcal{P} p_1)$ and obtain:

$$\beta(\chi_1 \to 3P) \approx \sum_{i=1,2,3} B_i^4 \int_0^1 dt\, t(1-t)^{3/2}(b_i - t)^{3/2} \left[\frac{B_i^2 t^2}{6 m_\chi^4} - \right.$$
$$\left. - \frac{1}{6} + \frac{B_i^2}{4 m_\chi^4}(1-t)(b_i - t) \right] \tag{27}$$

where $B_1 = (m_\chi - m_1)^2 - (m_2 + m_3)^2$ and $b_1 = \left[(m_\chi + m_1)^2 - (m_2 - m_3)^2 \right] / B_1$ and so on. Here we have neglected some terms $\sim m_i^2$.

The differences between β's corresponding to the different decays are mainly due to variation of B_i.

The amplitude of the decay $\chi_2 \to 3P$ can be written as

$$A(\chi_2 \to 3P) = \varepsilon_{\alpha\beta\mu\nu} \chi_{\alpha\delta} p_{1\beta} p_{2\mu} p_{3\nu} \{ p_{1\delta}\, g(p_1, p_2, p_3) +$$
$$+ \text{cyclic permutation}. \tag{28}$$

where $g(p_1, p_2, p_3) = - g(p_1, p_3, p_2)$. Taking into account only the first term in power series expansion,

$$g(p_1,p_2,p_3) = g_0\cdot(\mathcal{P}, p_2-p_3),$$ we obtain

$$\beta = \sum_{i=1,2,3} B_i^7 \int_0^1 dt\, \mathcal{F}_i^{5/2}(t)(t+\varepsilon_i)\left[1 - \frac{2B_i}{m_\chi^2}(t+\varepsilon_i)\right] \qquad (29)$$

where $\mathcal{F}_i = t(1-t)(\delta_i - t)(\gamma_i + t)(\varepsilon_i + t)^{-2}$, $\varepsilon_1 = (m_2+m_3)^2/B_1$, $\gamma_1 = 4m_2m_3/B_1$ and so on and the other notations are the same as in eq (26).

It follows from eqs (26) and (29) that the mass dependence is essential and should be taken into account when the SU_3 relations are tested.

4. The decays $\psi\to 2PV$ The SU_3 predictions for these decays can be obtained from eqs. (25), inserting the vector meson instead of the corresponding tensor one. The following results seem to be interesting

$$\begin{gathered} 4\gamma(\rho^0K^+K^-) = \gamma(K^{0*}K^-\pi^+ + c.c.), \\ \gamma(\omega\pi^+\pi^-) = 2\gamma(\phi K^+K^-), \\ \gamma(\phi K^+K^-) + \gamma(\phi\pi^+\pi^-) - \gamma(\omega K^+K^-) = \gamma(\rho^0K^+K^-). \end{gathered} \qquad (30)$$

Under assumption of the constant form factors the amplitude of the $\psi\to 2PV$ decay can be written as follows

$$\mathcal{A}(\psi\to 2PV) \sim \psi_\mu V_\nu\left[g_{\mu\nu} + x\frac{q_\mu q_\nu}{m_\psi^2} + y\frac{\tau_\mu\tau_\nu}{m_\psi^2}\right] \qquad (31)$$

where $q = p_1+p_2$, $\tau = p_1 - p_2$ and $p_{1,2}$ are the four momenta of the pseudoscalar mesons.

To find the mass corrections we need to perform the space integration of $|\mathcal{A}(\psi\to 2PV)|^2$. The corresponding expression is rather complicated and we will not write it down.

The calculations performed show that for the decays $\rho K\bar{K}$ and $K^*K\pi$ the correction factor $R = \beta(K^*K\pi)/\beta(\rho K\bar{K})$ is nearly constant independently of x and y and is bounded between 1 and 1.2. This probably does not depend on the concrete form of the amplitudes of the decays and is due to fact that the values of $(m_\psi - m_V)^2 - (m_1 + m_2)^2$ are the same for both decays. The study of these decays is therefore very convenient to check the SU_3 prediction (the first of eqs. (30)). As for the second of eqs. (30) the ratio $\beta(\phi K\bar{K})/\beta(\omega\pi\pi)$ can have any value between 0.05-0.45 and this is the accuracy of the theoretical prediction. The experiment[3] gives $2\Gamma(\phi K^+K^-)/\Gamma(\omega\pi^+\pi^-) = 0.25 \pm 0.15$. The last of eqs (30) is rather difficult to compare with experiment because of the large ambiguity of each term due to the correction factor.

We have not considered the decays $\psi \to 3\rho$ because the experiment seems to show that this channel is mainly saturated by the resonances $\psi \to V\rho$.

IV. Conclusion

We see that SU_3 gives more or less definite predictions if the matrix element of a decay is determined with a single invarriant amplitude, as e.g. $\psi \to VP$ or $\chi_1 \to 2V$. If several invariant amplitudes contribute to the matrix element the theoretical predictions are very uncertain depending upon the relative values of these amplitudes (for example the correction factors for the decays $\psi \to VT$ can vary from 0.2 to 1.4). To exclude the ambiguity not only the total width but some additional measurements are needed. For example to compare SU_3 predictions with experiment for the decays $\psi \to B_8\bar{B}_8$ one should know the formfactors f_1 and f_2 separately. This can be done through decay angular distribution measurement.

It is interesting that for the decays $\psi \to \rho^0 K^+ K^-$ and $\psi \to K^{0*} K^- \pi^+$ the relative correction factor is close to unity and so these decays are good candidates to check SU_3 prediction (30).

It is rather surprising that the strange channels of the decays $\psi \to VT$ and $\chi \to 2PV$ seem to be enhanced as compared to the nonstrange ones. Of course the accuracy of the experimental data is not sufficient enough but the tendency is rather clear. For the decay $\psi \to VT$ this enhancement can arise from the heavier masses of the strange particles. As is shown in Sec. II. 3 this can act in the opposite direction to that of the simple phase space corrections. For the decays $\chi \to 2PV$ however we failed to find such"kinematical" enhancement. A Possible reason for larger strange particle emission in the decays $\chi \to 2PV$ could be slower decrease of the formfactors for the decays into strange particles. More interesting however seems the possibility that charmed quarks prefer to interact with strange ones. If it were true the cross section of charmed particle production would be somewhat higher in the kaon beam than that in the pion beam. It should be noted however that this is very speculative and in the decay $\psi \to PV$ the strange particle emission is suppressed.

We would like to thank L.B.Okun without whose stimulating influence this paper never would be written.

Table Captions

Table I. The relative kinematical correction factors $R = \beta(B\bar{B})/\beta(N\bar{N})$ for the decays $\chi_1 \psi \to B_8 \bar{B}_8$. In the upper lines for χ_2 and ψ the possible intervals of variation of $R(x)$ are presented. For the ψ-decays R is defined by eqs. (4) and for that of χ_2 R is defined by eq.(6). In the lower lines the usual correction factors $K^{2\ell+1}(B\bar{B})/K^{2\ell+1}(N\bar{N})$ are presented.

Table II. The relative kinematical correction factors $R = \beta(B_{10}\bar{B}_{10})/\beta(\Delta\bar{\Delta})$ for the decays $\chi_{0,1} \to B_{10}\bar{B}_{10}$ In the upper lines the intervals of variation of R(x) are presented. These intervals are defined by eqs (9) and (11) for χ_0 and χ_1 decays respectively.

Table III. The kinematical correction factors R for the decays $\psi \to VT$ for different choices of the decay amplitude.

Table IV. The kinematical correction factors R for the decays $\chi_{0,1} \to 2V$. In the upper line for χ_0 the intervals of variation of R(x) accordingly to eq. (17) are presented and in the lower line the usual s-wave phase space correction factors are given.

Table I

$B_8 \bar{B}_8$		$\Lambda\bar{\Lambda}$	$\Sigma\bar{\Sigma}$	$\Xi\bar{\Xi}$
χ_0		0,74	0,63	0,45
χ_1		0,77	0,66	0,48
χ_2	Eq(6)	0.5-0.82	0.43-075	0.15-0.55
	K^3	0.77	0.67	0.49
ψ	Eq(4)	0.44-1.02	0.29-0.98	0.08-0.95
	K	0,87	0,80	0,67

Table II

$B_{10} \bar{B}_{10}$				
χ_0	Eq.(9)	0.2-0.62	0.03-0.28	5.10^{-5}-$2.7.10^{-2}$
	K^3	0.6	0.26	0.025
χ_1	Eq.(11)	0.2-0.75	0.04-0.43	0.001 - 0.12
	K^3	0.64	0.32	0.08

Table III

$\psi \to VT$	$K^* K^{**}$	ωf	$\phi f'$
$K^{1/2}$	0.9	1.01	0.78
$T_{\mu\nu}\psi_\mu V_\nu$	0.5	1.08	0.28
$F^{\psi}_{\alpha\beta} F^{V}_{\mu\nu} R_{\alpha\beta\mu\nu}$	0.77	1.0	0.51
See Eq.(14)	0.3-1.4	0.9-1.4	0.09-1.6

Table IV

$\chi \to 2V$		2ω	$2K^*$	2ϕ	$\omega\phi$
χ_0	Eq.(17)	I	0.41-1.0	0.17-1.02	0.41-1.0
	κ	I	0.95	0.9	0.95
χ_1	κ^5	I	0.60	0.35	0.60

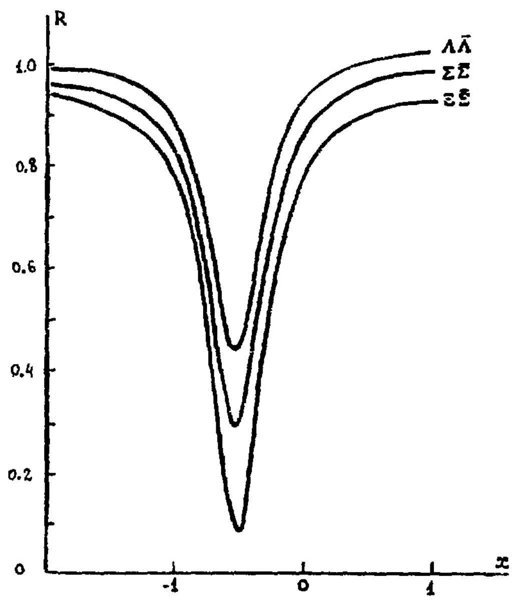


Fig.1.

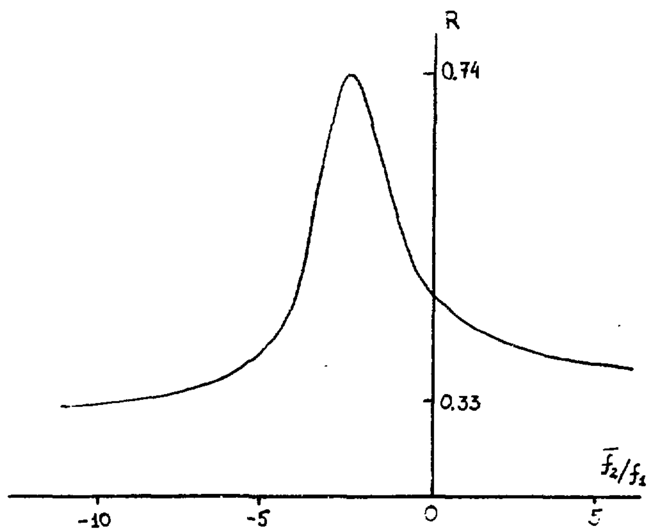
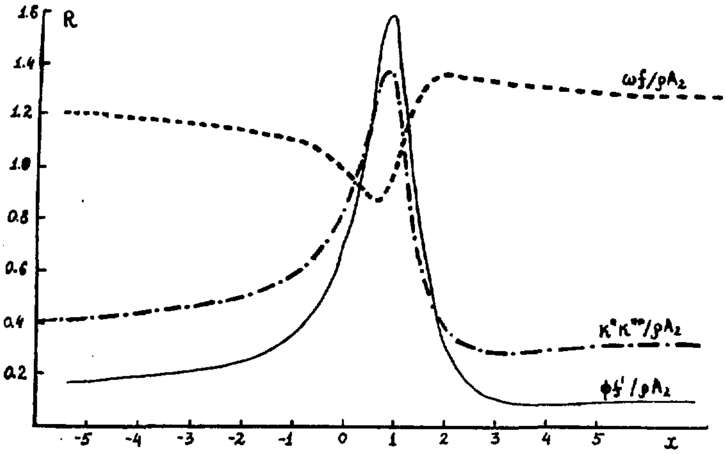


Fig.2



R
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$\omega f/\rho A_2$
$K^{*}K^{**}/\rho A_2$
$\phi f'/\rho A_2$
-5
-4
-3
-2
-1
0
1
2
3
4
5
x

Fig.3

Figure Captions

Fig. 1. The behaviour of R's as functions of x for the decays $\psi \to B_8 \bar{B}_8$ as is given by eq (4).

Fig. 2. The behaviour of R as the function of $\bar{f}_2/f_1$. (see eq (23)) for the decays $\chi_0 \to K^* K\pi$ and $\chi_0 \to \rho\pi\pi$

Fig. 3. The behaviour of $R(X)$ for the decays $\psi \to VT$.

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The manuscript received 20-th May 1977

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ДАЁТ ЛИ SU_3-СИММЕТРИЯ ПРЕДСКАЗАНИЯ ДЛЯ РАСПАДОВ НОВЫХ МЕЗОНОВ?

(на английском языке)
Ереванский физический институт

Тех.редактор А.С.Абрамян

Заказ 1059 ВФ- 03327 Тираж 299

Подписано к печати 26/УП-77г. Формат издания 30х40
1.8 уч.изд.л. Ц.12 к.

Издано Отделом научно-технической информации Ереванского физического института, Ереван-36, пер. Маркаряна 2